Web appendix for Trade Spill-overs of Fiscal Policy in the European Union: A Panel Analysis

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Contents

Part A. Data sources and procedures

Data sources are the Economic Outlook (EO) of the OECD Statistical Compendium; the International Financial Statistics (IFS) of the International Monetary Fund (IMF) Database; and the Direction of Trade Statistics (DOTS).

<u>Fiscal variables</u>

The EO provides time series at *annual* frequency for the following variables:

CGAA	=	Government Consumption
IGAA	=	Fixed Investment, Government
PCG	=	Deflator, Public Consumption (base year 1995 =100)
PIG	=	Deflator, Government Fixed Investment (base year 1995 =100)
TIND	=	Indirect Taxes
TSUB	=	Subsidies
TY	=	Direct Taxes
SSPG	=	Social Benefits Paid by Government
TRPG	=	Other Current Transfers Paid by Government
SSRG	=	Social Security Contributions Received by Government
TRRG	=	Other Current Transfers Received by Government

<u>Additional variables</u>

GDP	=	Gross Domestic Product (Market Prices), Value
PGDP	=	Deflator for *GDP* at Market Prices (base year 1995 =100)
IRS	=	Short term interest rate

From the above series, we construct the following variables:

Y	=	Real GDP = *GDP**100/*PGDP*
G	=	Real Public Spending = *CGAA**100/*PCG* + *IGAA**100/*PIG*
PY	=	Real Private GDP = *Y* – *G*
REVENUES	=	*TY* + *TIND* + *SSRG* + *TRRG*
TRANFERS	=	*TSUB* + *SSPG* + *TRPG*

NT = Real Net Taxes = (*REVENUES* – *TRANFERS*)*100/*PGDP*

Note that due to short data availability, for Ireland and the Netherlands *TRPG* and *TRRG* are not included in the calculation of *REVENUES* and *TRANFERS*.

In order to cyclically adjust net taxes, we follow Alesina *et al.* (2002) and for each component of revenues and transfers at time t we compute:

$$R_{it}^{CA} = R_{it}^{NCA}(Y_t^{TR}/Y_t)^{\xi_i},$$

where superscripts *CA*, *NCA* and *TR* denote, respectively, “cyclically adjusted”, “non cyclically adjusted” and “trend”, and ξ is the elasticity of component i with respect to real output. Elasticities are provided by Van den Noord (2000) and the OECD (2005). However, the OECD does not provide the transfers elasticity. Therefore, as in Alesina *et al.* (2002), we use the total primary expenditure elasticity and scale it up by the ratio of transfers to total primary spending. Additionally, we calculate trend GDP separately for each country by regressing log real GDP on a constant and a linear and a quadratic time trend.

Trade variables

The real bilateral export flows X_{ji} from country j to country i in a given year are taken from Bun and Klaassen (2003) updated with the years 2003 and 2004. They are constructed as the sum of the monthly real exports, where the latter is the nominal value of exports in exporter’s currency divided by the exporter’s price index. The nominal value of exports in exporter’s currency is obtained by converting the original dollar denominated export values of the DOTS. The real bilateral exchange rate RER_{ji} is the average of the monthly real rates computed using nominal rates and the same exporter’s price indices as used above. The real effective exchange rate *rer* (used in the sensitivity analysis for the fiscal block) is the weighted average of the log of RER_{ji} in index form, using export shares as weights. The trade integration dummies $EU_{ji,t}$ and $FTA_{ji,t}$ are based on the dating of the membership of the EU or a free trade agreement used in Bun and Klaassen (2003).

Variables used in the panel estimation

p	=	$\log(PGDP)$
y	=	$\log(Y)$
py	=	$\log(PY)$
i	=	IRS
rer	=	$\log(RER)$
g	=	$\log(G)$
nt^{CA}	=	$\log(NT^{CA})$
x_{ji}	=	$\log(X_{ji})$
rer_{ji}	=	$\log(RER_{ji})$

Country and data samples:

The "fiscal block" is estimated for 14 EU countries: Austria, Belgium, Denmark, Finland, France, Germany, Greece, Ireland, Italy, Netherlands, Portugal, Spain, Sweden and the United Kingdom. The estimation sample is 1965-2004. The only exceptions are Belgium (1970), Denmark (1971), Ireland (1977), the Netherlands (1969) and Portugal (1977). The "trade block" is estimated over the same period and accounts for all the bilateral trade relationships between the 14 countries above, leading to 182 country pairs. The trade variables for Belgium also include trade flows of Luxembourg.

References:

Bun, M.J.G. and F.J.G.M. Klaassen (2007). 'The Euro Effect on Trade is not as Large as Commonly Thought', *Oxford Bulletin of Economics and Statistics*, forthcoming.

OECD (2005). *Economic Outlook*, June, Paris.

Van den Noord, P. (2000). 'The Size and Role of Automatic Fiscal Stabilizers in the 1990s and Beyond', *OECD Working Paper*, No.230.

Part B.1. Model specification for the PVAR

The PVAR specification in its structural form is:

$$A_0 Z_{it} = A(L) Z_{it-1} + e_{it},$$

where Z_{it} is the $(m \times 1)$ vector of endogenous variables and A_0 is an $(m \times m)$ matrix with 1's on the diagonal. It contains the structural parameters that capture the contemporaneous relations among the endogenous variables. Further, e_{it} is the vector with the structural shocks. For the baseline model, $Z_{it} = \left[g_{it}, nt_{it}^{CA}, y_{it} \right]'$ and $e_{it} = \left[e_{it}^{g}, e_{it}^{nt,CA}, e_{it}^{y} \right]'$. By pre-multiplying the above equation by A_0^{-1}, we obtain its reduced form:

$$Z_{it} = B(L) Z_{it-1} + u_{it},$$

where $B(L) = A_0^{-1} A(L)$ and $u_{it} = A_0^{-1} e_{it}$ is the reduced-form residual vector. We can write out $A_0 u_{it} = e_{it}$ as:

$$\begin{pmatrix} 1 & -\alpha_{gt} & -\alpha_{gy} \\ -\alpha_{tg} & 1 & -\alpha_{ty} \\ -\alpha_{yg} & -\alpha_{yt} & 1 \end{pmatrix} \begin{bmatrix} u_{it}^{g} \\ u_{it}^{nt,CA} \\ u_{it}^{y} \end{bmatrix} = \begin{bmatrix} e_{it}^{g} \\ e_{it}^{nt,CA} \\ e_{it}^{y} \end{bmatrix},$$

where u_{it}^{g}, $u_{it}^{nt,CA}$ and u_{it}^{y} are the reduced-form residuals. The three identifying restrictions thus amount to imposing $\alpha_{gt} = \alpha_{gy} = \alpha_{ty} = 0$.

Part B.2. Combination of the responses for the two blocks

We take the extended fiscal block, which provides the most general case. For this block, we can write the impulse response functions for domestic output and the real effective exchange rate as:

$$\Delta y_{it} = \Psi(L) e_{it}^{f}, \; reer_{it} = \Phi(L) e_{it}^{f},$$

where $\Psi(L)$ and $\Phi(L)$ are lag polynomials, which both depend on the type of fiscal shock, and e_{it}^{f} is the discretionary fiscal shock ($e_{it}^{f} = e_{it}^{g}$ or $e_{it}^{f} = e_{it}^{nt,CA}$). The coefficients of $\Psi(L)$ and $\Phi(L)$ are functions of the parameters from the fiscal block. They show how a fiscal shock affects the two variables over time.

The trade block provides the link between domestic output and bilateral exports of the foreign country, where the response function is the distributed-lag function:

$$\Delta x_{ji,t} = D(L) \Delta y_{it} + C(L) \Delta rer_{jit},$$

where the coefficients in the lag polynomials $D(L)$ and $C(L)$ are functions of the parameters of the trade model.

Combining the previous expressions and assuming $\partial rer_{jit} / \partial reer_{it} = -1$,[1] we calculate the effects of the discretionary fiscal shock in country *i* on bilateral exports from country *j* to country *i* as:

$$\Delta x_{ji,t} = \left[D(L) \Psi(L) - C(L) \Phi(L) \right] e_{it}^{f}.$$

[1] This is the case when nominal exchange rate movements against third countries resulting from a domestic (policy) shock are roughly equal in percentage terms and domestic shocks have no or identical effects on third country inflation rates. In addition, the numbers we report are based on the estimated "average" response in the panel of exports to the bilateral real exchange rate, which may be close to the response of exports to a country to its real effective exchange rate.

A change in exports translates into a one-for-one direct change in country *j*'s GDP, Y_{jt}. We can write $Y_{jt} = Y_{jt}^D + X_{jt} = Y_{jt}^D + \sum_i X_{jit}$, where Y_{jt}^D is the component of *j*'s GDP that goes to domestic end users, X_{jit} is exports from country *j* to country *i* and X_{jt} is total exports by country *j*. Linearisation yields:

$$\Delta \log Y_{jt} = \left(Y_{jt}^D / Y_{jt} \right) \Delta \log Y_{jt}^D + \sum_i \left(X_{jit} / Y_{jt} \right) \Delta \log X_{jit}.$$

As before, using the small letters for the logs and using that the only source of change of Y_{jt} is the increase in exports from *j* to *i*, this expression reduces to:

$$\Delta y_{jt} = \left(X_{jit} / Y_{jt} \right) \Delta x_{jit} = \left(X_{jit} / X_{jt} \right) \left(X_{jt} / Y_{jt} \right) \Delta x_{jit}.$$

Substituting the expression for Δx_{jit} we end up with:

$$\Delta y_{jt} = \frac{X_{jit}}{X_{jt}} \frac{X_{jt}}{Y_{jt}} \left[D(L) \Psi(L) - C(L) \Phi(L) \right] e_{it}^f.$$

Of course, when the exchange rate channel is closed, as in our baseline case, then $C(L)\Phi(L) = 0$.

Part B.3. A budget squeeze

We want to compute the percent government spending reduction or net tax revenue increase that reduces the government deficit by 1% of GDP. We abstract from changes in interest payments on the public debt. These are likely to be of an order of magnitude smaller than the changes in spending or net taxes.[2] The percentage-point change in the government deficit to GDP ratio is then given by the percentage-point change in $\left(G_t - NT\right)_t / Y_t$, which is in turn approximated by

$$\begin{aligned}&\left(G_t / Y_t\right)\left[\hat{G}_t - \hat{Y}_t\right] - \left(NT_t^{NCA} / Y_t\right)\left[\hat{N}T_t^{NCA} - \hat{Y}_t\right] = \\ &\left(G_t / Y_t\right)\left[\hat{G}_t - \hat{Y}_t\right] - \left(NT_t^{NCA} / Y_t\right)\left[\left(\hat{N}T_t^{NCA} + \xi \hat{Y}_t\right) - \hat{Y}_t\right]\end{aligned}$$

where a hat denotes the percent deviation from the initial value. The move from the first to the second line makes use of the relation between cyclically and non-cyclically adjusted variables presented in Appendix A. With a discretionary government spending shock, $\hat{G}_t$ is equal to the shock and $\hat{Y}_t$ and $\hat{N}T_t^{CA}$ are the impulse responses that we computed earlier. With a discretionary net tax shock, $\hat{N}T_t^{CA}$ is equal to the shock and $\hat{Y}_t$ and $\hat{G}_t$ are the impulse responses that we computed earlier. To compute the instrument settings in Subsection 6.1 for a deficit reduction of 1% of GDP, we equate the second line of the above expression to –1, substitute into this line for G_t / Y_t and NT_t^{CA} / Y_t the averages of the ratios of government spending and cyclically-adjusted net taxes to GDP over all observations (0.241 and 0.217, respectively), and for ξ the average elasticity of net taxes to deviations of income from trend (2.10).[3] We also substitute a one-percent of

[2] A one-percent of GDP deficit reduction leads to a one-percent of GDP debt reduction at the end of the period in which the fiscal contraction was implemented. On average during the period, the debt-to-GDP ratio would be roughly lower by half a percent of GDP, an amount that needs to be multiplied by the annual interest rate to calculate its effect on the spending reduction or net tax increase.

[3] Preferably, when computing the effects of a German fiscal contraction, we would use the (latest) values of the corresponding German variables. However, the impulse responses that we feed into the computation are based on the estimates of the fiscal block on all observations and, for consistency, we thus stick with the numbers chosen here. This inevitably leads to inaccuracies, but the outcomes are primarily intended to give a sense of their order of magnitude.

GDP change in the policy instrument as well as the corresponding impulse responses in the other two variables, all expressed in percent of the variables themselves and all multiplied by the same scaling factor. We can then solve for the scaling factor and, thereby, for the instrument change and the changes in the other variables.

1 Part B.4. Consequences of Ignoring Spill-overs

The authorities aim at setting the policy instruments to affect the output. We compare the case where the policymakers do not take into account the spill-overs (and thus do not reach their target) with the case where they take proper account of the spill-overs (and thus do reach their output target).

Consider first our structural VAR:

$$A_0 Z_{it} = A(L) Z_{i,t-1} + e_{it},$$

where

$$Z_{it} = \begin{bmatrix} g_{it} \\ nt_{it}^{CA} \\ y_{it} \end{bmatrix}, \quad A_0 = \begin{bmatrix} 1 & 0 & 0 \\ -\alpha_{tg} & 1 & 0 \\ -\alpha_{yg} & -\alpha_{yt} & 1 \end{bmatrix}, \quad e_{it} = \begin{bmatrix} e_{it}^{g} \\ e_{it}^{nt,CA} \\ e_{it}^{y} \end{bmatrix}.$$

Hence,

$$Z_{it} = A_0^{-1} A(L) Z_{i,t-1} + A_0^{-1} e_{it},$$

where

$$A_0^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ \alpha_{tg} & 1 & 0 \\ \alpha_{tg}\alpha_{yt} + \alpha_{yg} & \alpha_{yt} & 1 \end{bmatrix},$$

With the use of f as the instrument (where $f = g$ or $f = nt^{CA}$), and applying a standard linearisation (see Appendix B.2.), we have that:

$$\Delta y_F = \beta_{yf} e_F^f + (X_{FG}/Y_F)\, \Delta x_{FG}, \tag{1}$$

$$\Delta y_G = \beta_{yf} e_G^f + (X_{GF}/Y_G)\, \Delta x_{GF}, \tag{2}$$

where $\beta_{yg} = (\alpha_{tg}\alpha_{yt} + \alpha_{yg})$ and $\beta_{y,nt^{CA}} = \alpha_{yt}$. Further, Δy_F (Δy_G) is the change in the natural log of French (German) GDP, x_{FG} (x_{GF}) is the natural log of exports from France (Germany) to Germany (France), X_{FG} (X_{GF}) is the level of French (German) exports to Germany (France) and Y_F (Y_G) is the level of French (German) output. Hence, X_{FG}/Y_F (X_{GF}/Y_G) is French (German) exports to Germany (France) as a share of French (German) GDP. Notice that an increase in exports adds one-for-one in an increase in output and that we ignore potential mutiplier effects associated with exports.

By export equation (2) in the main text, we have that:

$$\Delta x_{FG} = \beta_{20} \Delta y_G, \quad \Delta x_{GF} = \beta_{20} \Delta y_F, \tag{3}$$

where $\beta_{20} = 1.747$ is found in Table 2, column 1. Hence, we get

$$\Delta y_F = \beta_{yf} e_F^f + (X_{FG}/Y_F)\,\beta_{20} \Delta y_G, \tag{4}$$

$$\Delta y_G = \beta_{yf} e_G^f + (X_{GF}/Y_G)\,\beta_{20} \Delta y_F. \tag{5}$$

For further use, we provide some preliminary computations. Substituting (5) into (4), we obtain

$$\Delta y_F = \beta_{yf} e_F^f + (X_{FG}/Y_F)\,\beta_{20} \left[\beta_{yf} e_G^f + (X_{GF}/Y_G)\,\beta_{20} \Delta y_F\right],$$

which is rewritten as:

$$\Delta y_F = \beta_{yf} e_F^f + \beta_{yf}\,(X_{FG}/Y_F)\,\beta_{20} e_G^f + (X_{FG}/Y_F)\,(X_{GF}/Y_G)\,\beta_{20}^2 \Delta y_F,$$

or

$$\Delta y_F = \frac{\beta_{yf} e_F^f + \beta_{yf}\,(X_{FG}/Y_F)\,\beta_{20} e_G^f}{1 - (X_{FG}/Y_F)\,(X_{GF}/Y_G)\,\beta_{20}^2}. \tag{6}$$

Also

$$\Delta y_G = \frac{\beta_{yf} e_G^f + \beta_{yf}\,(X_{GF}/Y_G)\,\beta_{20} e_F^f}{1 - (X_{FG}/Y_F)\,(X_{GF}/Y_G)\,\beta_{20}^2}. \tag{7}$$

We consider now two scenarios: (1) both countries set their policy instruments with the intention of stimulating their economy by 1, and (2) France aims at stimulating its own output by 1, whereas Germany sets its instrument to reduce output by 1. Under both scenarios we compare the setting of the policy instruments and the realized output effects for the case where spill-overs are ignored with when the spill-overs are fully taken into account.

Scenario 1: Both countries implement a fiscal expansion

Spillover properly taken into account

When the spillover is properly taken into account, we have

$$\begin{aligned}\Delta y_F &= \beta_{yf} e_F^f + (X_{FG}/Y_F)\,\beta_{20}\Delta y_G, \\ \Delta y_G &= \beta_{yf} e_G^f + (X_{GF}/Y_G)\,\beta_{20}\Delta y_F.\end{aligned}$$

Substituting the second equation into the first one and imposing that $\Delta y_F = 1$ and $\Delta y_G = 1$:

$$e_F^f = \frac{1-(X_{FG}/Y_F)\,\beta_{20}}{\beta_{yf}},$$

or

$$e_G^f = \frac{1-(X_{GF}/Y_G)\,\beta_{20}}{\beta_{yf}}.$$

Spillover completely ignored

Assume now that both countries "aim" at increasing output by 1, but ignore the spillovers. Then, we have:

$$\begin{aligned}1 &= \beta_{yf} e_F^f \\ 1 &= \beta_{yf} e_G^f\end{aligned}$$

Hence,

$$e_F^f = \frac{1}{\beta_{yf}}, \quad e_G^f = \frac{1}{\beta_{yf}}.$$

Substituting these expressions into (6) and (7), we obtain the actual outcomes for output, when the spill-over is mistakenly ignored. That is:

$$\Delta y_F = \frac{1+(X_{FG}/Y_F)\,\beta_{20}}{1-(X_{FG}/Y_F)\,(X_{GF}/Y_G)\,\beta_{20}^2}.$$

$$\Delta y_G = \frac{1+(X_{GF}/Y_G)\beta_{20}}{1-(X_{FG}/Y_F)\,(X_{GF}/Y_G)\,\beta_{20}^2}.$$

Scenario 2: Germany implements a fiscal contraction, whereas France a fiscal expansion

Spillover properly taken into account

When the spillover is properly taken into account, we have

$$\begin{aligned}\Delta y_F &= \beta_{yf} e_F^f + (X_{FG}/Y_F)\,\beta_{20}\Delta y_G, \\ \Delta y_G &= \beta_{yf} e_G^f + (X_{GF}/Y_G)\,\beta_{20}\Delta y_F.\end{aligned}$$

Substituting the second equation into the first one and imposing that $\Delta y_F = 1$ and $\Delta y_G = -1$:

$$e_F^f = \frac{1 + (X_{FG}/Y_F)\,\beta_{20}}{\beta_{yf}},$$

or

$$e_G^f = -\frac{1 + (X_{GF}/Y_G)\,\beta_{20}}{\beta_{yf}}.$$

Spillover completely ignored

Assume now that both countries act without taking the spillovers into account. Then we have that:

$$\begin{aligned}1 &= \beta_{yf} e_F^f \\ -1 &= \beta_{yf} e_G^f\end{aligned}$$

Hence,

$$e_F^f = \frac{1}{\beta_{yf}}, \quad e_G^f = -\frac{1}{\beta_{yf}}.$$

Substituting these expressions into (6) into (7), we obtain the actual outcomes for output, when the spill-over is mistakenly ignored. That is:

$$\Delta y_F = \frac{1 - (X_{FG}/Y_F)\,\beta_{20}}{1 - (X_{FG}/Y_F)\,(X_{GF}/Y_G)\,\beta_{20}^2}.$$

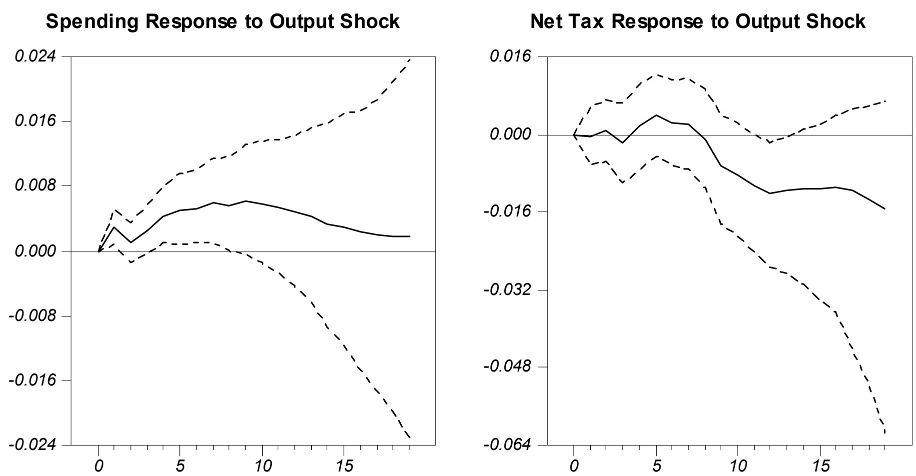
$$\Delta y_G = \frac{(X_{GF}/Y_G)\beta_{20} - 1}{1 - (X_{FG}/Y_F)\,(X_{GF}/Y_G)\,\beta_{20}^2}.$$

Appendix C: Additional empirical results

Appendix C.1. Construction of annual elasticity of G to Y based on quarterly VAR and re-estimation of annual model.

Quarterly 3-variable VAR for Germany

Given that we have non-interpolated quarterly data on net taxes only for Germany, we estimate the baseline three-variable VAR for Germany using pre-unification quarterly data. To cyclically adjust net taxes, we use the elasticity ($\alpha_{ty} = 0.90$) found by Perotti (2005).



There is no reaction of net taxes to GDP shocks. As for government spending, the reaction after one quarter is significant. However, then it becomes insignificant again, while the third-quarter reaction is on the border of significance.

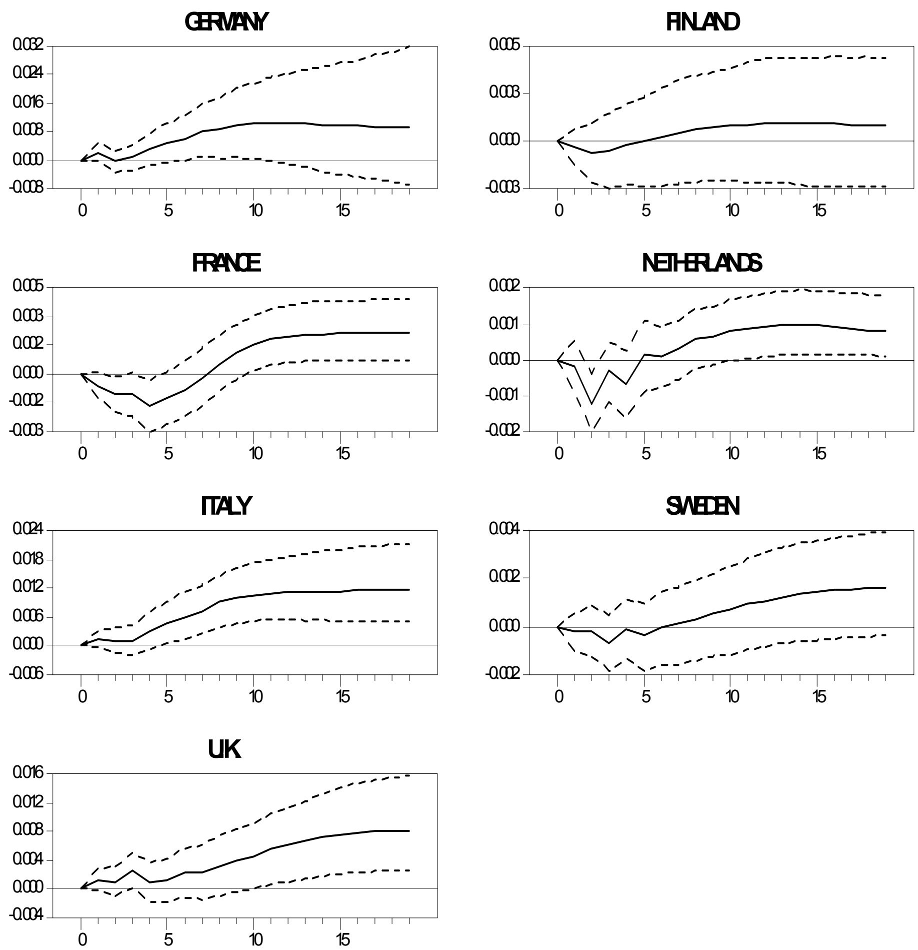
We use the procedure described in the Appendix D to transform quarterly estimates into the annual contemporaneous response parameters α_{gy}, α_{gt} and α_{ty} with 90% confidence intervals of, respectively, (-0.22, 0.69), (-0.13, 0.23) and (-3.07, 3.76).

Quarterly PVAR in government spending and output

Based on visual inspection of the post-1970 time series of government consumption and government investment provided by different issues of the OECD Economic

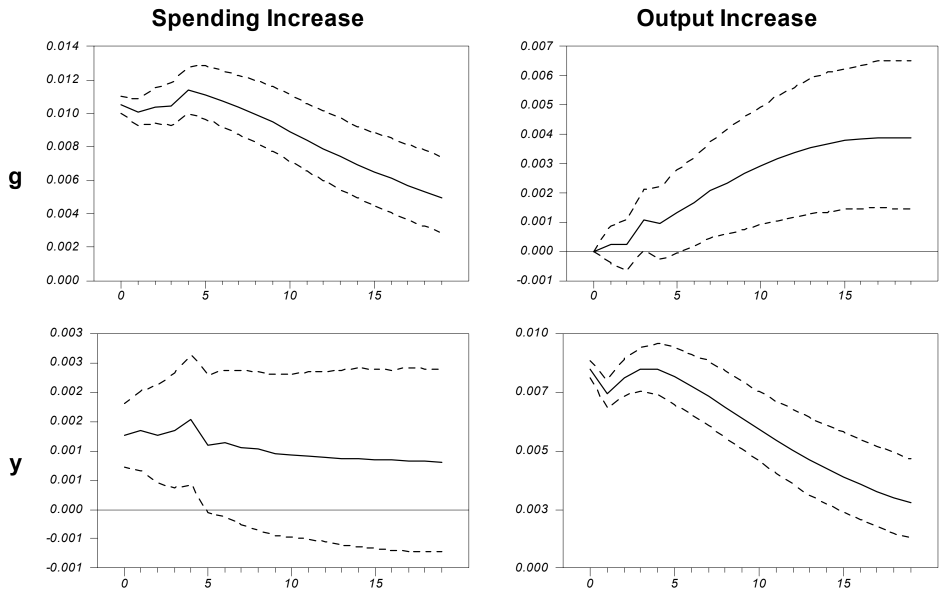
Outlook, Germany, France, Italy, Great Britain, the Netherlands, Sweden and Finland appear to produce a true quarterly data frequency for these two components. We set up a quarterly bivariate VAR model in public spending (consumption plus investment) and output, imposing a recursive structure according to which the former cannot react contemporaneously to output shocks. We first show the estimation results on an individual country basis (for the seven countries listed above):

Figure: Response of public spending to output shocks in VAR (2)



In four instances (Germany, France, the Netherlands and the U.K.) is there a significant within-year reaction of government spending to an output shock. This significance is at the border and exists only for one or two quarters, while, moreover, its sign is not unique. In two cases, the reaction of government spending is positive and in two cases it is negative. We thus decided to pool the estimation into a seven-country two-variable PVAR. The result is depicted in the following figure:

Figure: Response of public spending to output shocks in pooled VAR (2)



Using the approach described in Appendix D, on the basis of the estimates for the **quarterly** model, the estimated mean of the annualized output elasticity of government spending is 0.018 (median = 0.029; 5%-percentile = -0.082; 95%-percentile = 0.168).

On the basis of these figures, we re-estimate our **annual** baseline PVAR assuming $\alpha_{gy} = -0.082$ (scenario 1), $\alpha_{gy} = 0.018$ (scenario 2), $\alpha_{gy} = 0.168$ (scenario 3), respectively. The results are shown in the following figures:

Figure: impulse responses for the fiscal block (scenario 1 $\alpha_{gy} = -0.082$)

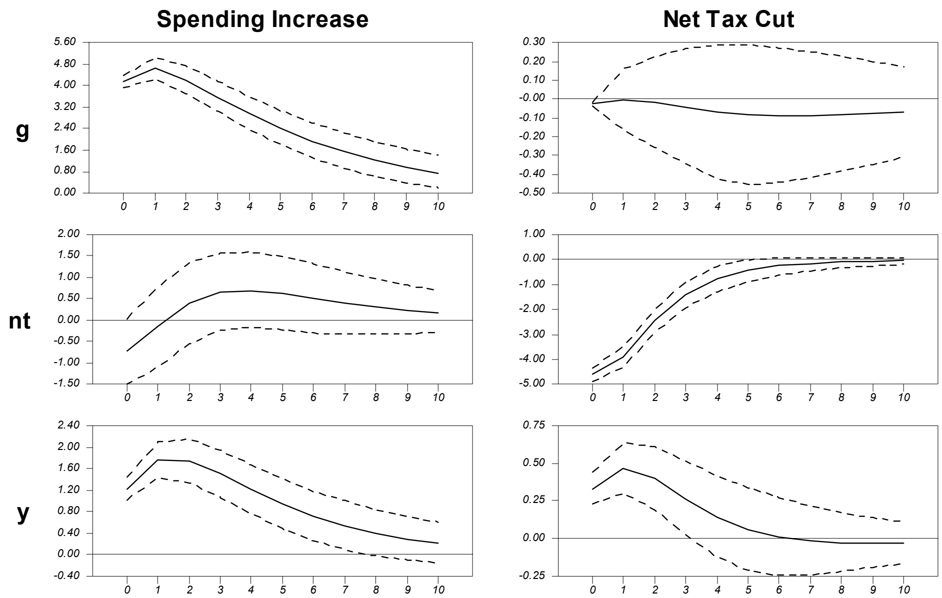


Figure: impulse responses for the fiscal block (scenario 2 $\alpha_{gy} = 0.018$)

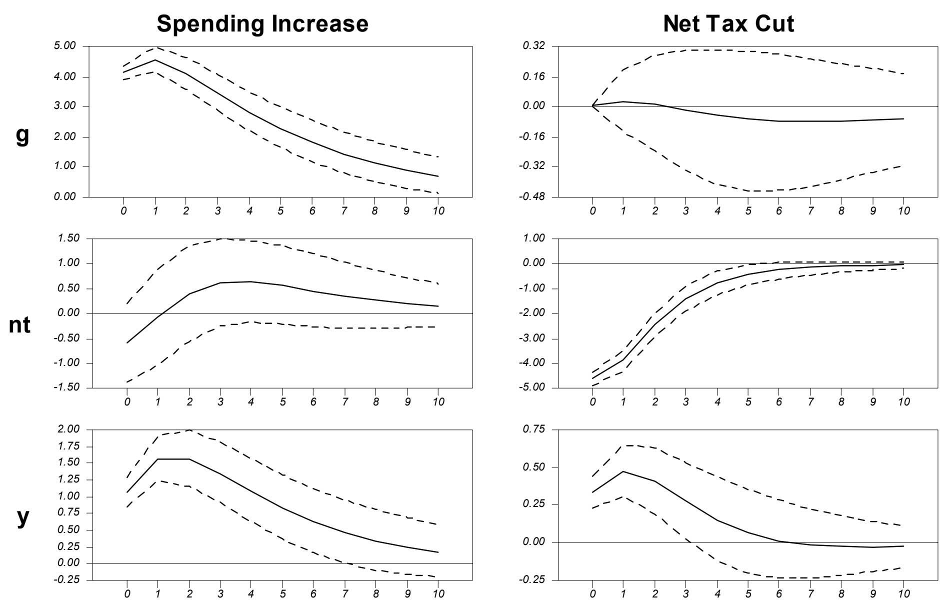
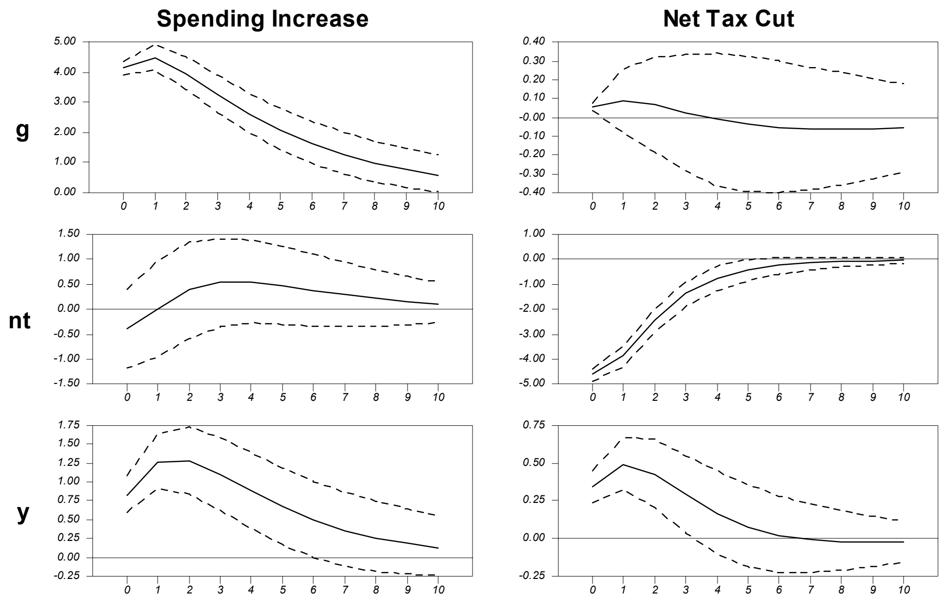
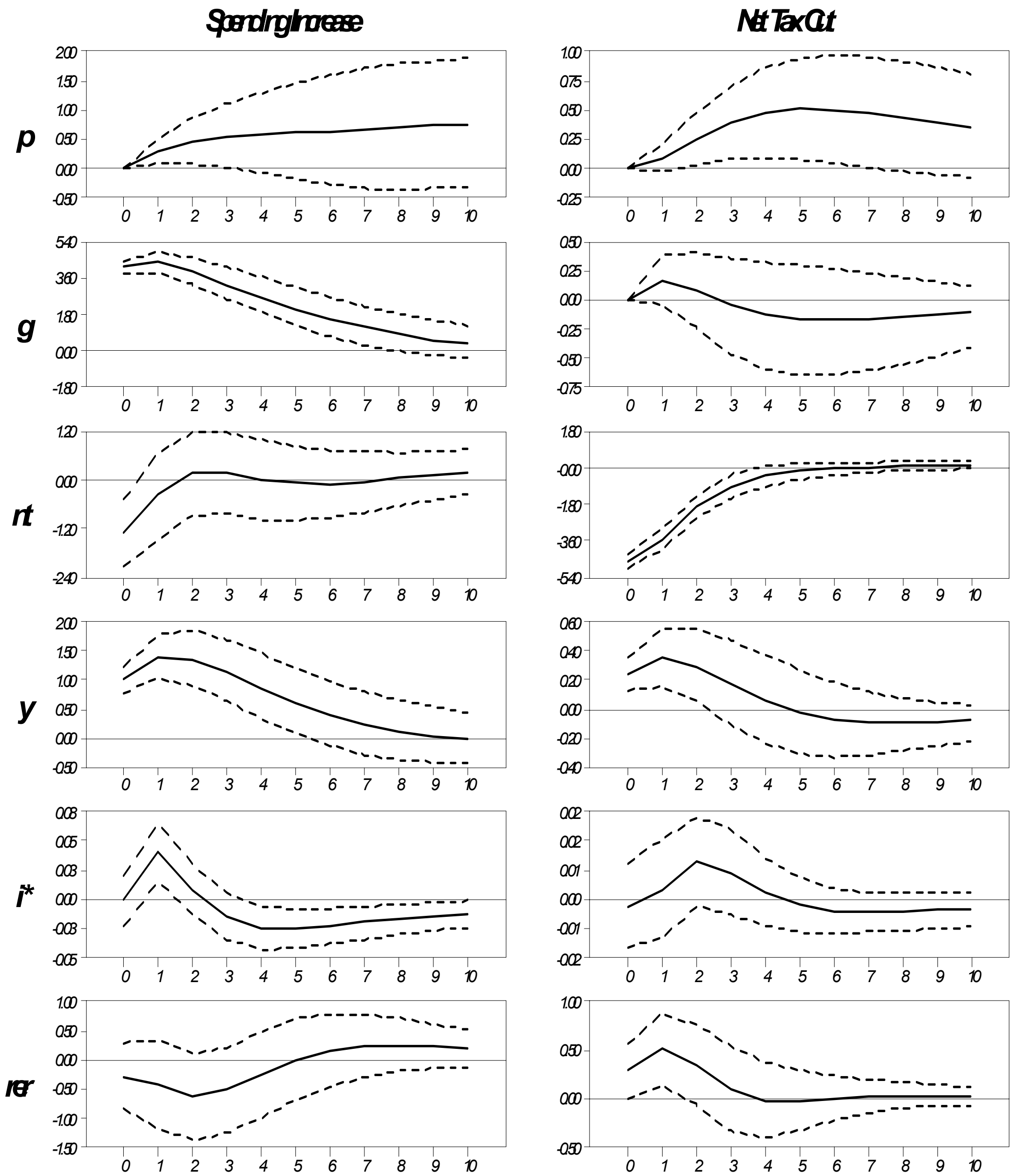


Figure: impulse responses for the fiscal block (scenario 3 $\alpha_{gy} = 0.168$)



Appendix C.2. Weighted average foreign ex-post real long-term interest rate in fiscal block

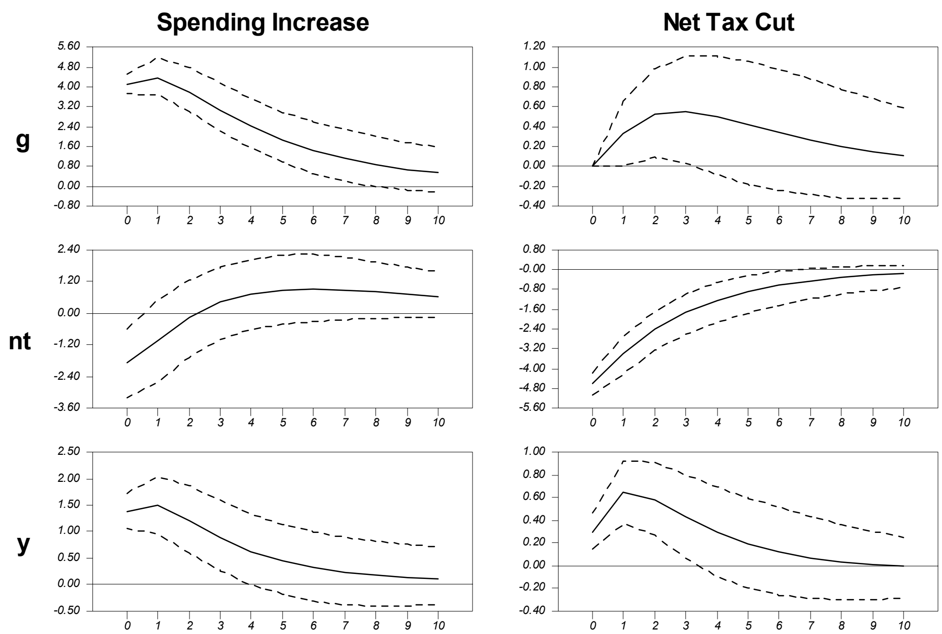
Figure: Impulse responses for the fiscal block; extended panel VAR with GDP-weighted average ex-post real long-term interest rate (i*)



Appendix C.3. Estimates for the core EU-5 countries

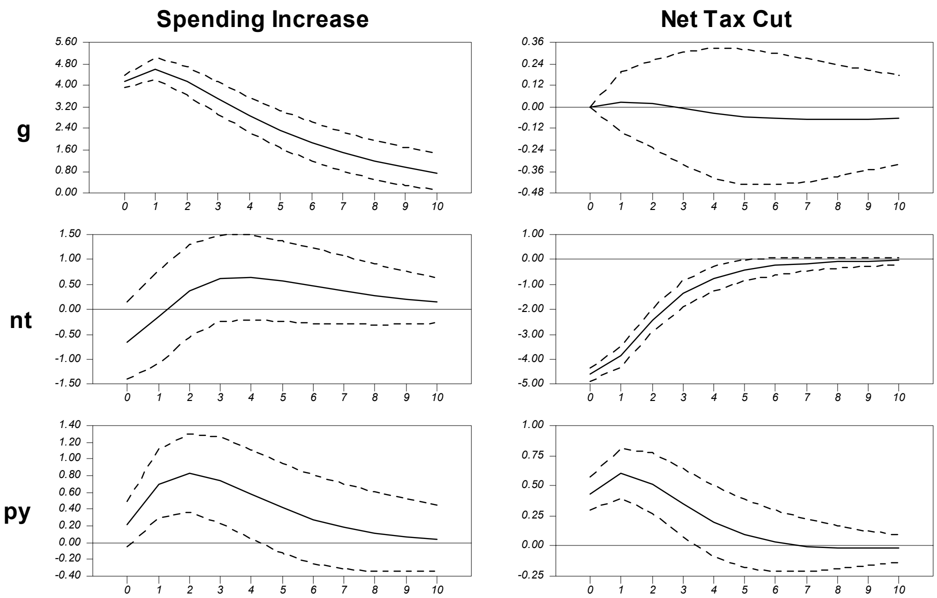
The impulse responses for the baseline specification for this sub-sample of countries are:

Figure: Impulse responses for the fiscal block Core EU 5



Appendix C.4. PVAR with private instead of total output

Figure: Impulse responses for the fiscal block (panel VAR with private output)



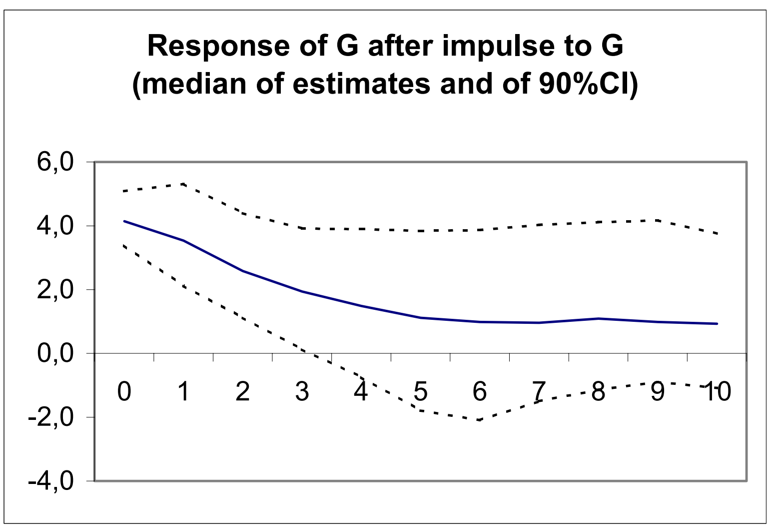
Notes: Confidence bands are the 5th and the 95th percentiles from Monte Carlo simulations based on 1,000 replications.

Appendix C.5. Individual country estimates

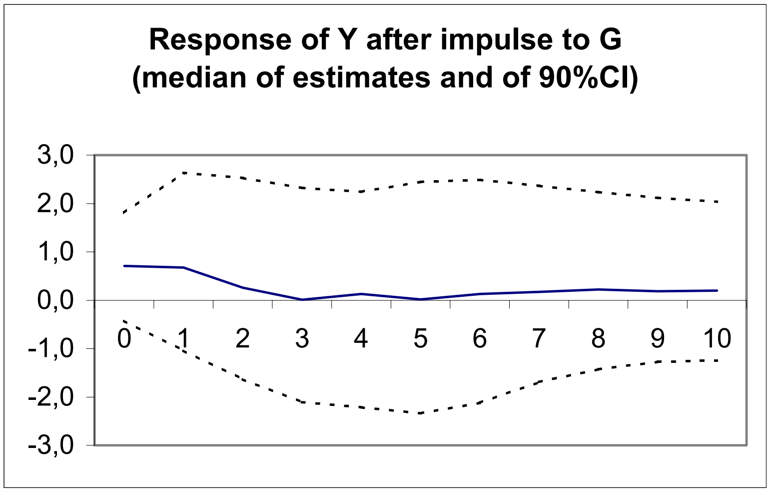
The following figures are based on the country-by-country estimations of the fiscal and trade blocks. Impulse responses and 90% confidence intervals are then computed, taking the mean and 5% and 95% quantiles of 1000 impulse response simulations. Hence, for each country we get three lines. Thus, we have three sets of 14 lines. Of each of these sets we take the median (actually the average of the 7th and 8th point in each response period). The median of the mean impulse response is depicted as the solid line. The medians of the lower and upper bounds are depicted as the dashed lines. The first two figures of each set of three figures are based on the fiscal block. The final figure in each set is based on the combination of the fiscal and trade block (and is also included in the paper).

Set 1

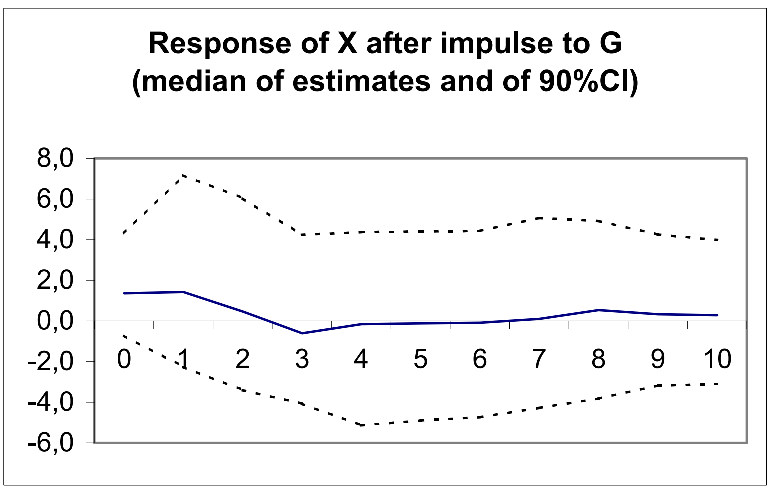
Response based on fiscal block:



Response based on fiscal block

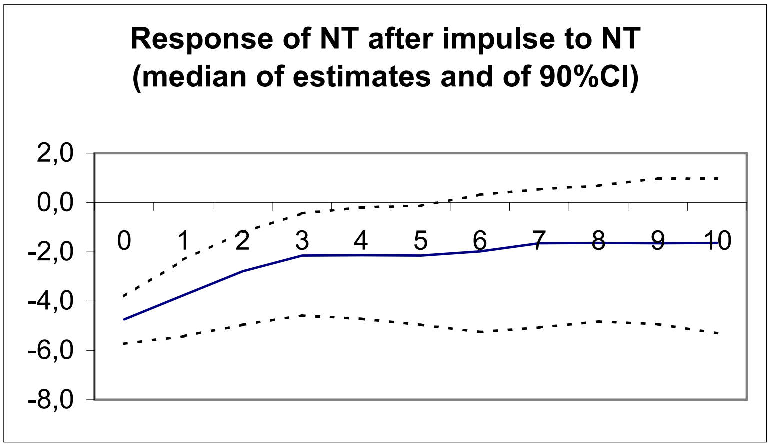


Response based on combination of fiscal and trade block

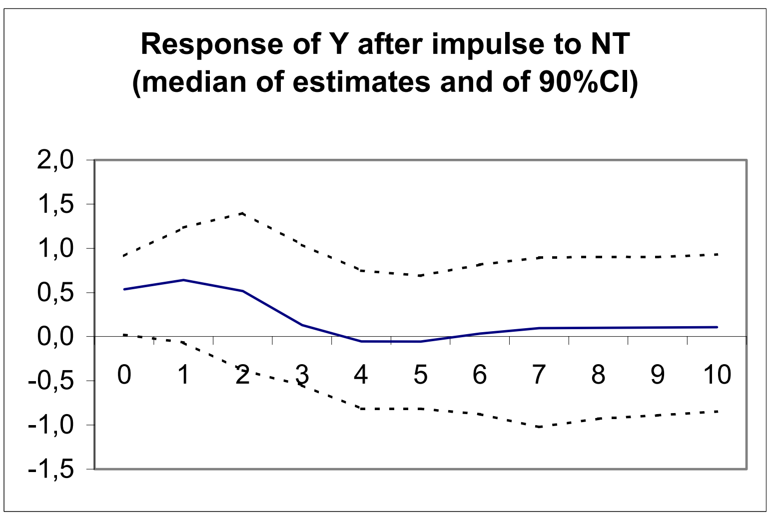


Set 2

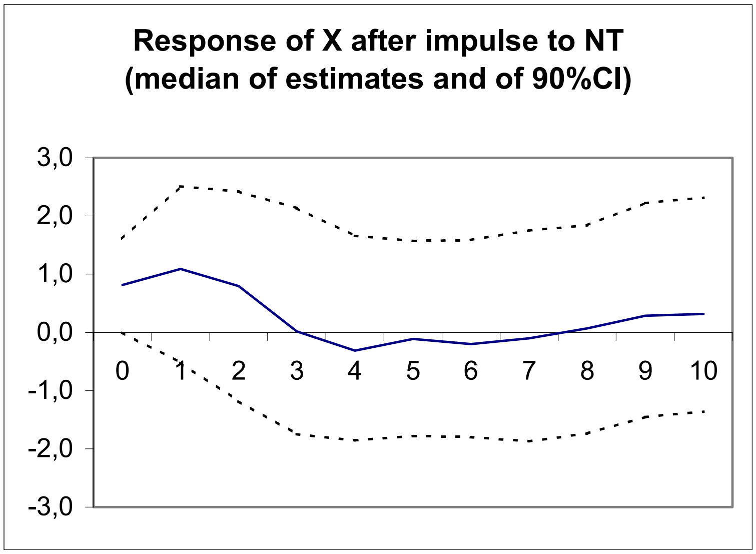
Response based on fiscal block:



Response based on fiscal block:



Response based on combination of fiscal and trade block:



Part D: Using quarterly SVAR to derive the contemporaneous effect of a fiscal impulse in annual SVAR

Let X_t be the $K-$vector of interest at the annual frequency, where we leave out the country indicator i for simplicity. We focus on the most general case in the paper, where $K = 3$ and $X_t = \left(g_t, nt_t^{CA}, y_t\right)'$, but the results for $K = 2$ and $X_t = (g_t, y_t)'$ follow in a similar way. The structural vector autoregression for X_t used in the paper is

$$A_0 X_t = A\left(L\right) X_{t-1} + E_t,$$

where

$$A_0 = \begin{bmatrix} 1 & -\alpha_{gt} & -\alpha_{gy} \\ -\alpha_{tg} & 1 & -\alpha_{ty} \\ -\alpha_{yg} & -\alpha_{yt} & 1 \end{bmatrix}.$$

The problem is how to obtain an estimate of the 6 unknown parameters in A_0. The idea of our solution is as follows. From the SVAR specification we know that feeding a shock of size E_t instead of 0 into the system leads to a change dX_t in X_t that fulfils

$$A_0 \cdot dX_t = E_t.$$

Suppose that we know the shock comes from variable k only ($k = 1, ..., K$), and that we know the resulting change in X_t, denoted by dX_t^k. Let $A_{0\neg k.}$ denote the submatrix of A_0 that contains the $K-1$ rows apart from row k. Then

$$A_{0\neg k.} \cdot dX_t^k = 0. \tag{1}$$

This gives $K-1$ equations. Doing so for each k, we get $\left(K-1\right) K$ equations. Because there are also $\left(K-1\right) K = 6$ unknown elements in A_0, we can solve for A_0, provided there are no singularities. In our case of $K = 3$, the K systems (1) are

$$\begin{cases} \begin{bmatrix} -\alpha_{tg} & 1 & -\alpha_{ty} \\ -\alpha_{yg} & -\alpha_{yt} & 1 \end{bmatrix} \begin{bmatrix} dg_t^1 \\ dnt_t^{CA1} \\ dy_t^1 \end{bmatrix} = 0 \\ \begin{bmatrix} 1 & -\alpha_{gt} & -\alpha_{gy} \\ -\alpha_{yg} & -\alpha_{yt} & 1 \end{bmatrix} \begin{bmatrix} dg_t^2 \\ dnt_t^{CA2} \\ dy_t^2 \end{bmatrix} = 0 \\ \begin{bmatrix} 1 & -\alpha_{gt} & -\alpha_{gy} \\ -\alpha_{tg} & 1 & -\alpha_{ty} \end{bmatrix} \begin{bmatrix} dg_t^3 \\ dnt_t^{CA3} \\ dy_t^3 \end{bmatrix} = 0 \end{cases},$$

which can be rewritten as one convenient block-diagonal system

$$\begin{bmatrix} dnt_t^{CA2} & dy_t^2 & & & & \\ dnt_t^{CA3} & dy_t^3 & & & & \\ & & dg_t^1 & dy_t^1 & & \\ & & dg_t^3 & dy_t^3 & & \\ & & & & dg_t^1 & dnt_t^{CA1} \\ & & & & dg_t^2 & dnt_t^{CA2} \end{bmatrix} \begin{bmatrix} \alpha_{gt} \\ \alpha_{gy} \\ \alpha_{tg} \\ \alpha_{ty} \\ \alpha_{yg} \\ \alpha_{yt} \end{bmatrix} = \begin{bmatrix} dg_t^2 \\ dg_t^3 \\ dnt_t^{CA1} \\ dnt_t^{CA3} \\ dy_t^1 \\ dy_t^2 \end{bmatrix}.$$

To operationalize this method we need values of dX_t^k that come from shocks to variable k. We generate such shocks from an SVAR for the vector $x_{t,q} = \left(g_{t,q}, nt_{t,q}^{CA}, y_{t,q}\right)'$ of quarterly values corresponding to the annual X_t, where $q = 1, ..., 4$ (note: the quarterly SVAR is identified, as its upper-triangular contemporaneous correlations are zero). Then we compute the annual changes dX_t^k by combining the four quarterly changes.

More precisely, let $x_{t,q}$ have an MA(∞) representation, so that for the quarters in year t we have

$$\begin{cases} x_{t,1} = & \psi_0\varepsilon_{t,1} + \psi_1\varepsilon_{t-1,4} + ... \\ x_{t,2} = & \psi_0\varepsilon_{t,2} + \psi_1\varepsilon_{t,1} + \psi_2\varepsilon_{t-1,4} + ... \\ x_{t,3} = & \psi_0\varepsilon_{t,3} + \psi_1\varepsilon_{t,2} + \psi_2\varepsilon_{t,1} + \psi_3\varepsilon_{t-1,4} + ... \\ x_{t,4} = & \psi_0\varepsilon_{t,4} + \psi_1\varepsilon_{t,3} + \psi_2\varepsilon_{t,2} + \psi_3\varepsilon_{t,1} + \psi_4\varepsilon_{t-1,4} + ... \end{cases},$$

where the ψ are $K \times K$ matrices and the ε are $K \times 1$ vectors of structural disturbances of the quarterly model. Because X_t and the $x_{t,q}$ are in logarithms, one cannot simply add $x_{t,1}, ..., x_{t,4}$ to obtain X_t. In fact,

$$X_t = \log\left[\exp\left(x_{t,1}\right) + \exp\left(x_{t,2}\right) + \exp\left(x_{t,3}\right) + \exp\left(x_{t,4}\right)\right],$$

where log and exp functions of vectors are defined elementwise. We now linearize around the value of $x_{t,q}$ that results from $\varepsilon_{t,1} = \varepsilon_{t,2} = \varepsilon_{t,3} = \varepsilon_{t,4} = 0$, denoted by $x^0_{t,q}$, so that

$$dX_t \approx \frac{\exp\left(x^0_{t,1}\right)}{\sum_{q=1}^{4} \exp\left(x^0_{t,q}\right)} dx_{t,1} + ... + \frac{\exp\left(x^0_{t,4}\right)}{\sum_{q=1}^{4} \exp\left(x^0_{t,q}\right)} dx_{t,4},$$

where $\exp\left(x^0_{t,q}\right) / \sum_{q=1}^{4} \exp\left(x^0_{t,q}\right)$ denotes the K-vector of elementwise divisions. We approximate these ratios by 1/4. As we want to know the effect of shocks in year t only, we have

$$\begin{cases} dx_{t,1} = & \psi_0 \varepsilon_{t,1} \\ dx_{t,2} = & \psi_0 \varepsilon_{t,2} + \psi_1 \varepsilon_{t,1} \\ dx_{t,3} = & \psi_0 \varepsilon_{t,3} + \psi_1 \varepsilon_{t,2} + \psi_2 \varepsilon_{t,1} \\ dx_{t,4} = & \psi_0 \varepsilon_{t,4} + \psi_1 \varepsilon_{t,3} + \psi_2 \varepsilon_{t,2} + \psi_3 \varepsilon_{t,1} \end{cases}.$$

This yields

$$dX_t \approx \frac{1}{4}\left[\psi_0 \varepsilon_{t,4} + \left(\psi_0 + \psi_1\right) \varepsilon_{t,3} + \left(\psi_0 + \psi_1 + \psi_2\right) \varepsilon_{t,2} + \left(\psi_0 + \psi_1 + \psi_2 + \psi_3\right) \varepsilon_{t,1}\right].$$

Hence, for each k, if we have quarterly shocks $\varepsilon_{t,1}, \varepsilon_{t,2}, \varepsilon_{t,3}$, and $\varepsilon_{t,4}$ where all elements except k are zero, we can compute (an approximate value of) the annual change dX^k_t, which can then be used in (1) to derive the corresponding annual contemporaneous correlations in A_0.

Finally, we describe the approach used to obtain the actual numbers. First, we estimate the quarterly SVAR; this yields estimates of the $K \times K$ diagonal covariance matrix Σ of the structural innovations ε, while a standard Monte Carlo method generates $S = 1000$ sets of relevant impulse response matrices $(\psi_0, ..., \psi_3)$. Second, for each k we draw $R = 1000$ vectors of uncorrelated quarterly shocks $\left(\varepsilon^k_{t,1}, \varepsilon^k_{t,2}, \varepsilon^k_{t,3}, \varepsilon^k_{t,4}\right)$ from the estimated error distribution $N\left(0, \widehat{\Sigma}_{kk}\right)$. Third, we combine one set of impulse reponses with one collection of K shock vectors to compute dX^k_t for all k and derive the unknown elements in A_0 from (1); for a each given combination $(\psi_0, ..., \psi_3)$ drawn above, we repeat this $R = 1000$ times. In the end, we have $SR = 1,000,000$ draws for A_0. We use the 5% and 95% quantiles to summarize their range.