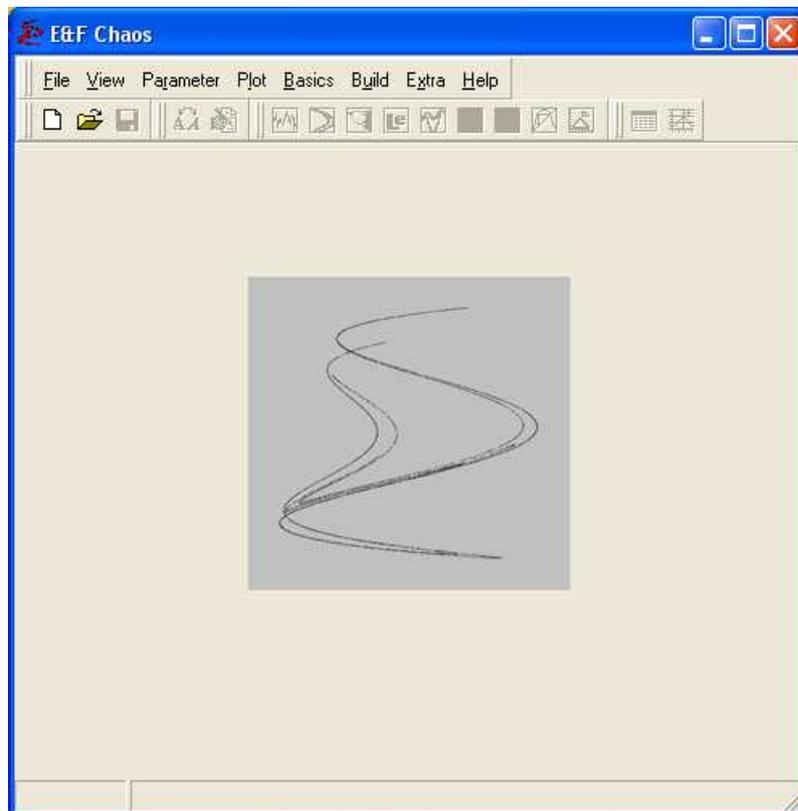


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# Manual E&F Chaos Program

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Cees Diks, Cars Hommes, Valentyn Panchenko and Marcelo Tyszler  
CeNDEF, University of Amsterdam



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(E&F Chaos version 1.00)



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# Chapter 1

## Introduction

### 1.1 About E&F Chaos

The E&F Chaos program is an open source Windows software package for numerical simulation of nonlinear dynamical systems, developed jointly by Cees Diks, Cars Hommes, Valentyn Panchenko and Roy van der Weide at the University of Amsterdam. The Delphi 4.0 source code is distributed under the GNU General Public License. The current version, E&F Chaos 1.00, allows the user to specify new models dynamically, without the need of a compiler, and to generate publication quality graphical output. The program comes with a large library of example models, mostly from economics and finance, that have already implemented. E&F Chaos runs under Windows 95 (2nd edition) or later, as well as under Wine on Linux systems.

### 1.2 Quickstart

You can download the program from the CENDEF webpage (<http://www.fee.uva.nl/cendef>). Currently the latest version is E&F Chaos 1.00. Simply unzip the files into a folder and the program is ready to use. Start the program by executing the file *EFCHAOS.EXE*. Once the program is running, you will get the E&F Chaos Window on your screen, containing several menus and submenus. You can select these menus or submenus as usual, either by using the mouse, by using the arrow keys followed by the Enter-key, or by using the Alt-key in combination with the appropriate letter.

The quickest way to start numerical simulations for one of the models is as follows. After starting the E&F Chaos program, select the  **Load Model**. In the program folder there is a library of pre-configured models. Select the *Discrete* subfolder to load a model based on a difference equation (or the *Continuous* subfolder to load a model based on a differential equation). Select one of the models, e.g. the Hénon Map, from the list of models. The corresponding model equations will be loaded into memory, and the program is ready for numerical model simulation. Next select the **Plot** menu from the main menu. A list of submenus will appear for numerical simulation: **Time Series**, **Phase Plot**, **Bifurcation Diagram**, **Delay Plot**, **Basin of Attraction**, **Parameter Basins for Periodic Cycles** and **Graphical Analysis**. The number of active submenus depends on the type of model that has been selected. Start playing and simulating, e.g. by selecting the **Time Series** option. You may also try other submenus for numerical simulation, e.g. **Phase Plot** or **Bifurcation Diagram**, and experiment a little with the program. The menu and submenu structure is more or less self-explanatory.

Chapter 2 contains a number of **tutorials** intended to get you familiar with most of the options of the program quickly. If you do run into trouble in playing with the program, please check Chapter 3 (**Reference Manual**).

### 1.3 Library of Models

A number of models, both in discrete and continuous time, have been implemented in the E&F Chaos program. These models include some of the standard models from the dynamical systems literature, such as the Logistic map, the Hénon map, and the Lorenz equations. Most of the models, however, come from economics and finance, and include standard models such as Cobweb and Asset Pricing Models. All implemented models can be opened through the  Load Model submenu and browsing through the preset library. The library starts at the subfolder *Models* and is subdivided in *Continuous*, *Discrete* with also a pre-configured folder *My Models*, in which you will find the higher dimensional model used in the **Tutorials** chapter.

### 1.4 About this Manual

This Manual refers to E&F Chaos version 1.0. Besides this introduction, there are two Chapters: **Tutorials** and **Reference Manual**. The **Tutorials** chapter is supposed to guide you through the main functionalities of the software, reproducing most of the figures in the PAPER. They are supposed to be followed in order and are increasing in complexity. The **Reference Manual** is supposed to provide a more extensive explanation of all options available. Each sub-item is written in order to be self-contained so one can simply go directly to the option of interest.

### 1.5 Final Remarks

The E&F Chaos program is still under development, and being improved regularly based on the feedback of users. As mentioned before, you should be able to find out easily how to work with most features of E&F Chaos. However, if you run into real trouble and cannot solve it on the basis of this manual, or find mistakes in the program, please contact us by e-mail: <C.G.H.Diks@uva.nl>. If you e-mail us, please be very precise in describing your particular problem. Enjoy E&F Chaos!

# Chapter 2

## Tutorials

This section contains 3 Tutorials. The first is intended for a first contact with the software and will guide you through the main functionalities, mostly based on a 1-dimensional example. The second tutorial will take you into a higher dimensional example and explore some more advanced options of the program. The third and final tutorials deals with more specific and advanced options.

In the tutorial you will be reproducing most of the figures in the paper *E&F Chaos: a user friendly software package for nonlinear economic dynamics* by Cees Diks, Cars Hommes, Valentyn Panchenko and Roy Van der Weide (Computational Economics, 2008, Forthcoming). At the time of writing this manual (June 2008) the paper has not appeared in print yet, but is already available on-line via OpenAccess at <http://www.springerlink.com/content/n442555742862636/fulltext.pdf>. In case you wish to refer to this paper before it appears in print, please do so by quoting its digital object identification number DOI 10.1007/s10614-008-9130-x.

For many more details on all aspects of E&F Chaos software consult Chapter 3 of this manual: **Reference Manual**.

### 2.1 First Tutorial: The Basics

#### 2.1.1 One-dimensional Example

To illustrate simulation of a 1-D system, we consider a simple economic example, the cobweb model with adaptive expectations. The dynamics of expected prices in this model are described by:

$$p_t^e = (1 - w)p_{t-1}^e + w \frac{a - \arctan(\lambda p_{t-1}^e)}{b} = f_{w,a,b,\lambda}(p_{t-1}^e). \quad (2.1)$$

Dynamics of (expected) prices in the cobweb model with adaptive expectations is thus given by a 1-D system  $x_t = f_{w,a,b,\lambda}(x_{t-1})$  with four model parameters. We next discuss how this model can be simulated in *E&F Chaos*.

#### 2.1.2 Loading the Model

A number of models, both in discrete and continuous time, are distributed with the E&F Chaos program. These models include some of the standard models from the dynamical systems literature, such as the Logistic map, the Hénon map, and the Lorenz equations.

Most of the models, however, come from economics and finance, and include standard models such as Cobweb and Asset Pricing Models. All implemented models can be opened through the  **Load Model** submenu and browsing through the preset library. Also, user-defined models previously saved can be retrieved by this submenu.

The first step is to call the  **Load Model** submenu from the **File** menu, as depicted in figure 2.1. You will be shown the Load window starting in the folder **Models**, as depicted

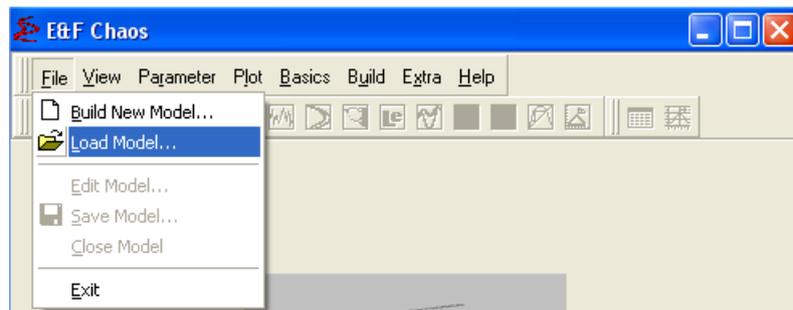


Figure 2.1: Load Model from File Menu

in figure 2.2. Find this path and file: **Discrete > Cob-Web Models > Adaptive**

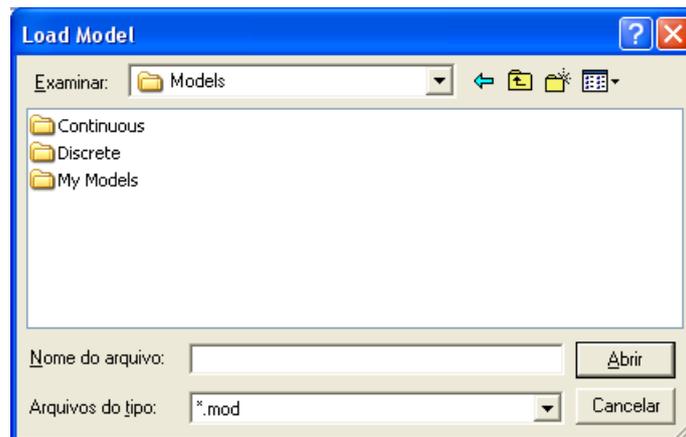


Figure 2.2: Load Model window

**Expectations > Cob-web S-shaped supply (arctan).mod**. This will load the referred model into memory.

### 2.1.3 Time Series

The most important menu in the *E&F Chaos* software is the **Plot** menu, where all graphical plot submenus are collected, which will be illustrated throughout these tutorials. Using the  **Time Series** submenu, an example of a chaotic price series can be illustrated.

First you need to make sure the parameters are with adequate values. Call the  **Change** submenu either by the icon on the toolbar or by the **Parameter** menu. Set the values to resemble figure 2.3.

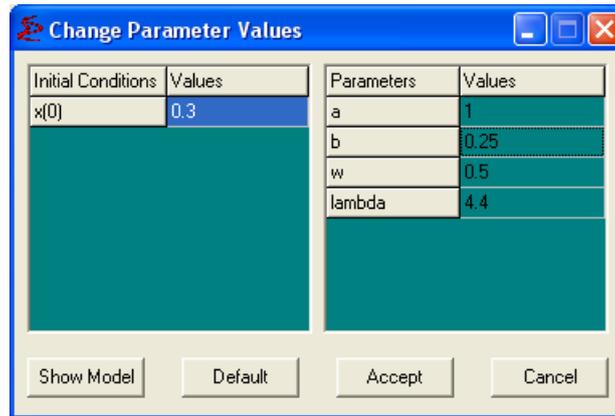


Figure 2.3: Change Parameters Window

Now call the  **Time Series** submenu also either by the icon on the toolbar or by the Plot menu. Press the **OK** button and you should obtain an output similar to figure 2.4.

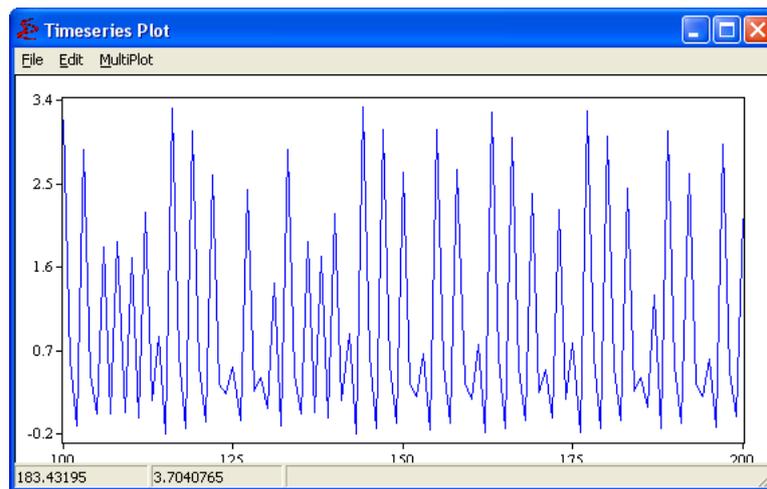


Figure 2.4: Time Series Output

It is possible to zoom in on these plots by dragging the mouse while pressing the left mouse-button, thus drawing a rectangle over the intended area. The original scaling can be obtained by double-clicking the left mouse button. Furthermore, the user can scroll the plotted object by moving the mouse while pressing both the CTRL-button and the left mouse button.

Each graphical window, such as  **Time Series**, has a submenu **Edit**, which enables the user to change parameter values, the initial state, the plot variable (only the expected price ' $x$ ' in this example), the number of points plotted, the number of points not plotted (so that a transient phase can be skipped) and the time step (so that e.g. each 10th period can be plotted). The  **Time Series** menu (as other Plot submenus) has additional options like **Connect Points** (if on, then subsequent points are connected by lines) and **Plot Squares** (if on, then each point is represented by a square). Scaling can be done either automatically (default) or manually. Also, there is a submenu **File** that allows users to save or print the

output.

### 2.1.4 Graphical Analysis

One-dimensional (1-D) systems  $x_t = f(x_{t-1})$  are relatively easy to analyze, since the graph of the corresponding 1-D generating map  $f$  essentially determines the dynamical behavior. The  **Graphical Analysis** submenu (only available for 1-D systems) gives the possibility for detailed investigations of the graph of the map  $f$  and higher order iterates  $f^k$ ,  $k \geq 2$  (i.e. applying the map  $f$   $k$  times).

After calling the submenu, adjust the settings as in figure 2.5.

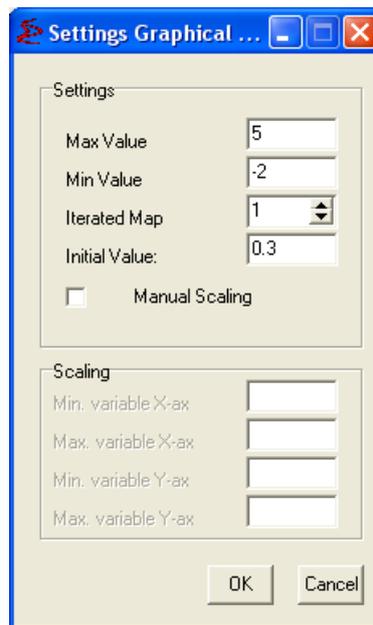


Figure 2.5: Graphical Analysis Submenu Settings

In the **Settings** submenu, the user can specify the range of the interval (i.e. minimum and maximum), the number of iterates and an initial state.

After pressing the **OK** button the output window will be shown, with the map in red. By clicking **Next Iteration(s)** the next steps in the graphical analysis (i.e. a horizontal line segment connecting the last point to the diagonal  $y = x$ , and a vertical line segment connecting this point to the graph of the map) appears. By repeatedly clicking it a ‘cobweb diagram’ as in Figure 2.6 appears, illustrating the dynamical behavior. Since the graphical analysis in this example does not converge, it suggests that the dynamical behavior is chaotic.

After clicking **Clear Form**, the graphical analysis can be repeated. Using the **Edit** submenu, the user can specify the number of time steps as an integer  $k$  (so that one click to **Next Iteration(s)** yields  $k$  steps in the graphical analysis).

### 2.1.5 Bifurcation Diagram

A powerful tool for investigating how the dynamical behavior of a nonlinear model depends on a single parameter is a bifurcation diagram. It shows the long run dynamical behavior as a function of a model parameter.

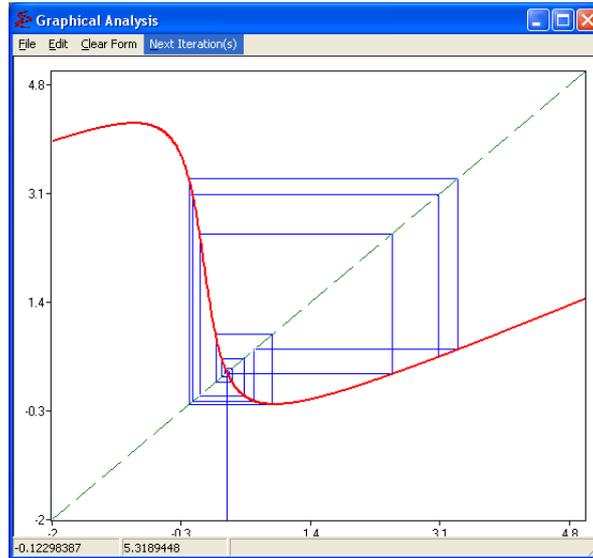


Figure 2.6: Graphical Analysis Iterations

After selecting the  **Bifurcation Diagram** submenu, the user can specify the parameter (or use the default parameter), its minimum and maximum value, the plot variable, the number of points to be plotted and the transient time (i.e. the number of points skipped before plotting is started). Adjust the settings according to figure 2.7. After pressing **OK** the bifurcation diagram is computed and shown, as in figure 2.8.

To provide different ways of dealing with the possibility of multiple attractors, in the **settings** there are several options for the initial conditions (the initial state variables) for each of the parameter values of the bifurcation diagram. There are three options: (i; default option) initialize on the same, fixed initial condition for each new parameter value, (ii) initialize on the last state found for the previous, slightly smaller parameter value, (iii) compute the diagram from ‘right to left’, i.e. from high to low values, with initial states as in (i) or (ii). Options (ii) and (iii) are intended to keep the orbit on the ‘same’ attractor as the parameter slowly changes, thus following the attractor as long as it exists.

Figure 2.8 shows a bifurcation diagram of the cobweb model with adaptive expectations with respect to the expectations weight factor  $w$ , illustrating the long run dynamics (100 iterations) after omitting a transient phase of 100 iterations. For small values of  $w$ ,  $0 \leq w \leq 0.27$ , prices converge to a stable steady state, while for high values of  $w$ ,  $0.77 < w \leq 1$  (close to naive expectations) prices converge to a stable 2-cycle with large amplitude. For intermediate  $w$ -values however, say for  $0.4 < w < 0.7$ , chaotic price oscillations of moderate amplitude arise. In particular, the chaotic price fluctuations for  $w = 0.5$  have been illustrated already in figure 2.6.

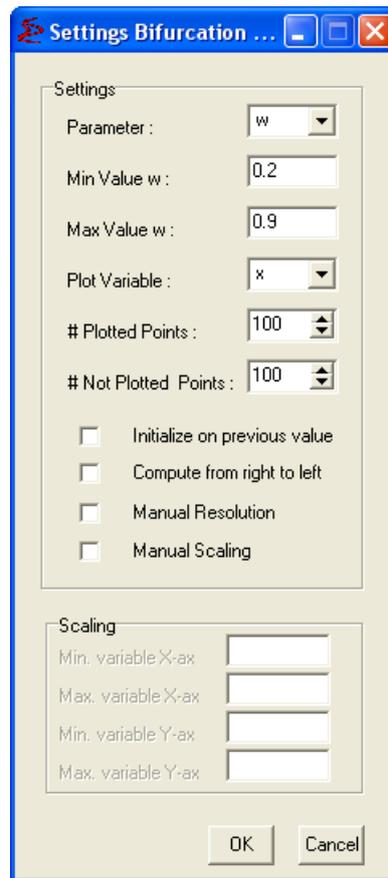


Figure 2.7: Bifurcation Diagram Submenu Settings

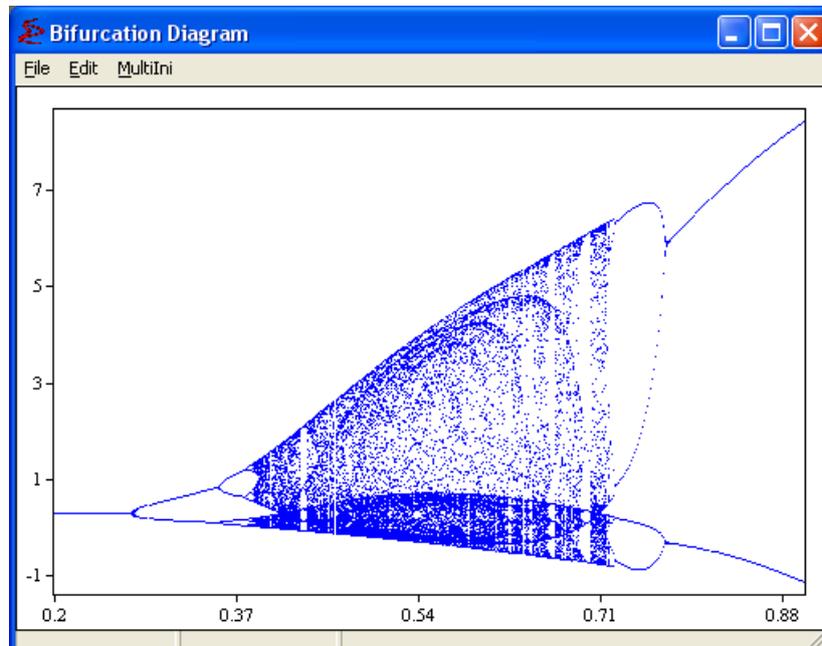


Figure 2.8: Bifurcation Diagram Output

## 2.2 Second Tutorial: More Advanced Plots

### 2.2.1 Parameter Basins for Periodic Cycles

An even more powerful tool in the numerical analysis of nonlinear dynamics are the  **Parameter Basins for Periodic Cycles** plots, sometimes also called 2-D bifurcation diagrams. A parameter basin plot assigns different colors in a 2-D parameter space to stable cycles of different periods.

Using the  **Parameter Basins for Periodic Cycles** submenu in *EEF Chaos*, the user can specify the number of periodic cycles to look for, the distance of convergence (epsilon), the two parameters, the number of transient iterations, the maximum distance (this is the distance after which the point is classified as “divergence”, in order to prevent overflows), the periods of the cycles to be detected, the minimum and maximum of the two parameter intervals and the resolution for both intervals. Finally, the user can specify the colors corresponding to each of the different stable cycles, the color for the non-convergence parameters (i.e. parameters for which points did not converge within distance epsilon) and the color for divergence parameters (i.e. parameters for which the trajectory diverges to infinity). For an initial run of the parameter basins, low values of the resolution (e.g. 200) and the transient time (e.g. 100) are recommended, because computation time rapidly increases with higher resolution. The default resolution is set to the maximum screen resolution.

Start by calling  **Parameter Basins for Periodic Cycles** submenu. Adjust your settings menu to resemble figure 2.9.

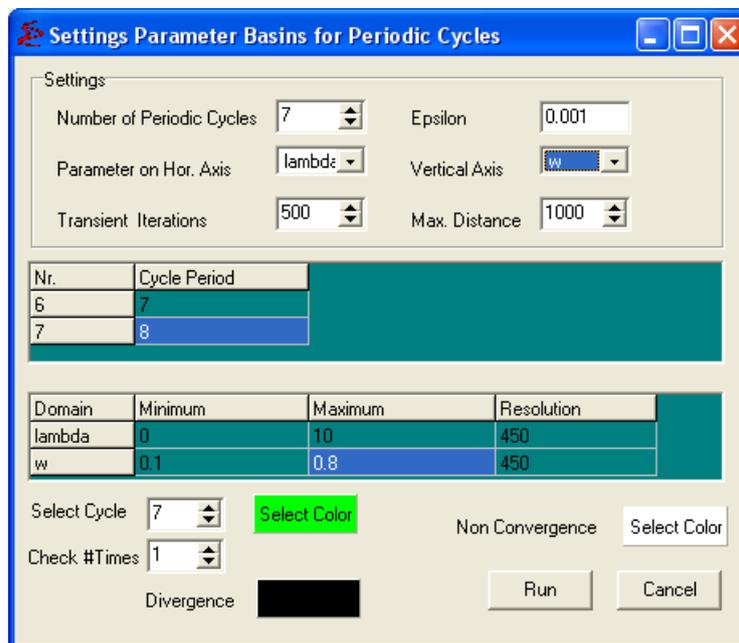


Figure 2.9: Parameter Basins for Periodic Cycles Submenu Settings

Specifically, use the following settings:

**Number of Periodic Cycles** : 7

**Parameter on Hor. Axis** :  $\lambda$

**Vertical Axis** :  $w$

**Transient Iterations** : 500 (if this is a first trial, you can leave it at 100, and change it later)

For the second block of information, define the following periods: 1,2,3,4,5,7,8 (therefore notice that you have to change the periods of Nr. 6 and 7 to periods 7 and 8, respectively).

In the third block of information adjust the domains in the following way:

$\lambda$  : 0 to 10

$w$  : 0.1 to 0.8

Finally use the following colors (in this order): (1) light blue; (2) dark blue; (3) yellow; (4) light purple; (5) orange; (6) red; (7) green.

After clicking the **Run** button, the computations start and a progress bar showing the fraction of computed points appears. With a 500 transient points, this should take around 5 minutes, depending on the computer. The user can interrupt the computations by clicking the **Stop** button. Figure 2.10 shows the output.

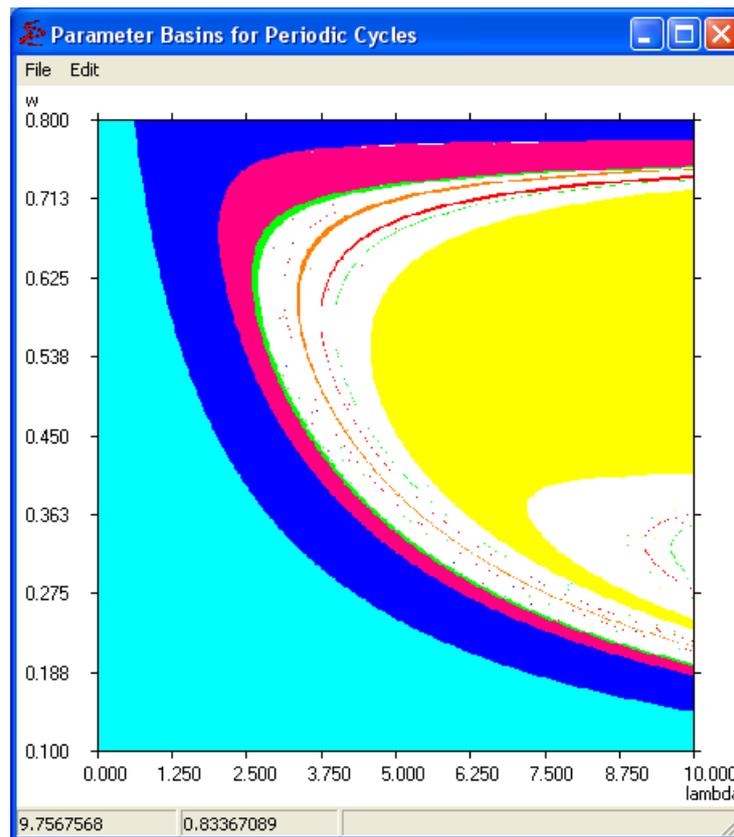


Figure 2.10: Parameter Basin for Periodic Cycles Output

Figure 2.10 presents a parameter basin with respect to the parameters  $(\lambda, w)$ , and assigns different colors to stable steady states and stable cycles of periods 2, 4, 8 (the first four cycles

in a period doubling bifurcation route to chaos) and periods 3, 5 and 7 (low order stable cycles of odd period).

Notice that the 1-D bifurcation diagram with respect to the parameter  $w$ , as illustrated in Figure 2.8, corresponds to a vertical line in the parameter basin plot, with fixed  $\lambda = 4.4$ . The reader should compare the bifurcation scenario's in both figures.

### 2.2.2 More Graphical Analysis

On a 1-D map it is also possible to use Graphical analysis to investigate bifurcations. The boundaries between different colors in the parameter basins in Figure 2.10 represent bifurcation curves in the 2-D  $(\lambda, w)$ -parameter space.

The parameters basins figure suggests that fixing  $\lambda = 6$  and increasing  $w$  leads to a *period doubling bifurcation* when the dynamics changes from a stable steady state (the light blue region) to a stable 2-cycle (the dark blue region).

First adjust the model parameters for  $\lambda = 6$  and  $w = 0.21$ . Call the  **Graphical Analysis** submenu again. This time set **Min** =  $-0.4$  and the **Max** =  $1$ . Also change the **Iterated Map** to 2. Your map should be just tangent to the 45 line. Now, from the **Settings** menu change  $w$  to  $0.25$ . The new map should show the cycle. Figure (2.11) shows the outputs.

You can try a similar thing for the third iterate. For this set **Min** =  $-0.5$  and the **Max** =  $2.5$ . Then set  $w$  to  $0.37$  first and then to  $0.385$ . Figure (2.12) shows the outputs.

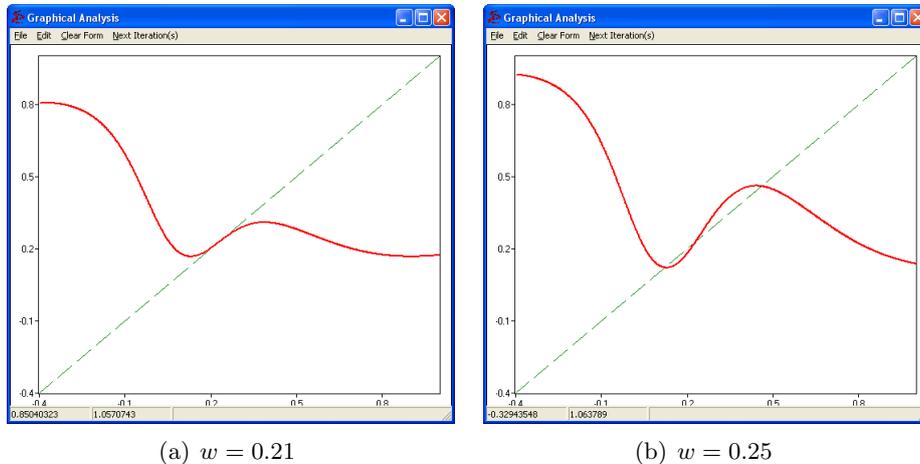


Figure 2.11: Graphical Analysis. Second Iterator

### 2.2.3 A higher dimensional example

For the remainder of the tutorial we consider a higher dimensional example of a nonlinear system. The model is an extension of the agent-based cobweb model with heterogeneous expectations (rational versus naive expectations) modified to allow for *asynchronous* updating of strategies.

The dynamics of the cobweb model with rational versus naive agents and asynchronous updating of strategies is given by 4 equations defining a nonlinear 4-dimensional (with state

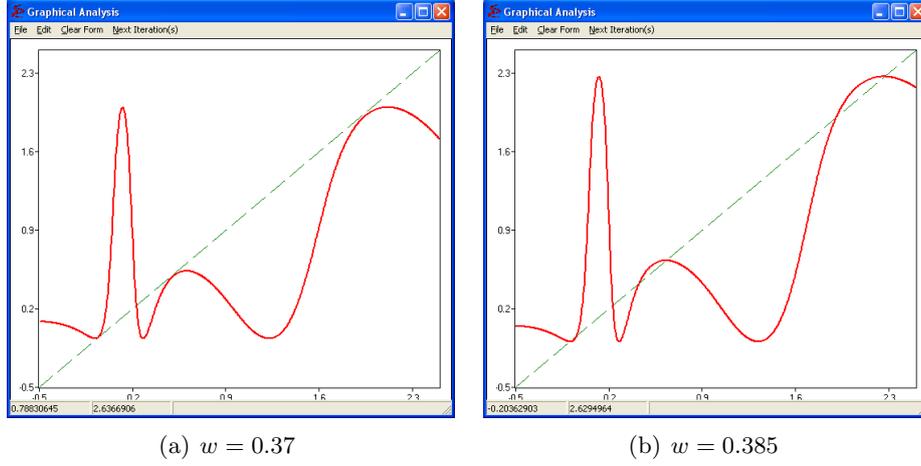


Figure 2.12: Graphical Analysis. Third Iterator

variables  $p_{t-1}$ ,  $n_{1,t-1}$ ,  $U_{1,t-1}$  and  $U_{2,t-1}$ ) discrete time dynamical system. They are, first indicating the price:

$$p_t = \frac{a - n_{2t}sp_{t-1}}{b + sn_{1t}}. \quad (2.2)$$

With  $n_{1t}$  and  $n_{2t}$  representing the fractions of type 1 and type 2 agents.

The fractions are determined by the past performance of the two forecasting strategies. The payoff  $\pi_{ht}$  of strategy  $h$  at time  $t$  is given by the realized profit of that strategy at time  $t$  minus the costs ( $C$ ) incurred:

$$\pi_{1t} = \frac{b}{2}p_t^2 - C, \quad \pi_{2t} = \frac{b}{2}p_{t-1}(2p_t - p_{t-1}). \quad (2.3)$$

The fitness measure  $U_{ht}$  of strategy  $h$  is a geometrically downweighted average of past profits, i.e.

$$U_{ht} = wU_{h,t-1} + (1 - w)\pi_{ht}, \quad (2.4)$$

where  $w \in [0, 1)$  is a memory parameter.

The dynamics of the fractions is given by a modified version of the discrete choice, logit probabilities:

$$n_{ht} = (1 - \delta)e^{\beta U_{h,t-1}} / Z_{t-1} + \delta n_{h,t-1}, \quad (2.5)$$

where  $Z_{t-1} = \sum_h e^{\beta U_{h,t-1}}$  is a normalization factor so that the fractions add up to 1.

This model is already implemented in the E&F Chaos program. To load it in memory find this path and file: **My Models > BH-cobweb-async.mod**.

## 2.2.4 Multiple Time Series

A convenient feature of the *E&F Chaos* software is that it can simultaneously plot different time series in a single graphics window. One can use this feature for example, to investigate and illustrate whether the model exhibits *sensitive dependence on initial conditions*, a key feature of a chaotic system.

To check this example first adjust the parameters so the initial value for  $p_0 = 2.2$ . Call the  **Time Series** submenu and set no transient points (**#Points Not Plotted**= 0). Pressing **OK** will plot the base graph in blue. Now, call the **Multiplot** menu and select the **Add** from the Red color, as indicated in Figure 2.13. This will plot a new series in Red exactly over the previous blue (since the parameters and setting are still the same). All the actions now will affect only the red curve.

Call the **Edit>Parameters...** submenu and change  $p_0 = 2.21$ . Your output should look like Figure 2.14(a).

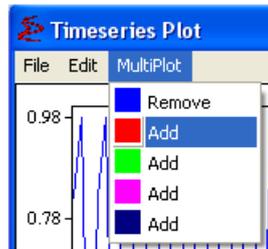


Figure 2.13: Multiplot submenu

The output shows simultaneous plots of price sequences with slightly different initial conditions. Since the two time series separate already after about 10 periods, the system exhibits sensitive dependence.

You can also plot different variables. Reset the parameters for  $p_0 = 2.2$ . Now go the **Settings** window and chose  $n_1$  as the plot variable. Your output should look like Figure 2.14(b).

We can see that the fraction of rational agents quickly increases, each time price deviations from the steady state become too large.

### 2.2.5 Phase and Delay Plot

For higher dimensional systems, Phase Plots and Delay Plots provide helpful tools to investigate the (long run) dynamical behavior and plot attractors of the system.

Examples are a stable steady state (single point attractor), a stable periodic orbit (finite number of points), a quasi-periodic attractor (“invariant circle”) and a strange (chaotic) attractor (fractal structure).

For this example adjust the parameters:  $\beta = 15$ ;  $a = 10$ ;  $\delta = 0.8$ .

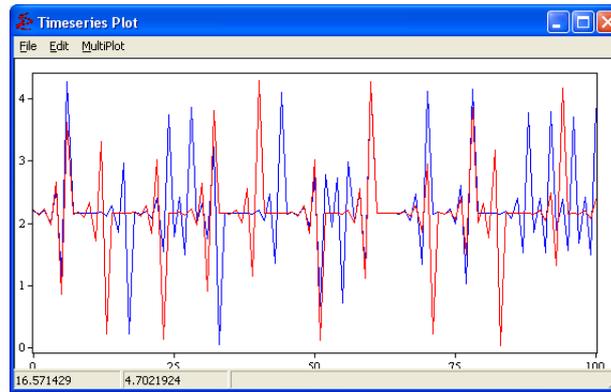
Call the  **Phase Plot** and you should obtain Figure 2.15. By calling the  **Delay Plot** you should obtain Figure 2.16

By holding the mouse button and drawing a rectangle, the user can zoom into the detailed fractal structure of the attractor.

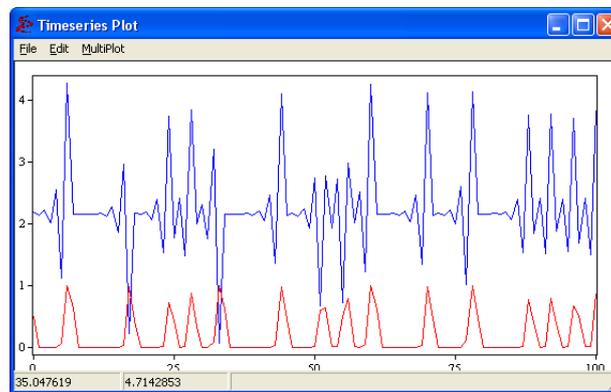
### 2.2.6 Largest Lyapunov Exponent

A powerful numerical tool to investigate whether the dynamical behavior is chaotic is a plot of the largest Lyapunov exponent, as a function of one of the model parameters.

After adjusting the parameter  $a = 6$ , call the  **Bifurcation Diagram** submenu and set **Min**= 0 and **Max**= 20. Your output should resemble Figure 2.17(a) that you can use to compare with the Largest Lyapunov Exponent output.



(a) Same Variable, different initial conditions ( $p = 2.2$ (blue) and  $p = 2.21$ (red))



(b) Different Variables,  $p$ (blue) and  $n1$ (red) ( $p = 2.2$ )

Figure 2.14: Multiple Time Series

Call now the  **Largest Lyapunov Exponent** submenu and set **Min**= 0, **Max**= 20 and **Iterations**= 5000. Your output should resemble Figure 2.17(b).

The bifurcation diagram shows a primary period doubling bifurcation, a secondary Hopf bifurcation of the 2-cycle followed by a bifurcation route to more complicated dynamical behavior. The Lyapunov exponent plot becomes positive for large values of  $\beta$ , providing numerical evidence that the dynamics becomes chaotic for large values of the intensity of choice.

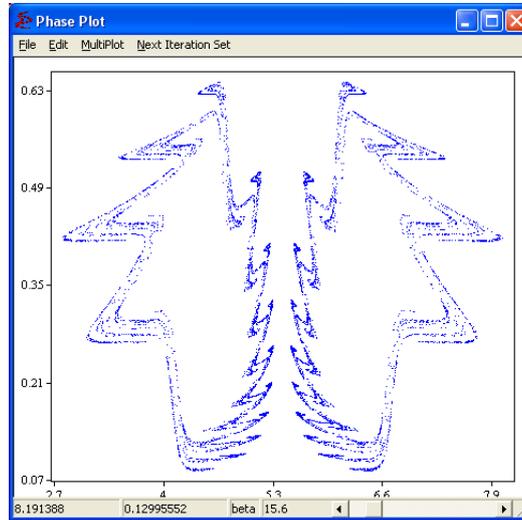


Figure 2.15: Phase Plot Output

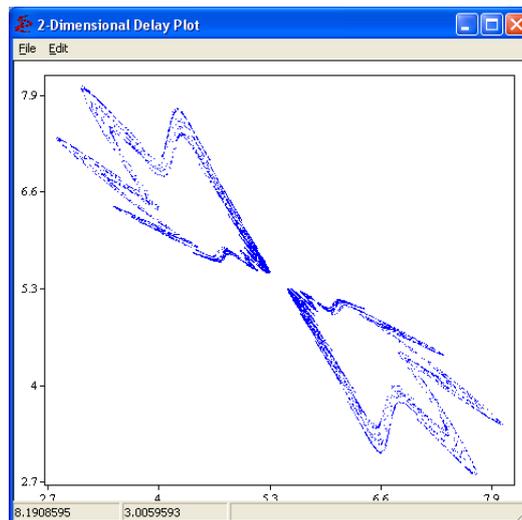
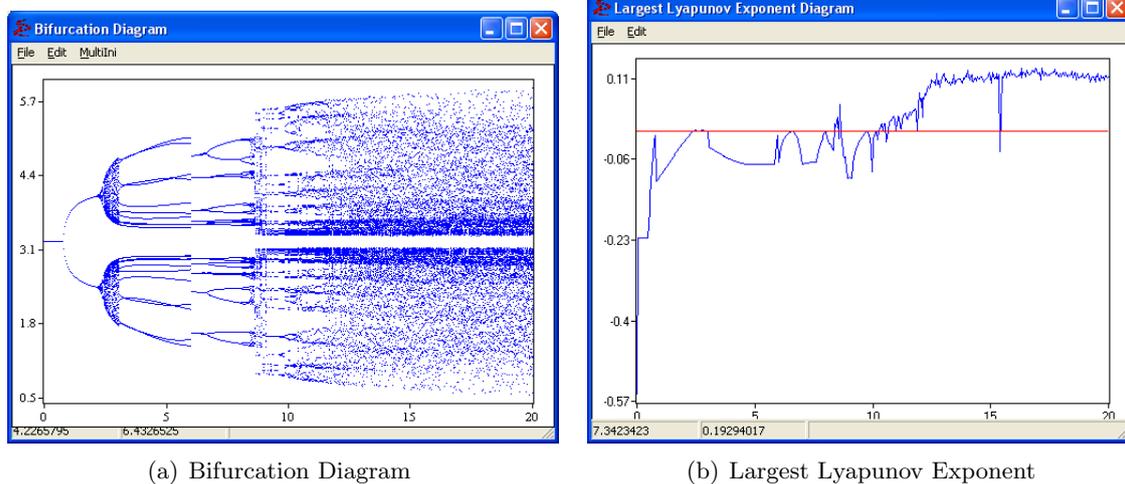


Figure 2.16: Delay Plot Output



(a) Bifurcation Diagram

(b) Largest Lyapunov Exponent

Figure 2.17: Bifurcation Diagram and Largest Lyapunov Exponent

## 2.3 Third Tutorial: Extra and Advanced Options

### 2.3.1 Approximation of the Unstable Manifold / Iterating Objects

The strange attractor in Figure 2.15 suggests that a near homoclinic tangency may also explain the complicated dynamics in the case of asynchronous updating of strategies. We can use the Iterating Objects submenu to approximate the unstable manifold of the steady state. The user can iterate circles as well as polygons over the attractor. Iterating small circles can e.g. illustrate the sensitive dependence and how quickly nearby points spread over a strange attractor. The option of Iterating a polygon and setting the number of vertices to two enables the user to iterate vectors. This option can then be used to iterate a small unstable eigenvector originating at the steady state thus approximating the unstable manifold of the steady state, as illustrated in Figure 2.19.

Call the  Iterating Objects submenu. Adjust your settings menu to resemble figure 2.18.

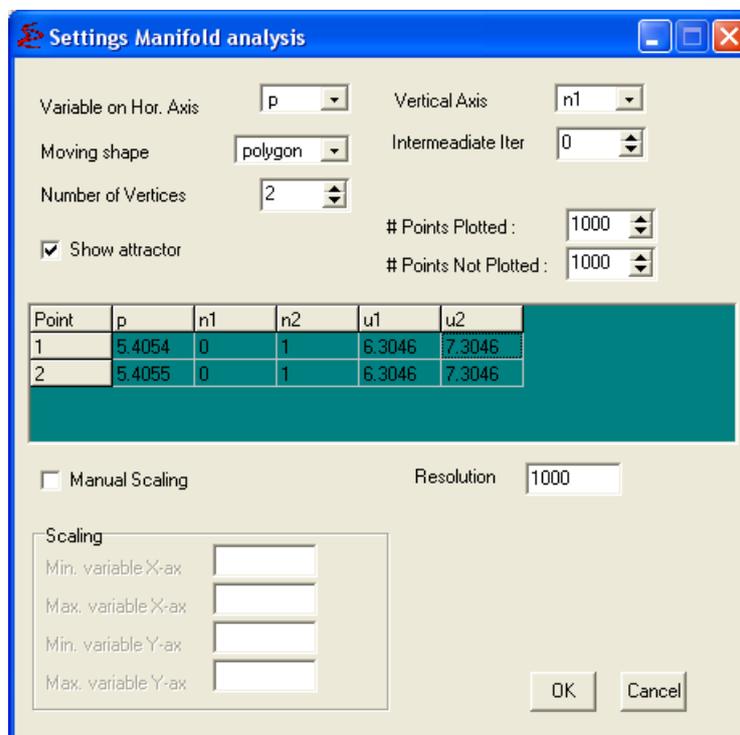


Figure 2.18: Iterating Objects Submenu Settings

Specifically, use the adjust the following settings:

**Number of Vertices** : 2

**Resolution** : 1000

For the second block of information, define the following points according to table 2.1. These are the endpoints of simple horizontal vector (the unstable direction) of length 0.0001 very close to the steady state.

Point	$p$	$n_1$	$n_2$	$u_1$	$u_2$
1	5.4054	0	1	6.3046	7.3046
2	5.4055	0	1	6.3046	7.3046

Table 2.1: Vector for Iterating Objects

After clicking the **OK** button, press the **Next Iteration Button** 22 times. Figure 2.19 shows the output.

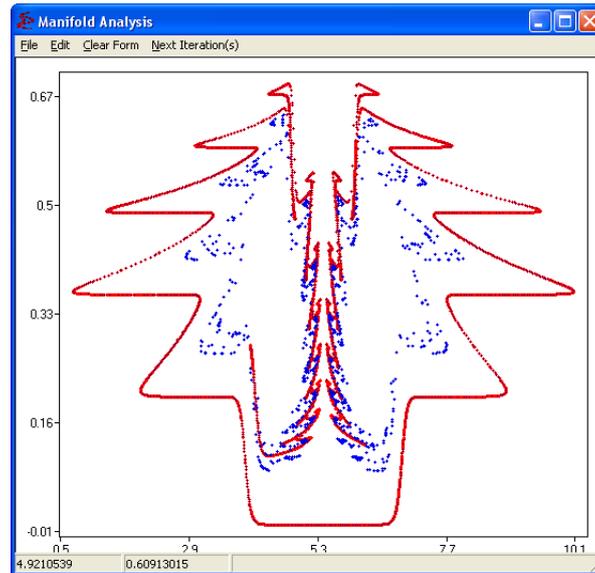


Figure 2.19: Phase Plot Output

### 2.3.2 Basins of Attraction

When there are multiple, co-existing attractors, the long run dynamical behavior depends on the initial state. The *basin of attraction* of an attractor is the set of initial conditions converging to that attractor. Using the basin of attraction menu, different colors are assigned to initial states converging to different attractors.

Figure 2.21 shows an example with two co-existing stable 4-cycles. Red points converge to one stable 4-cycle, while yellow points converge to the other (symmetrically opposite) stable 4-cycle.

In the basin of attraction window, the user can specify the number of attractors (i.e. stable cycles), the distance  $\varepsilon$  of convergence, the (two) variables for the initial state, the maximum number of transient iterations and the maximum distance of divergence (if exceeded, the point is colored as diverging to infinity). Furthermore, the user needs to specify the period  $P$  of each stable cycle and the coordinates of one point on each stable cycle. The latter can be entered either manually or with aid of the software (as will be explained below). Finally, the range of initial states and the resolution can be specified.

First adjust the parameters  $\beta = 2.74$ ;  $a = 0$ ;  $\delta = 0.8$ . Then call the  **Basins of Attraction** submenu. Adjust your settings menu to resemble figure 2.20.

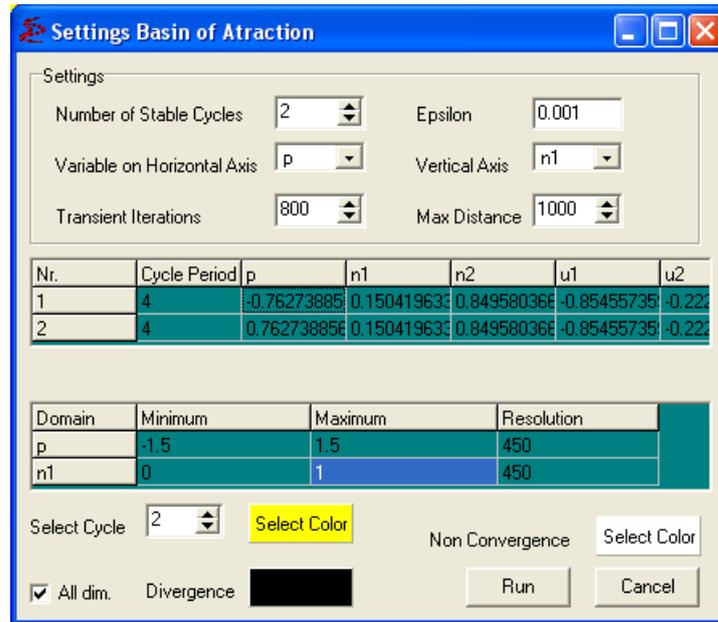


Figure 2.20: Basins of Attraction Submenu Settings

Specifically, use the following settings:

**Number of Stable Cycles** : 2

**Transient Iterations** : 800 (if this is a first trial, you can leave at 200, and change later)

For the second block of information, define the following points according to table 2.2. You can either type directly or use a special feature of the  Basins of Attraction submenu. For this reset the **Number of Stable Cycles** to 1, indicate the **Cycle Period** as 4 and **double-click** there. This will call the parameter window. Just accept and you will be taken to a Phase Plot diagram. Click the **Next Iteration Set** until you see one of the points indicated in table 2.2 at the second block of information (you can drag the Phase Plot window around so you can see both windows on screen at the same time). When this is done, close the Phase Plot and increase the **Number of Stable Cycles** to 2. This will create a “copy” of the point and you just have to change the sign of the  $p$  variable.

Nr.	Cycle Period	$p$	$n_1$	$n_2$	$u_1$	$u_2$
1	4	-0.76273885	0.150419633	0.849580366	-0.85455735	-0.222222
2	4	0.76273885	0.150419633	0.849580366	-0.85455735	-0.222222

Table 2.2: Cycle Points for Basins of Attractions

In the third block of information adjust the domains in the following way:

$p$  : -1.5 to 1.5

$n_1$  : 0 to 1

Use the following colors (in this order): (1) red; (2) yellow.

Finally check the box **All Dim.** After clicking the **Run** button, the computations start and a progress bar showing the fraction of computed points appears. With a 800 transient points, this should take around 15 minutes, depending on the computer. The user can interrupt the computations by clicking the **Stop** button. Figure 2.21 shows the output.

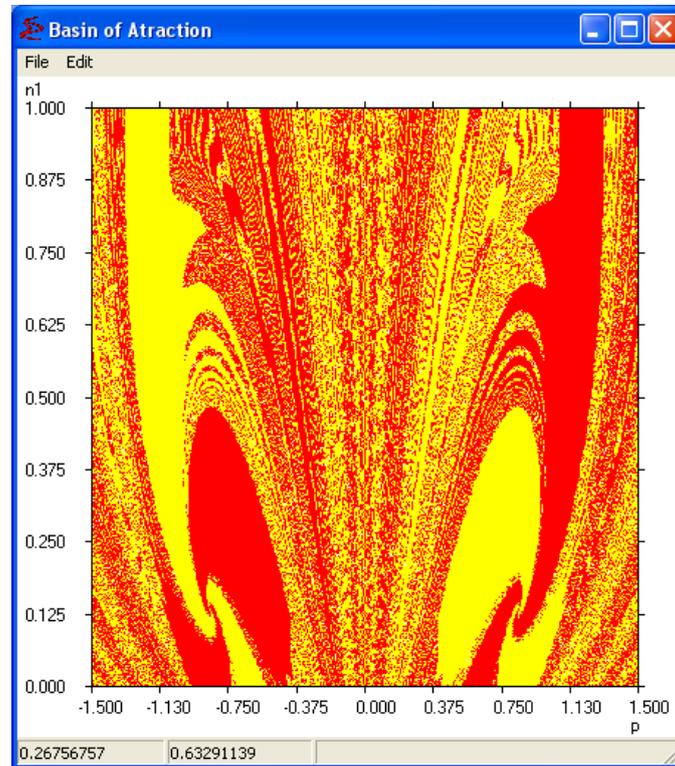


Figure 2.21: Basins of Attraction Output

### 2.3.3 Adding Noise

An important question, particularly relevant for economics, is how noise affects the dynamics of a nonlinear system. *E&F Chaos* has a built-in noise option intended for obtaining a quick impression of the effect of additive noise with various distributions on the state variables.

To illustrate this option re-load the file used for the 1-dimensional example as indicated in section 2.1.2 of this tutorial.

After selecting the submenu **Extra>Noise** the settings window will appear. Adjust the **distribution** to *normal* and the **Standard Deviation** to 0.1. Finally check the **Enable Noise Addition** box.

Follow the procedures to generate the bifurcation diagram as in Section 2.1.5 of this tutorial. Don't forget to check the parameter values ( $a = 1; b = 0.25; w = 0.5; \lambda = 4.4$ ). Figure 2.22 shows the output.

In each time step, a normally distributed random variable  $\varepsilon_t$ , with zero mean and standard deviation 0.1 has been added to the RHS of the dynamics in equation (2.1). The figure shows that the detailed structure of the bifurcation diagram disappears. However, in the presence of noise price fluctuations exhibit similar features as in the noise free case. For small values of

the parameter  $w$ ,  $0 \leq w \leq 0.3$ , price fluctuations are close to the steady state, for large values of  $w$ ,  $w > 0.8$ , prices exhibit up and down oscillations (a noisy 2-cycle), while for intermediate values of  $w$ ,  $0.3 < w < 0.7$  price fluctuations are more irregular.

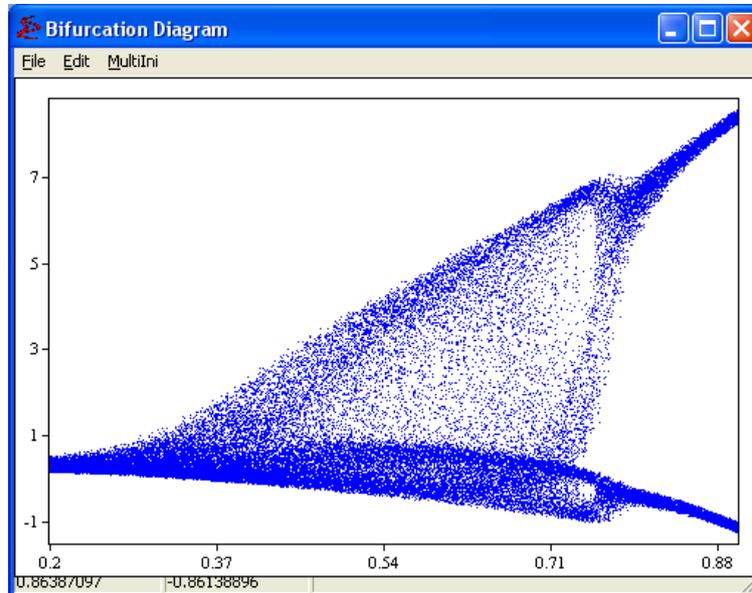


Figure 2.22: Bifurcation Diagram with noise On

### 2.3.4 Multiple Bifurcation Diagram

Another useful tool is to plot multiple Bifurcation Diagrams. For this option load the *Brock and Hommes Asset Pricing Model* with 2 types. The file can be found at **Discrete > Asset Pricing > BH model 2 types.mod**.

Initially plot the Bifurcation Diagram for  $\beta$  varying from 2 to 4. For higher precision adjust the number of **Plotted Points** to 500 and the number of **Not Plotted Points** to 1000. You should obtain an output as Figure 2.23.

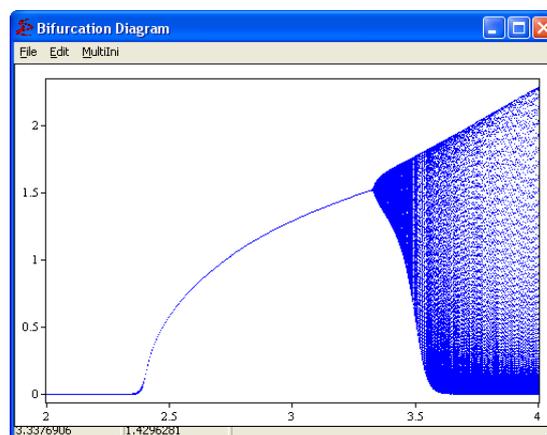


Figure 2.23: Bifurcation Diagram

You can see that close to  $\beta = 2.20$  there's a change. The steady state  $x = 0$  loses stability. We can check whether this is a pitch-fork bifurcation by plotting a diagram with negative initial values.

Add the new bifurcation diagram, using the **MultiIni** menu and choosing **Add** for the second color available. Change the initial values to  $x(0) = -0.2$  and  $y(0) = -0.1$ . This will draw a red bifurcation diagram, that is a mirror image of the first, revealing a typical pitch-fork bifurcation. Figure 2.24 shows the result.

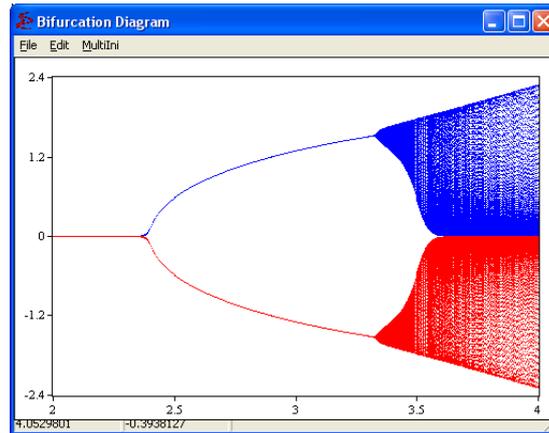


Figure 2.24: Multiple Bifurcation Diagram

### 2.3.5 Editing Code

Also as an advanced option on E&F Chaos, users can either create their own model or edit an existing one into a new model.

Start by loading the Logistic Map (**Logistic Map.mod**). By calling the  **Edit Model** submenu you should see figure 2.25.

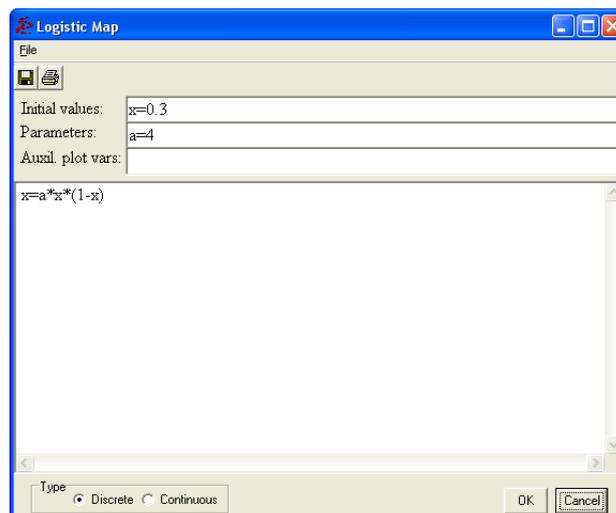


Figure 2.25: Logistic Map - Edit Model Window

The Logistic Map is defined by the equation  $x_{t+1} = ax_t(1 - x_t)$ . In the code note that the '=' sign indicates the assignment operator, meaning that the left hand side receives the value as evaluated from the right hand side. Therefore the code is simply written as `x = a*x*(1-x)`, indicating the new value of  $x$  is updated using the current one.

Suppose you want to change this into a Delayed Logistic Map. This is defined by the equation  $x_{t+1} = ax_t(1 - x_{t-1})$ . Now it is necessary to keep in memory the two last realizations of  $x$  in order to calculate the new one. As it is standard in this kind of analysis let's create an artificial variable  $y_{t+1} = x_t$ , and we have a multidimensional system with only 1 period lag. In the code you want the current  $x$  to keep the value of  $x_t$  and the current  $y$  to keep the value of  $x_{t-1}$ . In order to compute the new value of  $x$  while not losing the current observation you must write `x1=a*x*(1-y)`, indicating that the new  $x$  is updated using the current one and  $y$ , which is the previous  $x$ . After this you need to update the variables. First the previous  $x$  must be saved as the new  $y$ . Therefore write `y=x`. Finally the new  $x$  must be replaced as the one just calculated, so write `x=x1`. Since you have an extra variable,  $y$ , you need to add it to the box **Initial Values**, for example as `y=0.1`. Figure 2.26 shows the desired code.

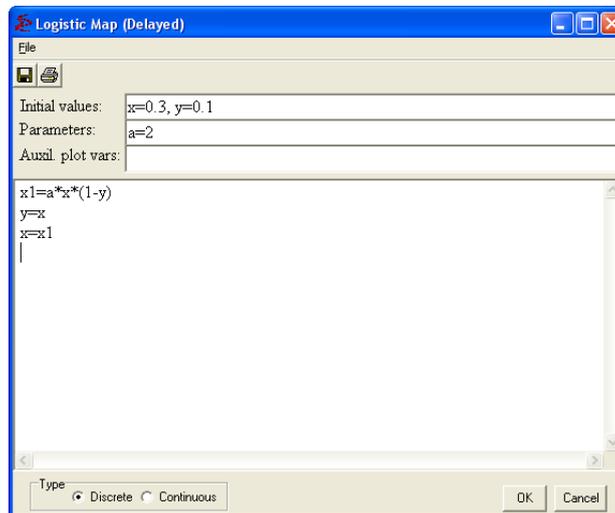


Figure 2.26: Delayed Logistic Map - Edit Model Window

Remember that the E&F Chaos do not have a "Save as..." option, therefore, if you save a model you will overwrite the current one. Avoid doing this by making a copy in advance of the file to be edited. The Delayed Logistic Map is also available at the Models Library as **Logistic Map (Delayed).mod**.



## Chapter 3

# Reference Manual

### 3.1 Foreword

Here you will find a detailed reference manual explaining all settings of the software. Each sub-item is supposed to be self-contained in a way you can simply read directly the option that interests you. No theory explanations will be included here. For a less technical description of the options and some theoretical remarks, check Chapter 2: Tutorials.

### 3.2 File

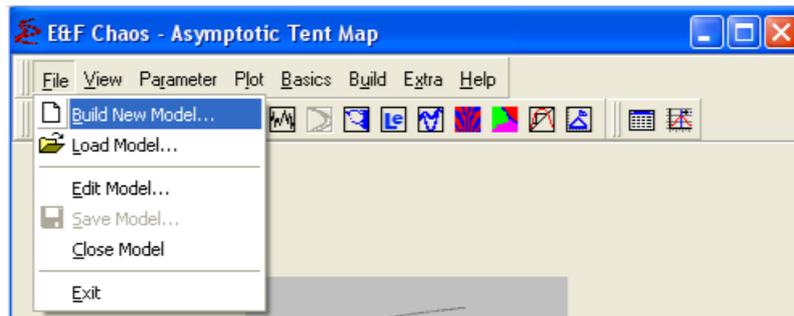


Figure 3.1: File Menu

The **File** menu can be used to control which model is in memory (being one of the preset models in the library or a user-defined model) by loading, saving and closing models, as well as by allowing the creation of a new model. As usual, it also has an option to exit the program.

#### 3.2.1 Build New Model

The  **Build New Model** submenu can be used to include a new model in the E&F Chaos program. After calling the  **Build New Model** submenu the window depicted in figure 3.2 will be shown. This window can be re-accessed at any time via the  **Edit Model** submenu. As default, the new model will be the following (as depicted in figure 3.2):

```
x1=a*x+y
```

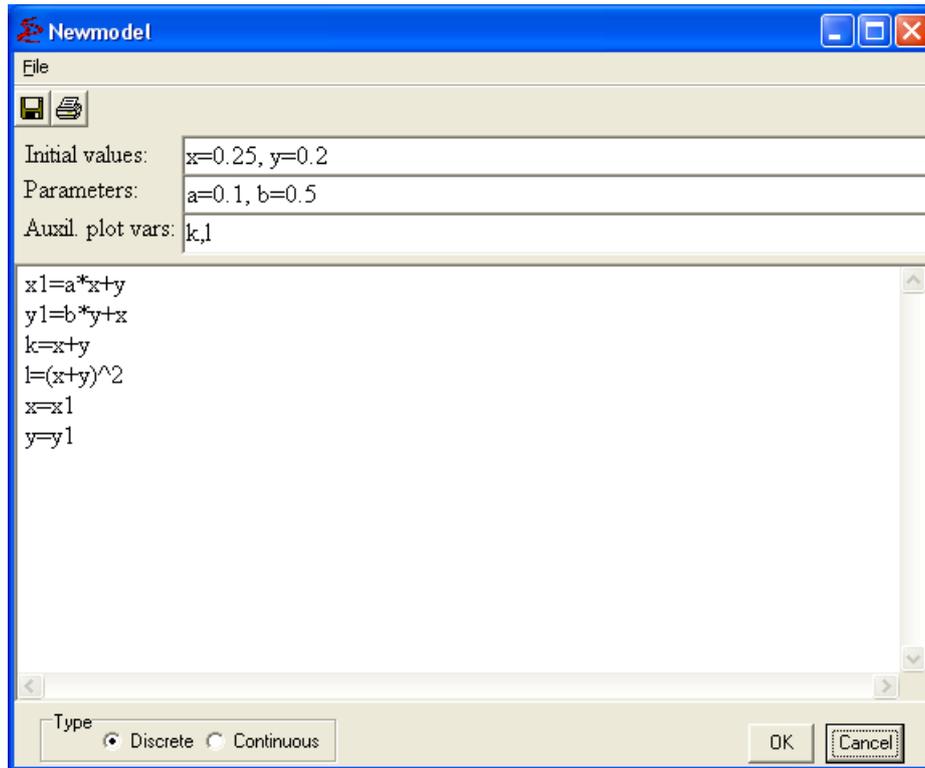


Figure 3.2: Build New Model submenu

```

y1=b*y+x
k=x+y
l=(x+y)^2
x=x1
y=y1

```

With initial values:  $x = 0.25$ ,  $y = 0.2$ ; parameters:  $a = 0.1$ ,  $b = 0.5$  and auxiliary plot variables  $k, l$ .

After building a new model, use the  **Save Model** submenu to save it on file (  **Save Model** will ask for a file name in case of a newly built model).

For more information on the model specification consult section 3.11: **Model specification: Instruction for E&F Chaos with LUA** at page 66.

**Warning:** Calling the  **Build New Model** submenu deletes the previous model from memory. Therefore to save the changes made, make sure to  **Save your model** before calling the  **Build New Model** submenu.

### 3.2.2 Load Model

A number of models, both in discrete and continuous time, have been implemented in the E&F Chaos program. These models include some of the standard models from the dynamical systems literature, such as the Logistic map, the Hénon map, and the Lorenz equations. Most of the models, however, come from economics and finance, and include standard models such

as Cobweb and Asset Pricing Models. All implemented models can be opened through the  Load Model submenu and browsing through the preset library. The library starts at the subfolder *Models* and is subdivided in *Continuous*, *Discrete* with also a pre-configured folder *My Models*. Moreover, user-defined models previously saved can be retrieved by this submenu.

**Warning:** Loading a model eliminates from memory the previous model. Therefore if you want the changes made to be saved, make sure to  Save your model before loading a new one.

### 3.2.3 Edit Model

The  Edit Model submenu can be used to make changes to the model current in memory. The window opened when the  Edit Model is called is similar to figure 3.2, however depicting the current model specification. For more information on the model specification consult section 3.11: **Model specification: Instruction for E&F Chaos with LUA.**

**Warning:** If you use  Save Model the current file will be overwritten. In case you select  Build New Model,  Load Model, Close Model or Exit, changes will be lost.

### 3.2.4 Save Model

 Save Model will save the model currently in memory. In case of a new model it will ask for a filename; in case of a previously saved model it will overwrite the current file.

**Warning:** When saving modifications on a previously saved model the file will be overwritten. Therefore if you want to keep the original file, make sure to make a copy of it before saving the changes.

### 3.2.5 Close Model

The Close Model submenu closes the current model (without saving), and thus removes the model equations from the memory.

**Warning:** When a model is closed it is not saved automatically. Therefore if you want to keep the changes made select the  Save Model before closing.

## 3.3 View

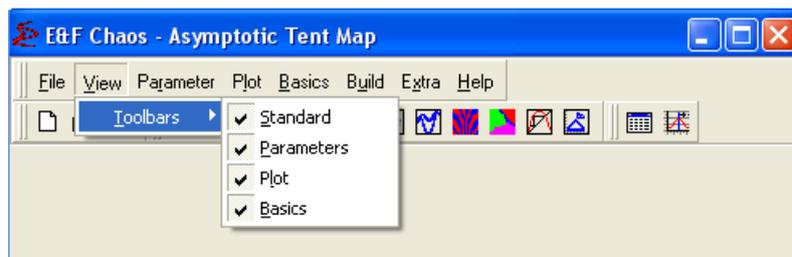


Figure 3.3: View Menu

### 3.3.1 Toolbars

The most common features of the program can also be found in the toolbars (the groups of pictograms just below the menu), providing a quicker access to frequently used options. Throughout this manual, the corresponding pictograms are placed together with the submenu name whenever this is applicable. In the **Toolbars** submenu, you can choose which toolbars should be visible and which not. By Default all are.

## 3.4 Parameter

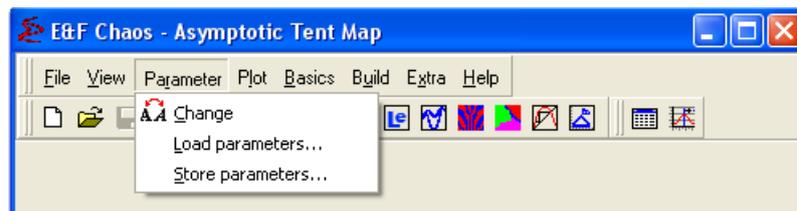


Figure 3.4: Parameter Menu

Through the **Parameter** menu it is possible to manage the current model parameters by changing the current values, loading previous parameter values or storing current values.

### 3.4.1 Change

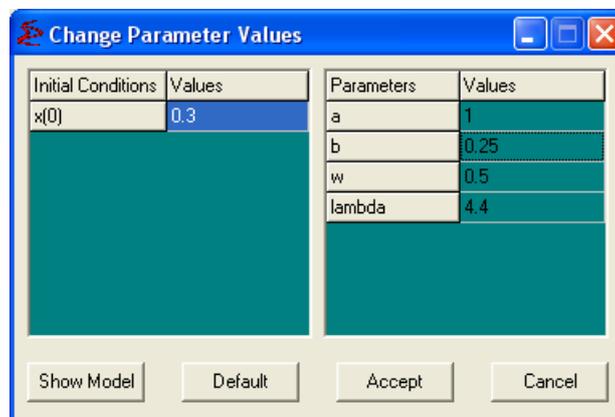


Figure 3.5: Change Parameters Window

The  **Change** submenu will call the window depicted in figure 3.5. On the left-hand side panel every variable of the model is listed and you can change the initial conditions for the numerical simulations. On the right-hand side panel the parameters are listed and can be changed. Four buttons are presented at the bottom:

**Show Model** Equivalent to calling the  **Edit Model** submenu. Check section 3.2.3 for details

**Default** Restore the parameters to the default values provided with the model specification (these will be the values loaded with the model)

**Accept** Accept the changes and close the window

**Cancel** Cancel the changes and close the window

Note that within other submenus it will also be possible to access the  **Change** submenu window.

### 3.4.2 Load Parameters

With the **Load Parameters** submenu it is possible to reload into memory a previously stored set of initial values and parameters. A previously saved file must be indicated for this purpose. The loaded initial values and parameters will overwrite the current values in memory.

**Warning:** Loading a parameter set that does not correspond to the current model in memory will lead to errors when numerical procedures are performed afterwards. Therefore make sure you load a proper file.

### 3.4.3 Store Parameters

By the **Store Parameters** submenu it is possible to save in file the current initial values and parameter values in memory. The system will ask you for a file name.

## 3.5 Plot

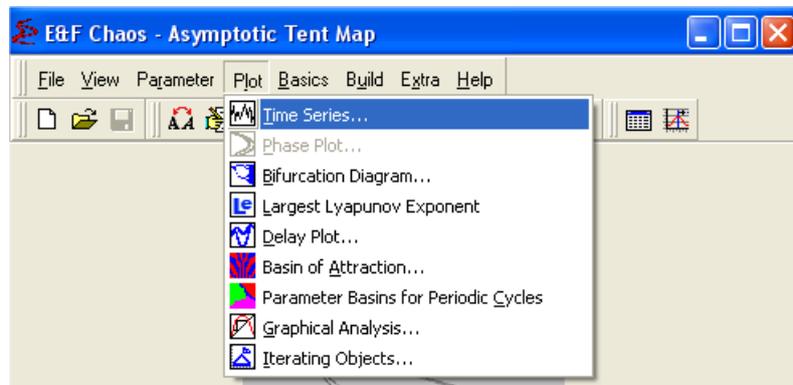


Figure 3.6: Plot Menu

The `Plot` menu is in fact the most important menu, and it will be described in more detail in this section. It contains the different submenus for numerical simulation of the model that has currently been selected by the user. In all cases, the output of the numerical simulations will be a graph, or a plot. It is possible to zoom in and out on these plots, drag them and save them in different formats. Fast access to all of the listed features is provided by the toolbars. Now we explore each of the submenus (the number of active submenus depends upon the type of model that has been selected).

### 3.5.1 Time Series

When the  `Time Series` submenu has been selected a settings window will appear, depicted in figure 3.7. Your options are:

**Plot Variable** Which variable will be analyzed

**# Points Plotted** Number of points in the intended simulation

**# Points Not Plotted** Number of transient time

**Time Step** Number of iterations between each data point. If you set, e.g. **Time Step** to 2, every 2 iterations will be reported (i.e. the second iterator function will be simulated)

Still on the settings window you can *check* as active or not the option to draw lines to **Connect Points** or to **Plot Squares** at each point. You can also activate a **Manual Scaling** that will enable the bottom part of the settings screen. When the **Manual Scaling** is active, there are four mandatory options:

**Min. variable X-ax** Time period where plotting starts

**Max. variable X-ax** Time period where plotting ends

**Min. variable Y-ax** Minimum value plotted for the variable of interest

**Min. variable Y-ax** Maximum value plotted for the variable of interest

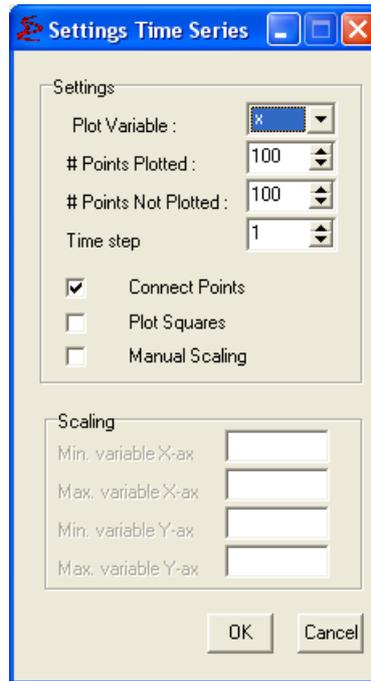


Figure 3.7: Time Series Submenu Settings

**Warning:** Be careful when using manual scaling. The *X-axis*, for example, refers to plotted time here, but not to the number of generated time points. So for example, if you define a **# Points Not Plotted** as 100, but define **Max. variable X-ax** smaller than 100, you will see nothing in your output.

After clicking the **OK** button, a Window with the time series plot will appear, depicted in figure 3.8. The **Cancel** button closes the window.

It is possible to zoom in the plot by drawing a rectangle with the mouse over the intended area with the left-button on hold. The original scaling can be re-obtained by double-clicking the left mouse button. Furthermore, you can scroll the plotted object by moving the mouse while pressing both the CTRL-button and the left mouse button.

At the top of the window there are three submenus:

**File** With the following suboptions:

**Save Data as** Allows you to save the generated series as a *.dat* file

**Save as** Allows you to save the output as a *.bmp*, *.emf* or *.wmf* file

**Create EPS** Allows you to save the output as an *.eps* file. This will trigger an extra window with self-explanatory options and an advanced menu. Details in the section 3.10: Create EPS options (Postscript Figure).

**Print** Allows you to immediately print the output. This will also trigger an extra window, with self-explanatory options.

**Close** Closes the current window

**Edit** With two options:

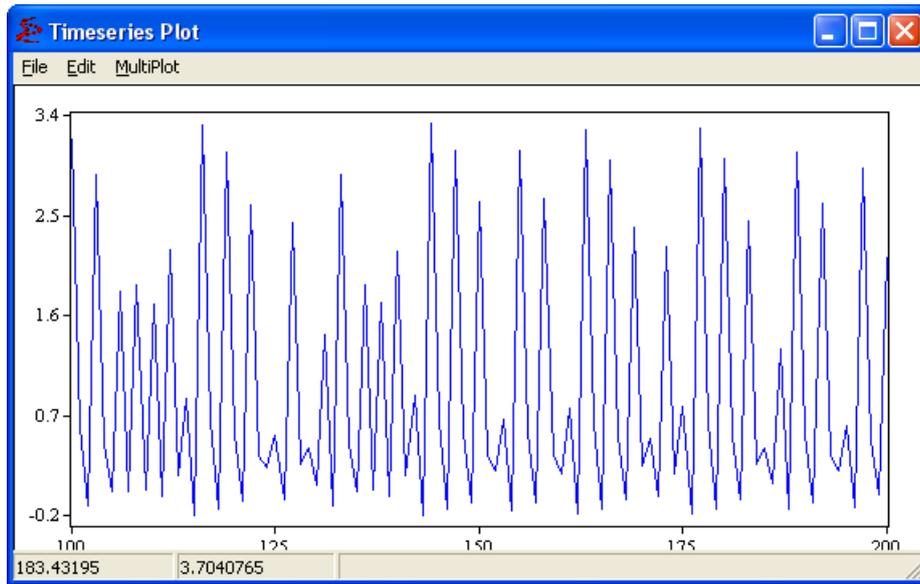


Figure 3.8: Time Series Output

**Parameters** Calls the  **Change** submenu window. Check section 3.4.1 for details.

**Settings** Returns to the window depicted in figure 3.7, allowing you to reset the simulation

**Multiplot** The **Multiplot** submenu allows you to plot different time series (up to five) over each other, as depicted in figure 3.9.

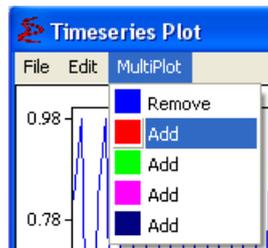


Figure 3.9: Multiplot submenu

Initially the Blue Series is the only one plotted. Via the **Multiplot** submenu you can **Add** a new series by selecting the **Add** option corresponding to a color not currently in use. Once this is done the current plotted series are frozen and a new **Settings** window will appear in which you can set the details for the new time series. The new plot will be put over the old ones. Once this is done you can do all operations (change settings and parameters) affecting only the new plot. You can always **Remove** another series by selecting the **Remove** corresponding to the color of the series you want to remove.

As depicted in figure 3.10, by right-clicking another menu will appear. The extra options in this menu are: **Change Background Color**, **Change Line Color**, **Show Grid**, all under the **Edit Graph Submenu**; **Copy to Clipboard as BMP** and **Copy to Clipboard as WMF**.

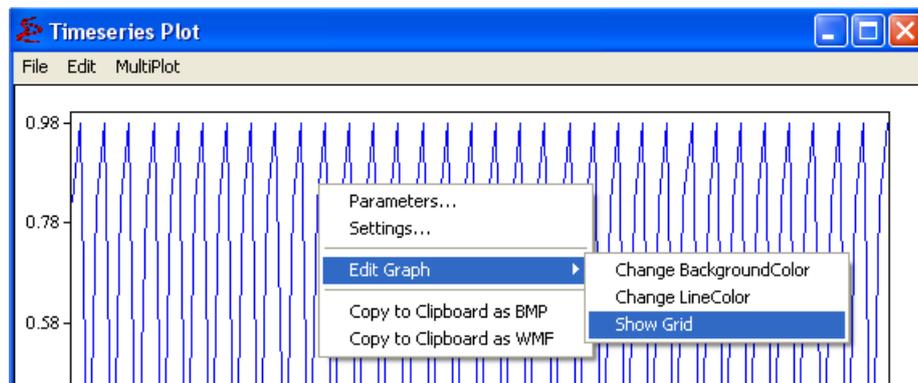


Figure 3.10: Time Series Output window, Right Click Menu

### 3.5.2 Phase Plot

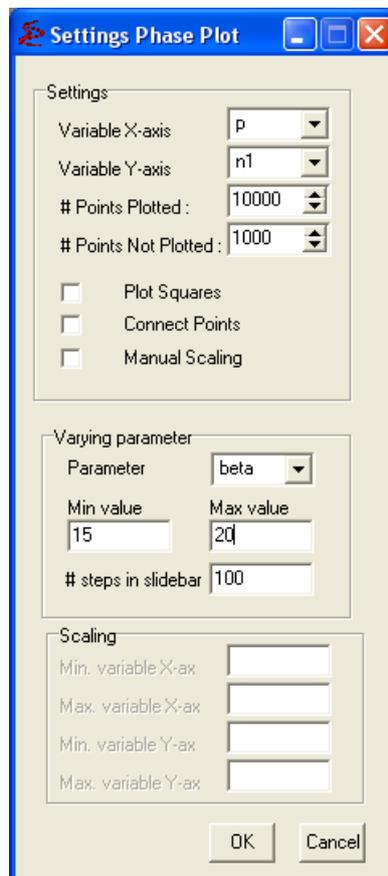


Figure 3.11: Phase Plot Submenu Settings

The  Phase Plot submenu will be available only for higher dimensional models. When the  Phase Plot submenu has been selected a settings window will appear, depicted in figure 3.11. Your options are:

**Variable X-Axis** Which variable will be plotted on the X-axis

**Variable Y-Axis** Which variable will be plotted on the Y-axis

**# Points Plotted** Number of points in the intended simulation

**# Points Not Plotted** Number of transient time

Still on the settings window you can *check* as active or not options to **Plot Squares** at each point or to draw lines to **Connect Points**. You can also activate a **Manual Scaling** that will enable the bottom part of the settings screen. When the **Manual Scaling** is active, there are four mandatory options:

**Min. variable X-ax** Minimum plotted value for the variable at the X-axis

**Max. variable X-ax** Maximum plotted value for the variable at the X-axis

**Min. variable Y-ax** Minimum value plotted for the variable at the Y-axis

**Min. variable Y-ax** Maximum value plotted for the variable at the Y-axis

Additionally, under the *Varying Parameter* are you can choose:

**Parameter** Which parameter will be possible to vary

**Min value** Minimum value for the varying parameter

**Max value** Maximum value for the varying parameter

**# Steps in sidebar** In how many steps to move from minimum to maximum.

After clicking the **OK** button, a Window with the Phase Plot will appear, depicted in figure 3.12. The **Cancel** button closes the window.

It is possible to zoom in the plot by drawing a rectangle with the mouse over the intended area with the left-button on hold. The original scaling can be re-obtained by double-clicking the left mouse button. Furthermore, you can scroll the plotted object by moving the mouse while pressing both the CTRL-button and the left mouse button.

By using the sidebar on the bottom-right part of the screen the user can change one of the parameters, as specified in the Varying Parameter part of the Settings screen.

At the top of the window there are four submenus:

**File** With the following suboptions:

**Save Data as** Allows you to save the generated series as a *.dat* file

**Save as** Allows you to save the output as a *.bmp*, *.emf* or *.wmf* file

**Create EPS** Allows you to save the output as an *.eps* file. This will trigger an extra window with self-explanatory options and an advanced menu. Details in the section 3.10: Create EPS options (Postscript Figure).

**Print** Allows you to immediately print the output. This will also trigger an extra window, with self-explanatory options.

**Warning:** When the Phase Plot is printed or saved, care should be taken that the **# Points Plotted** is not too high (Default 10000), because otherwise it will contain too many pixels, which will make printing of the phase plot very slow and/or the saved file very large.

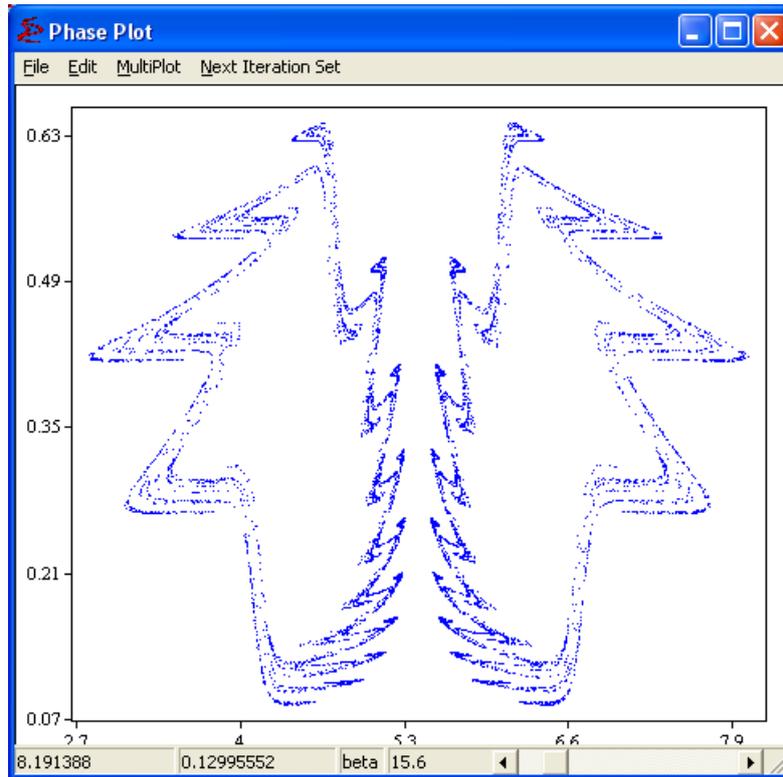


Figure 3.12: Phase Plot Output

**Close** Closes the current window

**Edit** With two options:

**Parameters** Calls the  Change submenu window. Check section 3.4.1 for details.

**Settings** Returns to the window depicted in figure 3.11, allowing you to reset the simulation

**Multiplot** The Multiplot submenu allows you to plot different phase plots (up to five) over each other, as depicted in figure 3.13.

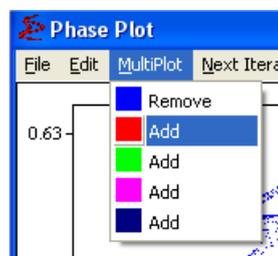


Figure 3.13: Multiplot submenu

Initially the Blue Series is the only one plotted. Via the **Multiplot** submenu you can **Add** a new plot by selecting the **Add** option corresponding to a color not currently in

use. Once this is done the current plot is frozen and a new **Parameter** window will appear in which you can set the details for the new diagram. The new diagram will be put over the old ones. Once this is done you can do all operations (change settings and parameters) affecting only the new plot. You can always **Remove** another diagram by selecting the **Remove** corresponding to the color of the series you want to remove.

**Next Iteration Set** Runs one more set of iteration plotting **# Points Plotted** more points.

As depicted in figure 3.14, by right-clicking another menu will appear. The extra options in this menu are: **Change Background Color**, **Change Line Color**, **Show Grid**, all under the **Edit Graph** Submenu; **Copy to Clipboard as BMP** and **Copy to Clipboard as WMF**.

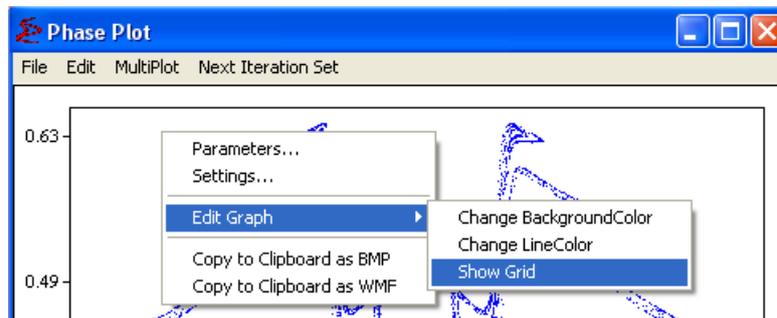


Figure 3.14: Phase Plot Output window, Right Click Menu

### 3.5.3 Bifurcation Diagram

When the  **Bifurcation Diagram** submenu has been selected a settings window will appear, depicted in figure 3.15. Your options are:

**Parameter** Which parameter will be varied during the analysis

**Min Value** Smallest Value of the range of the Parameter

**Max Value** Largest Value of the range of the Parameter

**Plot Variable** Which variable will plotted for each value of the parameter analyzed

**# Points Plotted** Number of points in the intended simulation

**# Points Not Plotted** Number of transient time

**Warning:** For high values of **# Plotted Points** and/or **# Not Plotted Points** computation of the bifurcation diagram may be time consuming. In this case, a “Program is computing” message with a progress bar will appear, showing the percentage of computation time that has elapsed.

Still on the settings window you can *check* as active or not options to **Initialize on Previous Value** or to **Compute from Right to Left**. If the first option is active, then for each new parameter value in the diagram, the last plotted point will be chosen as the Initial Condition. In that way an attractor can be “followed”, to see how it bifurcates as the parameter increases. If this option is switched off, then for each new parameter value in the diagram the same Initial

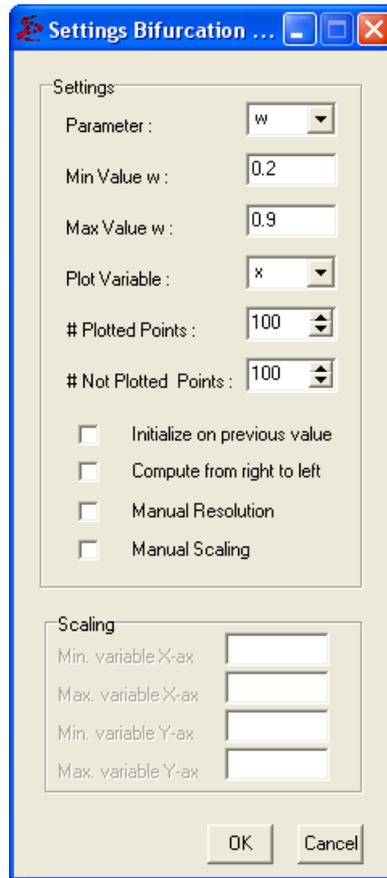


Figure 3.15: Bifurcation Diagram Submenu Settings

Conditions will be used (which can be changed by the user in the Edit/Parameters submenu of the Bifurcation Diagram Window). The second check box option forces the diagram to be calculated from maximum value to the minimum instead of the (default) order of the minimum to the maximum.

Moreover you can also activate a **Manual Resolution** (the number of steps that the program will go from the minimum to the maximum) that will enable a small box with the current value.

Finally, you can also activate a **Manual Scaling** that will enable the bottom part of the settings screen. When the **Manual Scaling** is active, there are four mandatory options:

**Min. variable X-ax** Minimum plotted value for the parameter

**Max. variable X-ax** Maximum plotted value for the parameter

**Min. variable Y-ax** Minimum value plotted for the outcome variable

**Max. variable Y-ax** Maximum value plotted for the outcome variable

**Warning:** The manual scale will affect only the Y-ax, since the X-ax will be determined by the settings of the parameter. Still, you need to define values for it, otherwise the program will not proceed.

After clicking the **OK** button, a Window with the bifurcation diagram will appear, depicted in figure 3.16. The **Cancel** button closes the window.

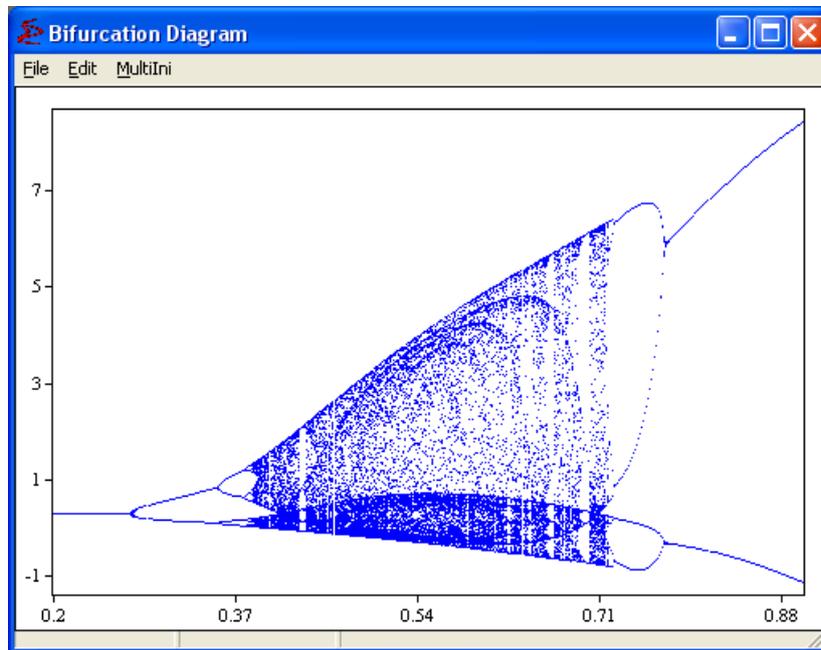


Figure 3.16: Bifurcation Diagram Output

It is possible to zoom in the plot by drawing a rectangle with the mouse over the intended area with the left-button on hold. The original scaling can be re-obtained by double-clicking the left mouse button. Furthermore, you can scroll the plotted object by moving the mouse while pressing both the CTRL-button and the left mouse button.

At the top of the window there are three submenus:

**File** With the following suboptions:

**Save Data as** Allows you to save the generated series as a *.dat* file

**Save as** Allows you to save the output as a *.bmp*, *.emf* or *.wmf* file

**Create EPS** Allows you to save the output as an *.eps* file. This will trigger an extra window with self-explanatory options and an advanced menu. Details in the section 3.10: Create EPS options (Postscript Figure).

**Print** Allows you to immediately print the output. This will also trigger an extra window, with self-explanatory options.

**Warning:** Care should be taken when printing a bifurcation diagram, or saving it as a file. The **# Plotted Points** should not be too high (Default 100), because the picture may contain too many pixels making printing very slow and/or the saved file very large.

**Close** Closes the current window

**Edit** With two options:

**Parameters** Calls the  Change submenu window. Check section 3.4.1 for details.

**Settings** Returns to the window depicted in figure 3.15, allowing you to reset the simulation

**MultiIni** The **MultiIni** submenu allows you to plot different bifurcation diagrams (up to five) over each other, as depicted in figure 3.17.

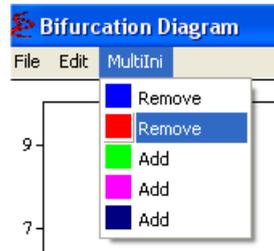


Figure 3.17: MultiIni submenu

Initially the Blue Series is the only one plotted. Via the **MultiIni** submenu you can **Add** a new diagram by selecting the **Add** option corresponding to a color not currently in use. Once this is done the current plotted diagram is frozen and a new **Parameter** window will appear in which you can set the details for the new diagram. The new diagram will be put over the old ones. Once this is done you can do all operations (change settings and parameters) affecting only the new plot. You can always **Remove** another diagram by selecting the **Remove** corresponding to the color of the series you want to remove.

As depicted in figure 3.18, by right-clicking another menu will appear. The extra options in this menu are: **Change Background Color**, **Change Line Color**, **Show Grid**, all under the **Edit Graph** Submenu; **Copy to Clipboard as BMP** and **Copy to Clipboard as WMF**.

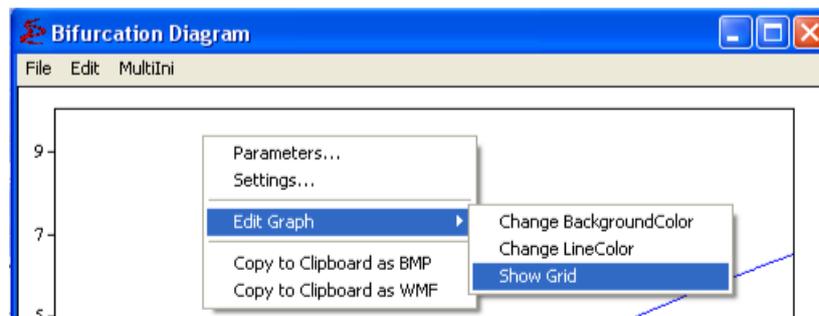


Figure 3.18: Bifurcation Diagram Output window, Right Click Menu

### 3.5.4 Largest Lyapunov Exponent

When the  Largest Lyapunov Exponent submenu has been selected a settings window will appear, depicted in figure 3.19. Your options are:

**Parameter** Which parameter will be varied during the analysis

**Min Value** Initial Value of the range of the Parameter

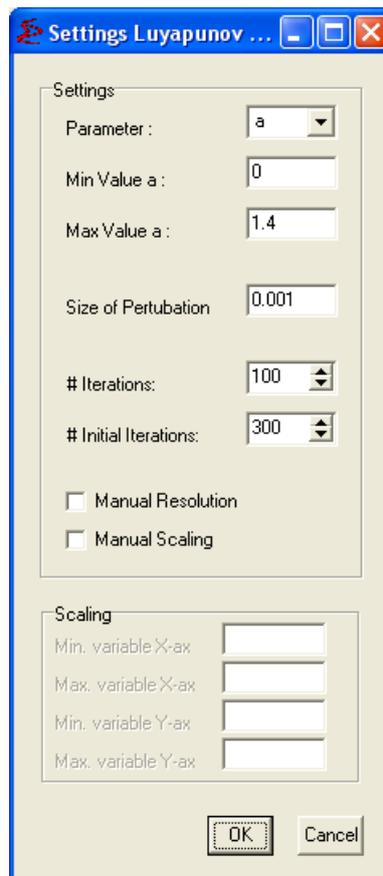


Figure 3.19: Largest Lyapunov Exponent Submenu Settings

**Max Value** Final Value of the range of the Parameter

**Size of Perturbation** Size of the disturbance on the computation

**# Iterations** Number of iterations in the simulation

**# Initial Iterations** Number of transient time

You can also activate a **Manual Resolution** (the number of steps that the program will go from the minimum to the maximum) that will enable a small box with the current value.

Finally, you can also activate a **Manual Scaling** that will enable the bottom part of the settings screen. When the **Manual Scaling** is active, there are four mandatory options:

**Min. variable X-ax** Minimum plotted value for the parameter

**Max. variable X-ax** Maximum plotted value for the parameter

**Min. variable Y-ax** Minimum value plotted for the outcome variable

**Max. variable Y-ax** Maximum value plotted for the outcome variable

**Warning:** The manual scale will affect only the Y-ax, since the X-ax will be determined by the settings of the parameter. Still, you need to define values for it, otherwise the program will not proceed.

After clicking the **OK** button, a Window with the Largest Lyapunov diagram will appear, depicted in figure 3.20. The **Cancel** button closes the window.

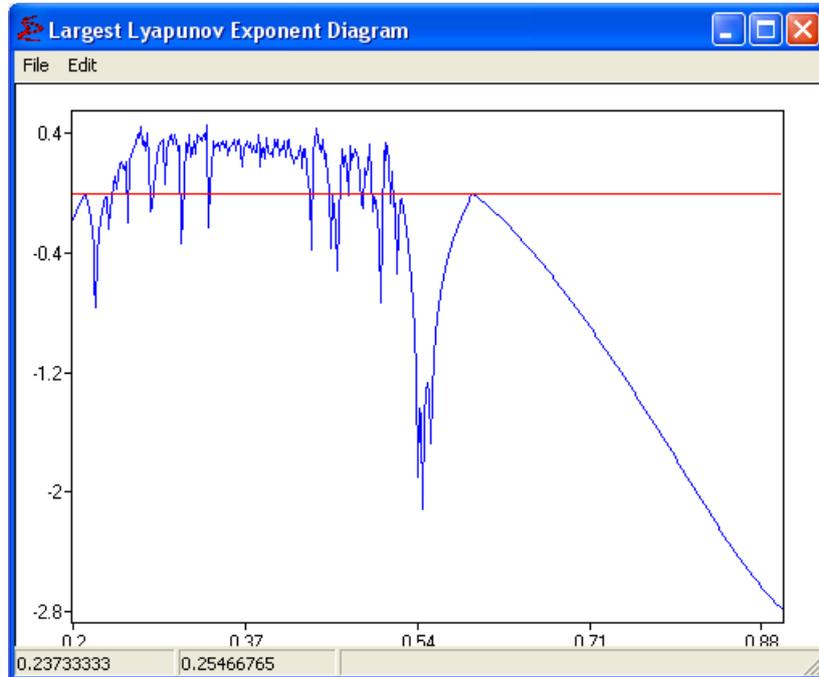


Figure 3.20: Largest Lyapunov Exponent Output

It is possible to zoom in the plot by drawing a rectangle with the mouse over the intended area with the left-button on hold. The original scaling can be re-obtained by double-clicking the left mouse button. Furthermore, you can scroll the plotted object by moving the mouse while pressing both the CTRL-button and the left mouse button.

At the top of the window there are two submenus:

**File** With the following suboptions:

**Save Data as** Allows you to save the generated series as a *.dat* file

**Save as** Allows you to save the output as a *.bmp*, *.emf* or *.wmf* file

**Create EPS** Allows you to save the output as an *.eps* file. This will trigger an extra window with self-explanatory options and an advanced menu. Details in the section 3.10: Create EPS options (Postscript Figure).

**Print** Allows you to immediately print the output. This will also trigger an extra window, with self-explanatory options.

**Close** Closes the current window

**Edit** With two options:

**Parameters** Calls the  Change submenu window. Check section 3.4.1 for details.

**Settings** Returns to the window depicted in figure 3.19, allowing you to reset the simulation

As depicted in figure 3.21, by right-clicking another menu will appear. The extra options in this menu are: **Change Background Color**, **Change Line Color**, **Show Grid**, all under the **Edit Graph** Submenu; **Copy to Clipboard as BMP** and **Copy to Clipboard as WMF**.

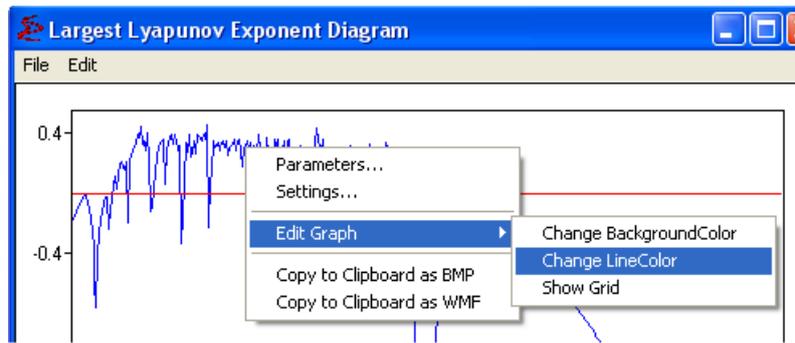


Figure 3.21: Largest Lyapunov Exponent Output window, Right Click Menu

### 3.5.5 Delay Plot

The structure of the  **Delay Plot** is very similar to that of the  **Phase Plot** submenu. This submenu enables the user to make delay plots of  $x_t$  versus  $x_{t-d}$ , where  $d$  is the Delay Time, with default  $d = 1$ .

When the  **Delay Plot** submenu has been selected a settings window will appear, depicted in figure 3.22. Your options are:

**Variable** Which variable will be plotted

**Delay Time** Which will be the delay of the variable (the lagged variable will be plotted on the X-axis)

**# Points Plotted** Number of points in the intended simulation

**# Points Not Plotted** Number of transient time

Still on the settings window you can *check* as active or not options to **Plot Squares** at each point or to draw lines to **Connect Points**. You can also activate a **Manual Scaling** that will enable the bottom part of the settings screen. When the **Manual Scaling** is active, there are four mandatory options:

**Min. variable X-ax** Minimum plotted value for the variable on the X-axis

**Max. variable X-ax** Maximum plotted value for the variable on the X-axis

**Min. variable Y-ax** Minimum value plotted for the variable on the Y-axis

**Max. variable Y-ax** Maximum value plotted for the variable on the Y-axis

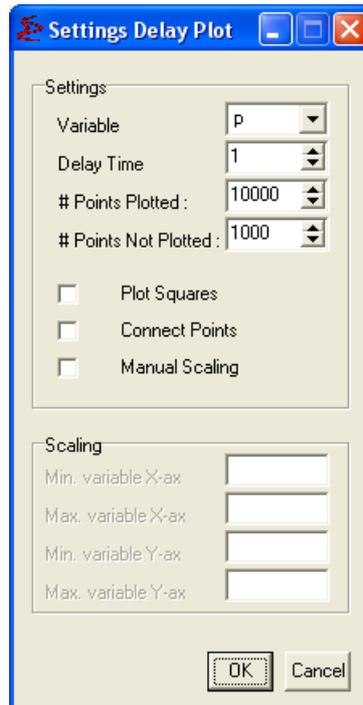


Figure 3.22: Delay Plot Submenu Settings

After clicking the **OK** button, a Window with the Phase Plot will appear, depicted in figure 3.23. The **Cancel** button closes the window.

It is possible to zoom in the plot by drawing a rectangle with the mouse over the intended area with the left-button on hold. The original scaling can be re-obtained by double-clicking the left mouse button. Furthermore, you can scroll the plotted object by moving the mouse while pressing both the CTRL-button and the left mouse button.

At the top of the window there are two submenus:

**File** With the following suboptions:

**Save Data as** Allows you to save the generated series as a *.dat* file

**Save as** Allows you to save the output as a *.bmp*, *.emf* or *.wmf* file

**Create EPS** Allows you to save the output as an *.eps* file. This will trigger an extra window with self-explanatory options and an advanced menu. Details in the section 3.10: Create EPS options (Postscript Figure).

**Print** Allows you to immediately print the output. This will also trigger an extra window, with self-explanatory options.

**Warning:** When the Phase Plot is printed or saved, care should be taken that the **# Points Plotted** is not too high (Default 10000), because otherwise it will contain too many pixels, which will make printing of the phase plot very slow and/or the saved file very large.

**Close** Closes the current window

**Edit** With two options:

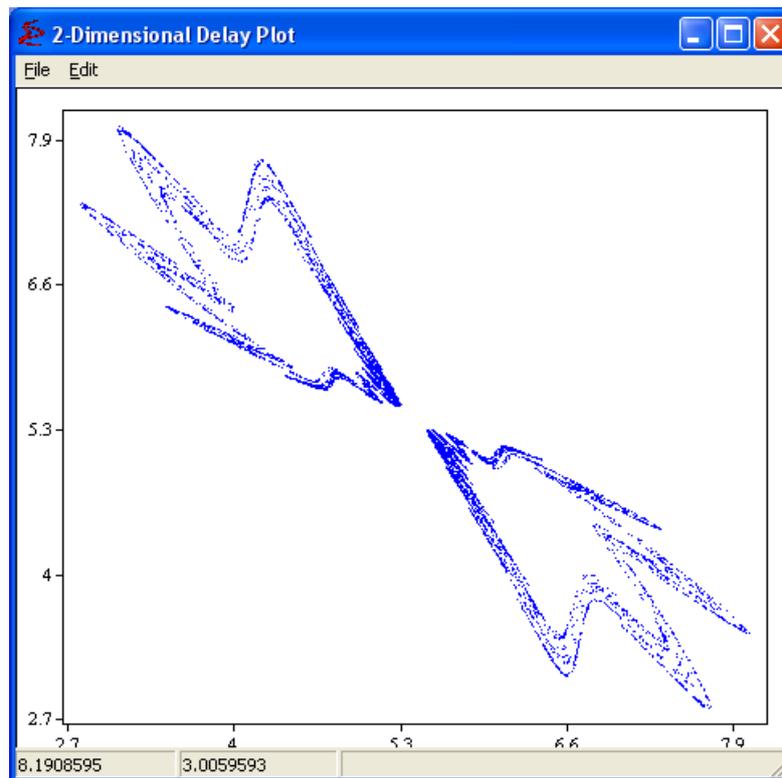


Figure 3.23: Delay Plot Output

**Parameters** Calls the  Change submenu window. Check section 3.4.1 for details.

**Settings** Returns to the window depicted in figure 3.22, allowing you to reset the simulation

As depicted in figure 3.24, by right-clicking another menu will appear. The extra options in this menu are: Change Background Color, Change Line Color, Show Grid, all under the Edit Graph Submenu; Copy to Clipboard as BMP and Copy to Clipboard as WMF.

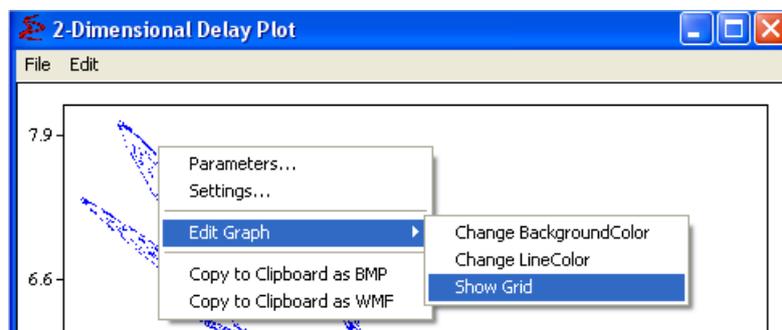


Figure 3.24: Delay Plot Output window, Right Click Menu

### 3.5.6 Basin of Attraction

This is an advanced submenu which should be used with care. It enables the user to plot the basins of attraction of different co-existing stable cycles, that is, combinations of initial values that converge to the specified stable cycles.

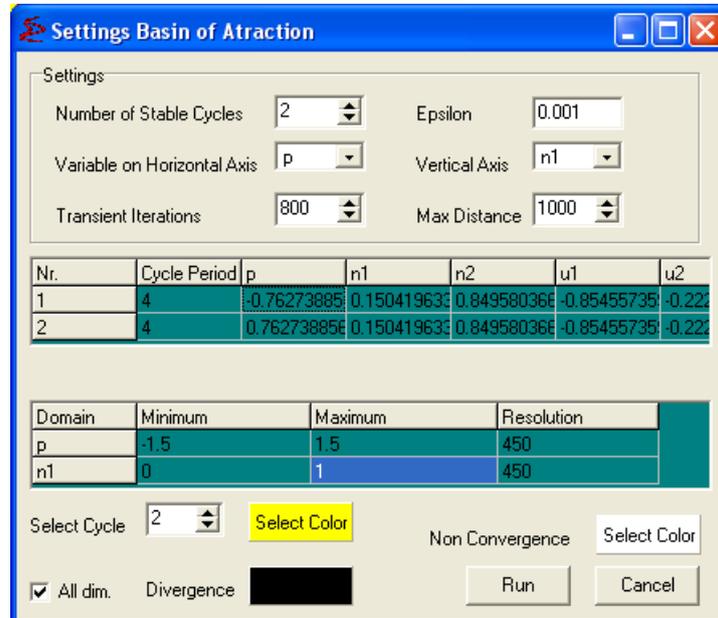


Figure 3.25: Basins of Attraction Submenu Settings

When the  Basins of Attraction submenu has been selected a settings window will appear, depicted in figure 3.25. Your options are, for *Settings*:

**Number of Stable Cycles** How many different Stable Cycles you want to include in the numerical analysis

**Epsilon** Value to test convergence distance

**Variable on Hor. Axis** Which variable will be plotted on the X-axis

**Vertical Axis** Which variable will be plotted on the Y-axis

**Transient Iterations** Number of transient time

**Max Distance** Distance after which the point is classified as divergence

The second block is the most important and relevant for this menu. It asks you to specify which are the stable cycles. For each stable cycle to be tested you need to specify its order (**Cycle Period**) and then the coordinates for all variables of one of the points in the stable cycle. These coordinates can be either typed directly (and you can get then, for example, from the  Spreadsheet menu - section 3.6.1) or you can use the aid of the  Phase Plot: by double-clicking on variable a menu with the parameters will be opened and after

clicking **OK** the Phase Plot output will be called. After a few iterations, when you close the menu the coordinates will be automatically saved.

The third block of information concerns the range of the variables. For each you need to specify the range (**minimum** and **maximum**) and the **resolution** (the number of steps that the program will go from the minimum to the maximum).

**Warning:** For high values of **Transient Times** and/or **Resolution** computation may be time consuming. In a first run, the Resolution should not be set too high (e.g. 100), Epsilon not too small (e.g. 0.01) and the Maximum Distance not too high (e.g. 1000). A “Program is computing” message with a progress bar will appear, showing the percentage of computation time that has elapsed.

The final block of information is mostly concerned with the colors. At the **Select Cycle** button you first need to change the stable cycle number (according to the second block of information) and you can change the color by clicking on the **Select Color** button. You can do a similar choice of color for the **Non-Convergence** and **Divergence**.

Lastly, you can check the **All dim.** option. If this option is checked convergence is checked regarding all variables in the system. If not, it is checked using only the two variables on the plot.

After clicking the **Run** button, a Window with the diagram will appear, depicted in figure 3.26, after computation is completed. The **Cancel** button closes the window.

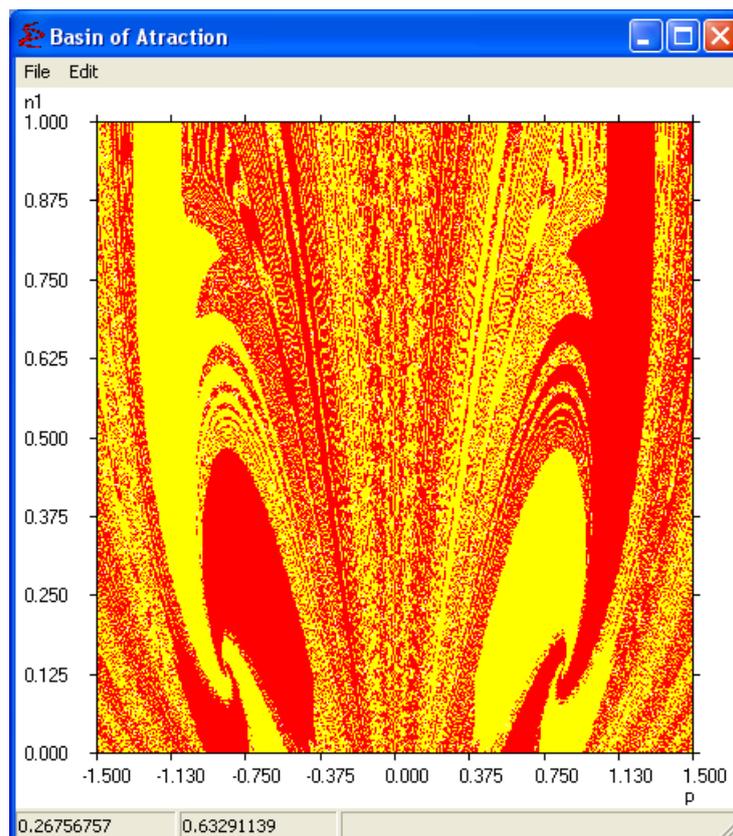


Figure 3.26: Basins of Atraction Output

At the top of the window there are two submenus:

**File** With the following suboptions:

**Save Data as** Allows you to save the generated series as a *.dat* file

**Save as** Allows you to save the output as a *.bmp*, *.emf* or *.wmf* file

**Create EPS** Allows you to save the output as an *.eps* file. This will trigger an extra window with self-explanatory options and an advanced menu. Details in the section 3.10: Create EPS options (Postscript Figure).

**Print** Allows you to immediately print the output. This will also trigger an extra window, with self-explanatory options.

**Close** Closes the current window

**Edit** With two options:

**Parameters** Calls the  **Change** submenu window. Check section 3.4.1 for details.

**Settings** Returns to the window depicted in figure 3.25, allowing you to reset the simulation

### 3.5.7 Parameter Basins for Periodic Cycles

This is another advanced submenu which should be used with care. It enables the user to plot Parameter Basins of stable Periodic Cycles, that is, regions in the parameter space converging to stable cycles of a particular period. Each color corresponds to a given period.

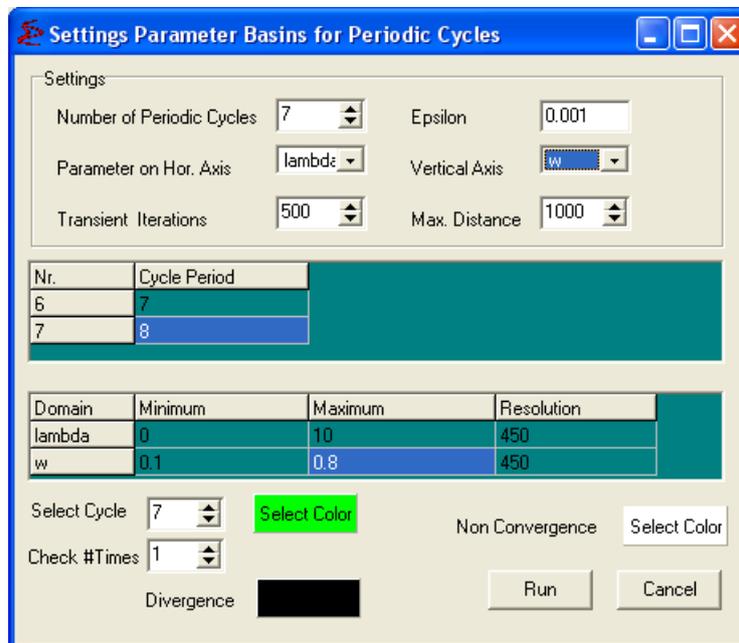


Figure 3.27: Parameter Basins for Periodic Cycles Submenu Settings

When the  **Parameter Basins for Periodic Cycles** submenu has been selected a settings window will appear, depicted in figure 3.27. Your options are, for *Settings*:

**Number of Periodic Cycles** How many different Periodic Cycles you want to test for

**Epsilon** Value to test convergence distance

**Parameter on Hor. Axis** Which parameter will be plotted on the X-axis

**Vertical Axis** Which parameter will be plotted on the Y-axis

**Transient Iterations** Number of transient time

**Max Distance** Distance after which the point is classified as divergence

The second block of information asks you to define which are the periods for each cycle. As default, the usual numbering will be followed, but you can change this (for example, you can define the cycle nr. 2 as period of order 3).

The third block of information concerns the range of the parameters. For each you need to specify the range (**minimum** and **maximum**) and the **resolution** (the number of steps that the program will go from the minimum to the maximum).

**Warning:** For high values of **Transient Times** and/or **Resolution** computation may be time consuming. A “Program is computing” message with a progress bar will appear, showing the percentage of computation time that has elapsed.

The final block of information is mostly concerned with the colors. At the **Select Cycle** button you first need to change the cycle number (according to the second block of information) and you can change the color by clicking on the **Select Color** button. You can do a similar choice of color for the **Non-Convergence** and **Divergence**.

Lastly, there is the **Check # Times** button. This option specifies how many times the convergence to a periodic cycle is verified. If it is 1 convergence is indicated after reaching the same point after one cycle period. If it is more, the same point must be reached after the specified number of cycle period times .

After clicking the **Run** button, a Window with the diagram will appear, depicted in figure 3.28, after computation is completed. The **Cancel** button closes the window.

At the top of the window there are two submenus:

**File** With the following suboptions:

**Save Data as** Allows you to save the generated series as a *.dat* file

**Save as** Allows you to save the output as a *.bmp*, *.emf* or *.wmf* file

**Create EPS** Allows you to save the output as an *.eps* file. This will trigger an extra window with self-explanatory options and an advanced menu. Details in the section 3.10: Create EPS options (Postscript Figure).

**Print** Allows you to immediately print the output. This will also trigger an extra window, with self-explanatory options.

**Close** Closes the current window

**Edit** With two options:

**Parameters** Calls the  **Change** submenu window. Check section 3.4.1 for details.

**Settings** Returns to the window depicted in figure 3.27, allowing you to reset the simulation

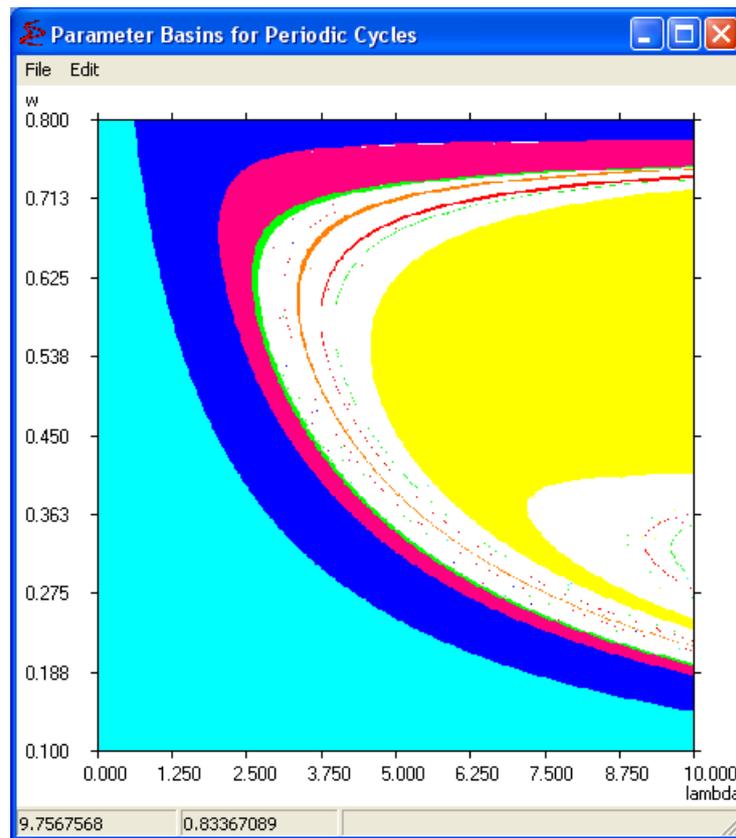


Figure 3.28: Parameter Basin for Periodic Cycles Output

### 3.5.8 Graphical Analysis

The  **Graphical Analysis** submenu will be available only for 1-D models.

When the  **Graphical Analysis** submenu has been selected a settings window will appear, depicted in figure 3.29. Your options are:

**Max Value** Maximum value shown for the Variable of Interest

**Min Value** Minimum value shown for the Variable of Interest

**Iterated Map** Number of iterations between each data point. If you set, e.g. **Time Step** as 2, every 2 iterations will be reported (i.e. the second iterator function will be simulated)

**Initial Value** Initial value for the simulations

**Warning:** Note that the **Initial Value** for the simulations is used instead of the  $x_0$  set in the *Parameters*.

You can also activate a **Manual Scaling** that will enable the bottom part of the settings screen. However for this menu this will be ineffective since the scaling will be defined by the **Max Value** and **Min Value** defined above.

After clicking the **OK** button, a Window for the Graphical analysis will appear, depicted in figure 3.30. The **Cancel** button closes the window.

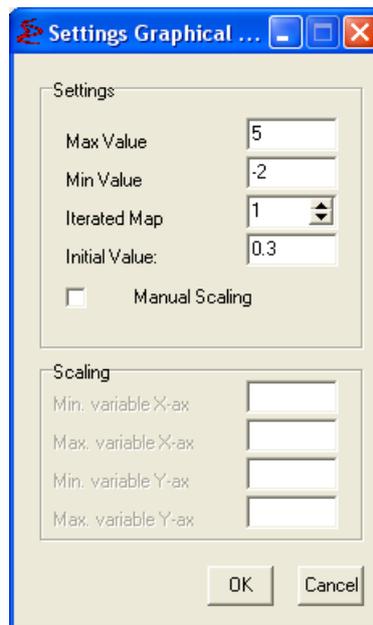


Figure 3.29: Graphical Analysis Submenu Settings

In the output window the curve in Red indicates the iterator plotted, the dotted line the identity line and the blue lines indicate the iteration process.

It is possible to zoom in the plot by drawing a rectangle with the mouse over the intended area with the left-button on hold. The original scaling can be re-obtained by double-clicking the left mouse button. Furthermore, you can scroll the plotted object by moving the mouse while pressing both the CTRL-button and the left mouse button.

At the top of the window there are three submenus:

**File** With the following suboptions:

**Save Data as** Allows you to save the generated series as a *.dat* file

**Save as** Allows you to save the output as a *.bmp*, *.emf* or *.wmf* file

**Create EPS** Allows you to save the output as an *.eps* file. This will trigger an extra window with self-explanatory options and an advanced menu. Details in the section 3.10: Create EPS options (Postscript Figure).

**Print** Allows you to immediately print the output. This will also trigger an extra window, with self-explanatory options.

**Close** Closes the current window

**Edit** With three options:

**Parameters** Calls the  **Change** submenu window. Check section 3.4.1 for details.

**Settings** Returns to the window depicted in figure 3.29, allowing you to reset the simulation

**# Time Steps** Defines how many iterations are carried for every click of **Next Iteration** button

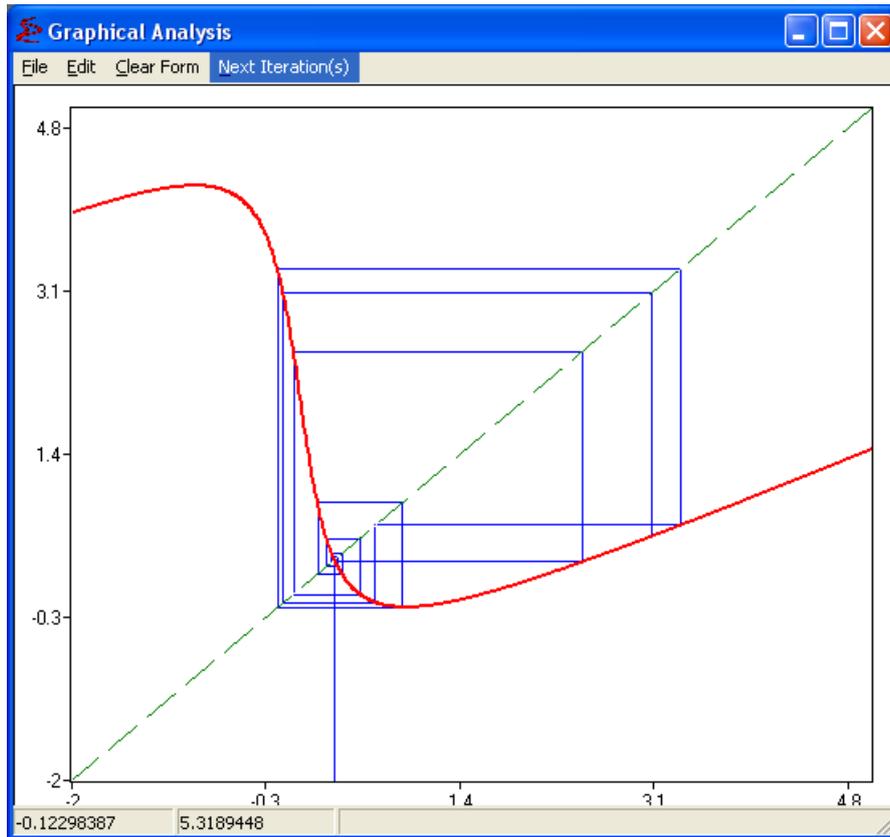


Figure 3.30: Graphical Analysis Iterations

**Clear Form** Clean the blue lines that indicates the simulated iterations

**Next Iteration(s)** Runs new iteration(s) with steps defined in the **# Time Steps** under the **Edit** menu.

As depicted in figure 3.31, by right-clicking another menu will appear. The extra options in this menu are: **Change Background Color**, **Change Line Color**, **Show Grid**, all under the **Edit Graph** Submenu; **Copy to Clipboard as BMP** and **Copy to Clipboard as WMF**.

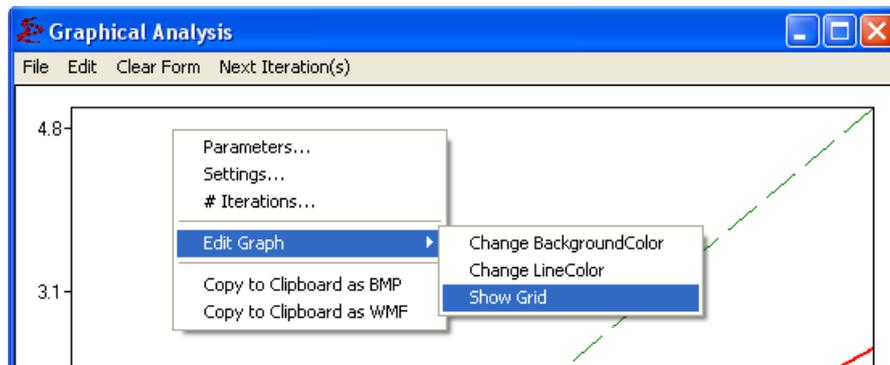


Figure 3.31: Graphical Analysis Output window, Right Click Menu

### 3.5.9 Iterating Objects

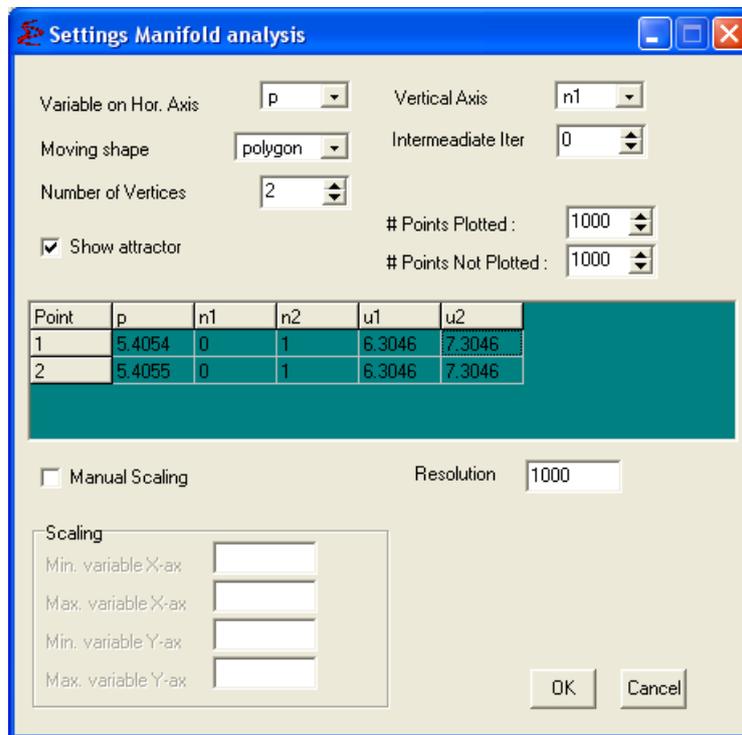


Figure 3.32: Iterating Objects Submenu Settings

When the  **Iterating Objects** submenu has been selected a settings window will appear, depicted in figure 3.32. Your options are:

**Variable on Hor. Axis** Which variable will be plotted on the X-axis

**Vertical Axis** Which variable will be plotted on the Y-axis

**Moving Shape** Which shape will be iterated: *polygon* or *circle*. Different options will be shown for each case:

**Polygon:**

**Number of Vertices** How many vertices will be on the Polygon

**Circle:**

**Radius of the circle** Radius for the iterating circle

**Intermediate Iter** Number of intermediate iterations for which the points are not plotted

**Show Attractor** If the output window will show the attractor or not. If this is on, you can set two extra options:

**# Points Plotted** Number of points plotted for the attractor

**# Points Not Plotted** Number of transient points for the attractor

The second block of information asks for coordinates. If you selected *polygon* you have to specify the coordinates for each vertex; if you select *circle* you have to specify the coordinates of the circle center.

You can also activate a **Manual Scaling** that will enable the bottom part of the settings screen. When the **Manual Scaling** is active, there are four mandatory options:

**Min. variable X-ax** Minimum plotted value for the variable on the X-axis

**Max. variable X-ax** Maximum plotted value for the variable on the X-axis

**Min. variable Y-ax** Minimum value plotted for the variable on the Y-axis

**Max. variable Y-ax** Maximum value plotted for the variable on the Y-axis

Finally, you can set the **Resolution** (the number of point taken equally into the polygon or around the circle).

After clicking the **OK** button, a Window with the diagram will appear, depicted in figure 3.33. The **Cancel** button closes the window.

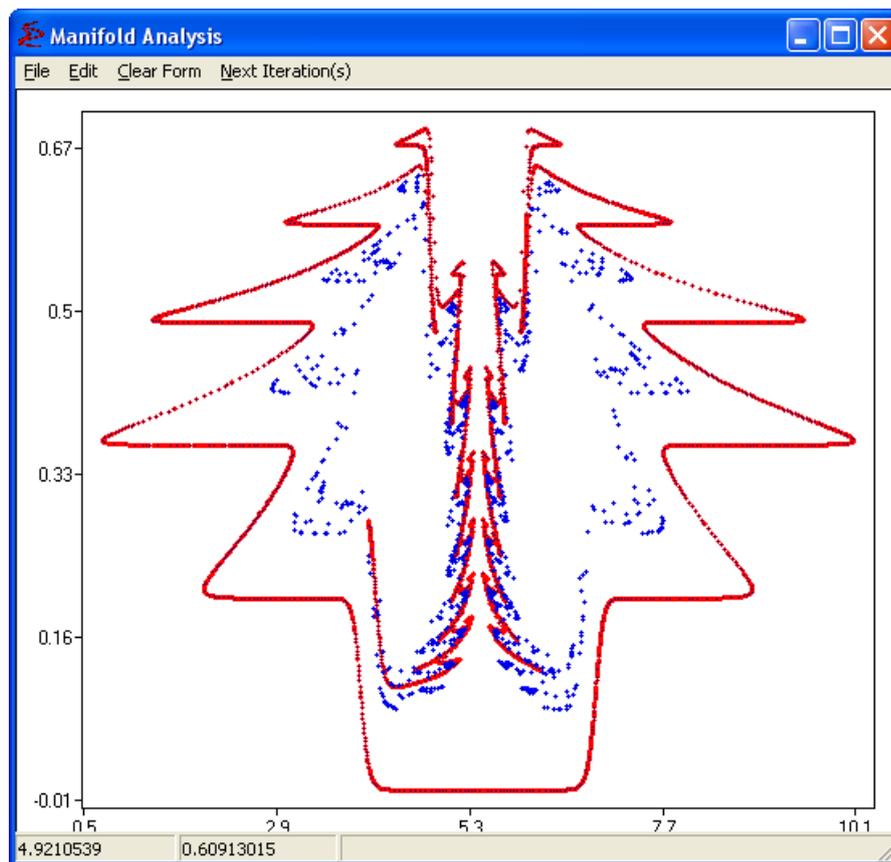


Figure 3.33: Iterating Objects Output

At the top of the window there are four submenus:

**File** With the following suboptions:

**Save Data as** Allows you to save the generated series as a *.dat* file

**Save as** Allows you to save the output as a *.bmp*, *.emf* or *.wmf* file

**Create EPS** Allows you to save the output as an *.eps* file. This will trigger an extra window with self-explanatory options and an advanced menu. Details in the section 3.10: Create EPS options (Postscript Figure).

**Print** Allows you to immediately print the output. This will also trigger an extra window, with self-explanatory options.

**Close** Closes the current window

**Edit** With three options:

**Parameters** Calls the  **Change** submenu window. Check section 3.4.1 for details.

**Settings** Returns to the window depicted in figure 3.32, allowing you to reset the simulation

**# Time Steps** Defines how many iterations are carried for every click of **Next Iteration** button

**Clear Form** Resets the simulation

**Next Iteration(s)** Runs new iteration(s) with steps defined in the **# Time Steps** under the **Edit** menu.

As depicted in figure 3.34, by right-clicking another menu will appear. The extra options in this menu are: **Change Background Color**, **Change Line Color**, **Show Grid**, all under the **Edit Graph** Submenu; **Copy to Clipboard as BMP** and **Copy to Clipboard as WMF**.

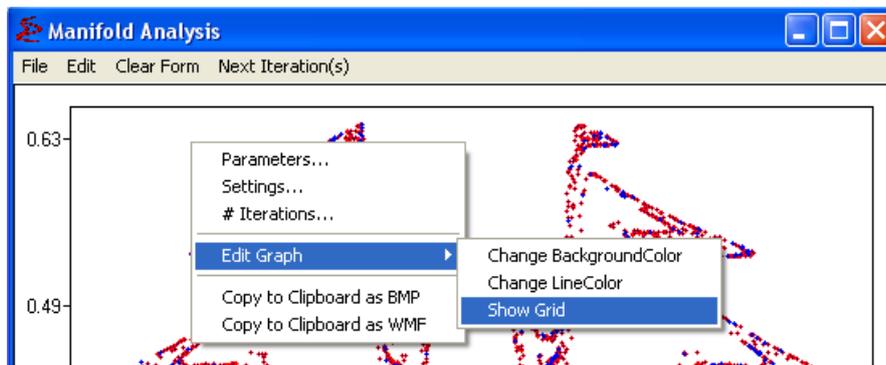


Figure 3.34: Iterating Objects Output window, Right Click Menu

## 3.6 Basics

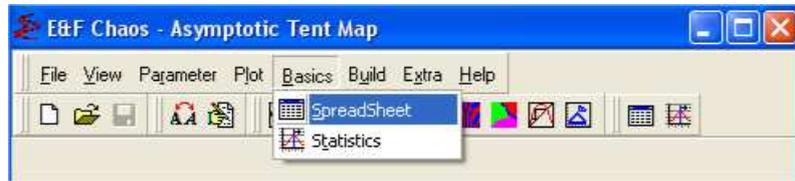


Figure 3.35: Basics Menu

As the menu name suggest, the **Basics** menu offers some basic simulations and statistics on the model, namely generating time series in a spreadsheet and calculating some basic statistics of the generated time series.

### 3.6.1 SpreadSheet

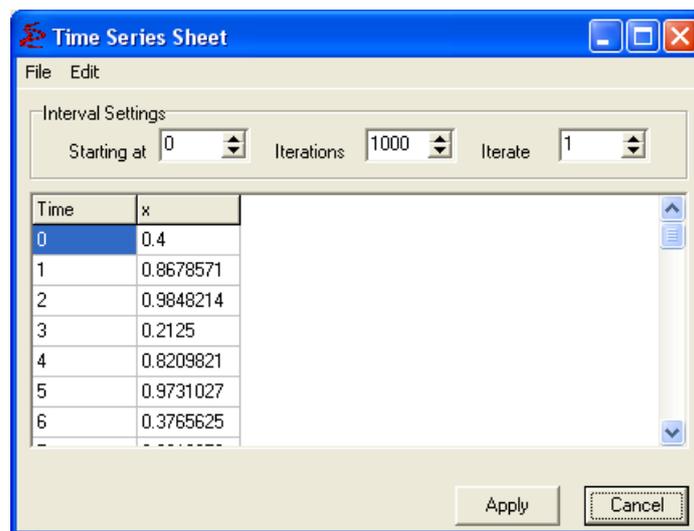


Figure 3.36: Spreadsheet submenu

The  **Spreadsheet** submenu yields the possibility to view the numerical values of the time series of all variables on the screen. After calling the **Spreadsheet** submenu the window depicted in figure 3.36 will appear. At the top section of the window screen you will find the settings options for the time series:

**Starting at** First period reported. Equivalent to the number of initial points you want to discard

**Iterations** Number of data points you want generated, including the initial one

**Iterate** Number of iterations between each data point. If you set, e.g. **Iterate** as 2, every 2<sup>nd</sup> iteration will be reported (i.e. the second iterator of the function will be simulated)

The section in the middle will show the results of the last simulation. The button **Apply** runs the simulation, and the button **Cancel** closes the window. At the top of the window there are two submenus:

**File** With two suboptions:

**Save as** Allows you to save the output as a *.txt* file

**Close** Similar to pressing the **Cancel** button: closes the window

**Edit** With a unique option, **Parameters**. Calls the  **Change** submenu window. Check section 3.4.1 for details.

### 3.6.2 Statistics

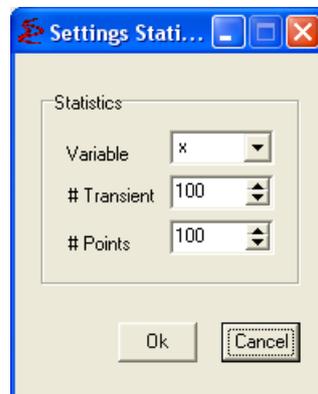


Figure 3.37: Statistics submenu

The  **Statistics** submenu yields some basic statistics of a time series. When called it brings the window depicted in figure 3.37. The choices are:

**Variable** Which variable will be analyzed

**# Transient** Number of transient (not analyzed) points

**# Points** Number of points in the analyzed sample

The button **Ok** runs the simulation and generate the output window with statistics, depicted in figure 3.38, and the button **Cancel** closes the window.

At the output window there are a few menu options:

**File** With two suboptions:

**Save as** Allows you to save the output as a *bitmap* file

**Close** Similar to pressing the **Cancel** button: closes the window

**Edit** With two suboptions:

**Parameters** Calls the  **Change** submenu window. Check section 3.4.1 for details.

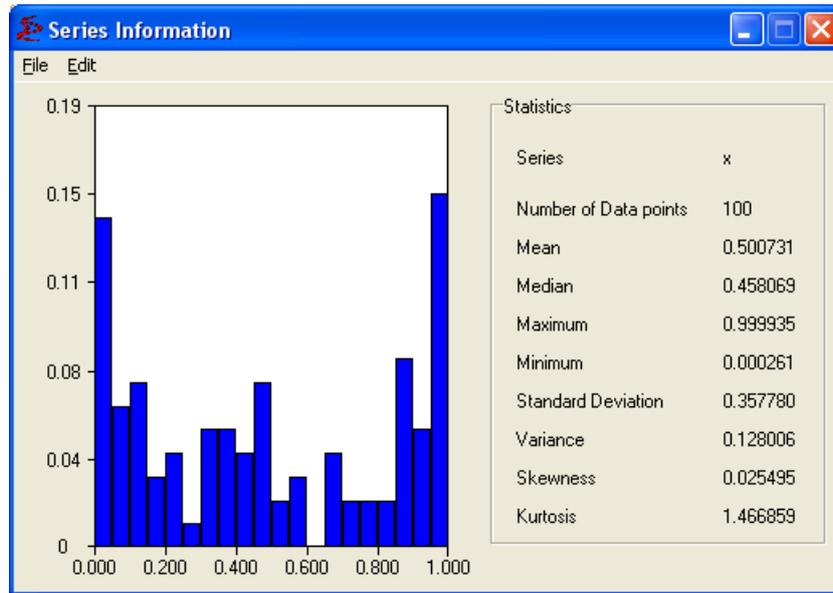


Figure 3.38: Statistics Output window

**Settings** Returns to the initial window (depicted in figure 3.37), allowing you to reset the simulation

Also, at the output window, by right-clicking with the mouse, there is an extra option Copy to Clipboard as BMP, as indicated in figure 3.39.

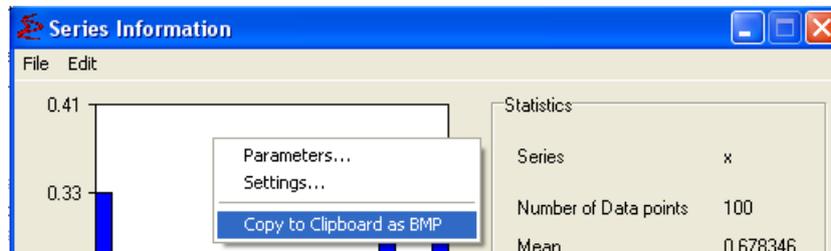


Figure 3.39: Statistics Output window, Right Click Menu

## 3.7 Build

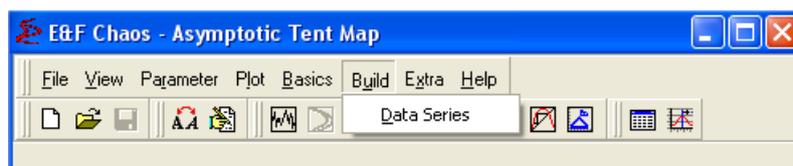


Figure 3.40: Build Menu

The **Build** menu, with a unique submenu, also gives the possibility to save a time series on a file in ASCII format.

### 3.7.1 Data Series

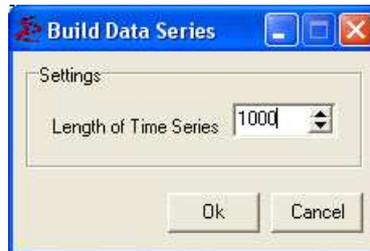


Figure 3.41: Data Series submenu

When called, the submenu **Data Series** calls the window depicted in figure 3.41. There the only option is the length of the time series (starting at period 1). When you press the button **Ok**, the system will ask for a file name. The button **Cancel** closes the window.

For more possibilities on the generated time series, use the **Basics** menu. For more details consult section 3.6.

## 3.8 Extra



Figure 3.42: Extra Menu

The Extra menu currently has only one, but important, submenu: **Noise**. This yields the possibility of adding noise to the numerical simulations.

### 3.8.1 Noise

After selecting the submenu **Noise** a window as depicted in figure 3.43 will appear. You can add noise from three **distributions**: *uniform*, *normal* or *double exponential*. One can also modify the **Standard Deviation** of the noise (mean will always be 0) and select the **variable** to which the noise should be added.

The next options allow the setting of the number of **Increments between noise addition** (number of time steps after which a noise term is added) and the **Probability of noise addition**.

Next it is possible to set the **Random Seed**. As the last option on the screen, it is possible to fix the **Random Seed for Each Run** (a random seed will be selected for the first run and then kept constant).

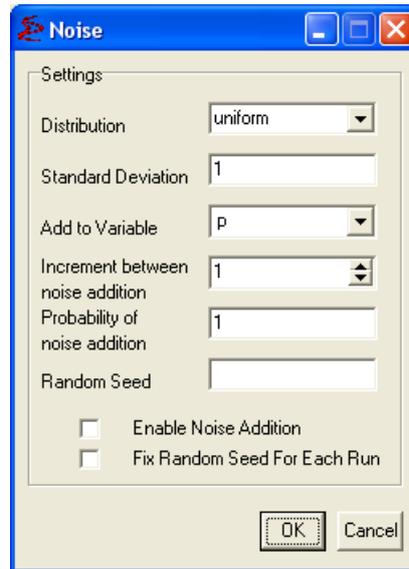


Figure 3.43: Noise submenu

In order to add noise to the model, the **Enable Noise Addition** option box has to be activated. Once the noise has been activated, dynamic noise will be added to the variable selected in the Noise Window. Noise will be added in all subsequent simulations, until the **Enable Noise Addition** box is switched off again.

In the bottom left of the main window a **Noise Off** or **Noise On** message will indicate the current noise state.

## 3.9 Help

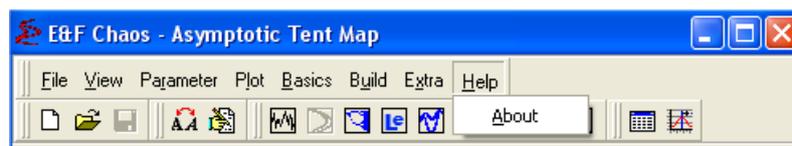


Figure 3.44: Help Menu

The **Help** submenu just gives information about the program.

### 3.9.1 About

Gives the basic information about the E&F Chaos program, depicted in figure 3.45.

## 3.10 Create EPS options (Postscript Figure)

All output generated with one of the Plot submenus can be saved as an *.eps* file. Although the options available may slightly vary from submenu to submenu, a typical window is depicted in figure 3.46.

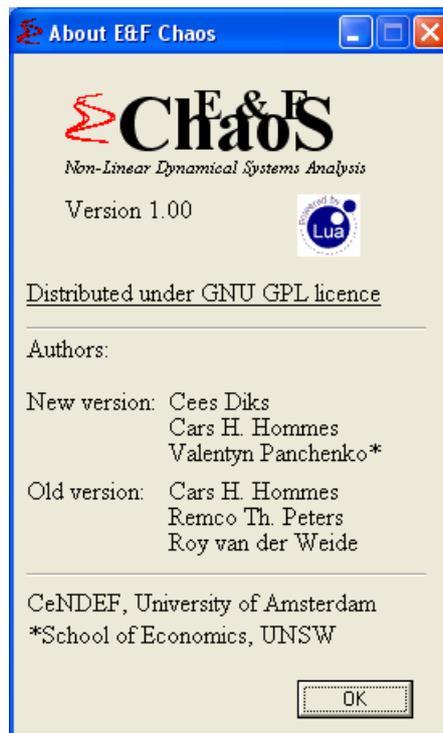


Figure 3.45: About submenu

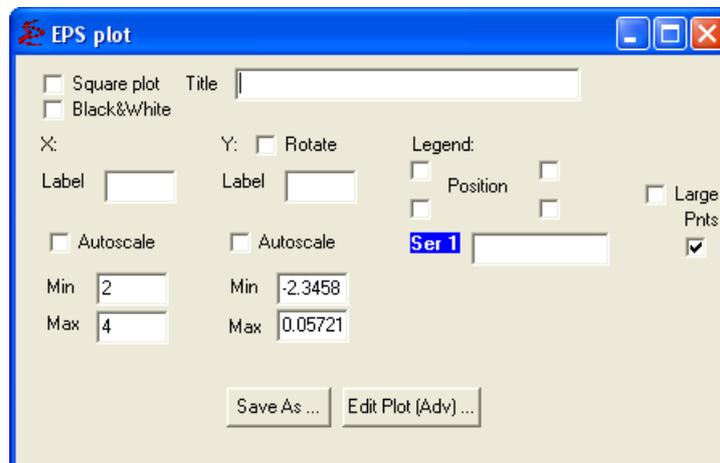


Figure 3.46: Create EPS submenu

The options are as follows. Depending on the submenu in use some of them may not be available:

**Square Plot** When checked forces the output graph to a square

**Black & White** When checked bypasses the original colors and generates black and white output

**Title** When filled adds the text on top of the output

**X:** Under **X:** there are the following options:

**Label** When filled adds the text centrally below the horizontal axis

**Autoscale** When checked hides the scale input boxes and determines it automatically from the plot

**Min** When autoscale is not enabled, allows you to set the lowest number on the horizontal axis

**Max** When autoscale is not enabled, allows you to set the largest number on the horizontal axis

**Y:** Under **Y:** there are the following options:

**Label** When filled adds the text centrally to the left of the vertical axis

**Rotate** When checked rotates the **Label** 90° counter clockwise

**Autoscale** When checked hides the scale input boxes and determines it automatically from the plot

**Min** When autoscale is not enabled, allows you to set the lowest number on the vertical axis

**Max** When autoscale is not enabled, allows you to set the largest number on the vertical axis

**Legend** Under **Legend:** there are the following options:

**Position** If one of the 4 checkboxes around the word *Position* is checked the legend is positioned accordingly (Top-Left, Top-Right, Bottom-Left or Bottom-Right). If no box is checked the legend is not plotted

**Series** For each series available it is possible to define the label for the legend (if no label is defined nothing will be shown in the legend)

**Series** Also besides each series available there are checkboxes that whether the series will be plotted and their appearance:

**Large** (one single box for the whole output) if checked the plotted series and legend markers are plotted bigger than on screen plot (standard is *pointsize* = 0.2, large is *pointsize* = 0.6)

**Line** If checked the series is plotted with a smoothed line.

**Pnts** If checked the series is plotted with points.

At the bottom of the window there are two buttons. **Save As...** uses the options indicated in the windows and saves the output after asking for a file name. **Edit Plot (Adv)...** brings a new window, depicted in figure 3.47, where the *GNUplot* code can be customized. For example, one could change the last line on figure 3.47 to increase the size of points.

This window has the following submenus:

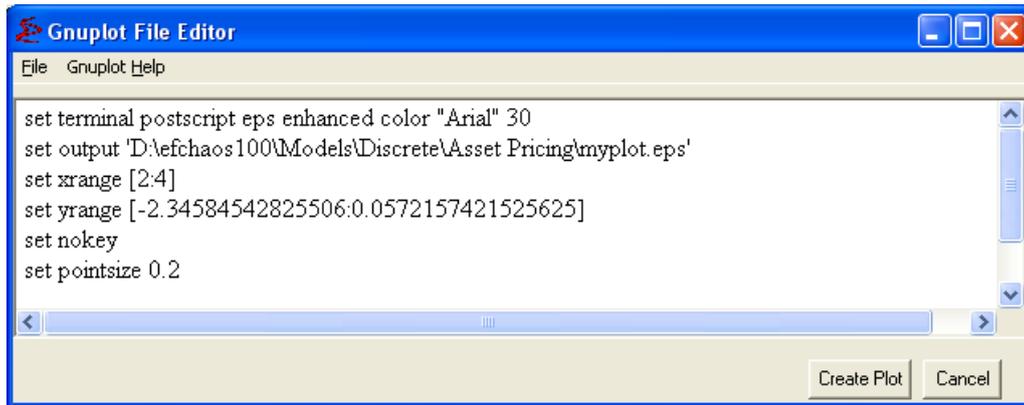


Figure 3.47: GNUplot Editor Window

**File** With the following suboptions:

**Save Current Text as** Allows you to save the current code

**Close** Closes the code window

**GNUplot Help** Provides a Help with details about *GNUplot*. Covering the *GNUplot* code is out of the scope of this manual.

### 3.11 Model specification: Instruction for E&F Chaos with LUA

The language for model implementation is LUA. To access their web page go to the **Help** menu, **About** submenu and click on the LUA logo ([www.lua.org](http://www.lua.org)). Alternatively you can type the web-address on your browser. In this section we include a brief overview of the Math Library in section 3.11.1.

To create or edit a model you first need to follow the procedures indicated in either section 3.2.1 **Build New Model** or section 3.2.3 **Edit Model**, respectively.

You have to specify the **Variables** and their **Initial Values**, the **Parameters** and their Initial Values and the **Auxiliary Plot Variables**.

When you create a new model, it contains an automated code as a model proxy. It can (and should) be adjusted in any suitable way. Any internal variables can be added without declaration. If you would like to plot some variable you have to specify it either as variable or as auxiliary plot variable. Loops and math functions can be used.

After you are done with editing click **OK**. If the model is implemented correctly (in terms of LUA syntax), you can run further commands. Otherwise a syntax error message will appear and you have to review the model.

It is important to mention that the operations are executed in the order they are typed. Therefore if you have a dynamic model in which variables affect each other make sure you write the program in a way that the correct time domains are used on the update rule. For example suppose, in a very simplistic way that, in your model you have:

$$x_{t+1} = \frac{\exp(\lambda(3y_t - 1))}{\exp(\lambda(3y_t - 1)) + 1}$$

$$y_{t+1} = \frac{\exp(\lambda(3x_t - 1))}{\exp(\lambda(3x_t - 1)) + 1}$$

The correct specification in LUA would be:

```
x_ = math.exp(lambda*(3*y -1))/(math.exp(lambda*(3*y -1))+1)
y_ = math.exp(lambda*(3*x -1))/(math.exp(lambda*(3*x -1))+1)

x = x_
y = y_
```

Note that first the update is done to intermediate variables and then the state variables are update. If instead the code would be written as:

```
x = math.exp(lambda*(3*y -1))/(math.exp(lambda*(3*y -1))+1)
y = math.exp(lambda*(3*x -1))/(math.exp(lambda*(3*x -1))+1)
```

The first update would be done correctly, but not the second, since the variable  $x$  would have already been changed. This would correspond to a model:

$$x_{t+1} = \frac{\exp(\lambda(3y_t - 1))}{\exp(\lambda(3y_t - 1)) + 1}$$

$$y_{t+1} = \frac{\exp(\lambda(3x_{t+1} - 1))}{\exp(\lambda(3x_{t+1} - 1)) + 1}$$

### 3.11.1 Math Library

Here we briefly reproduce the math commands available from LUA. For a more comprehensive reference check <http://lua-users.org/wiki/MathLibraryTutorial> or <http://www.lua.org/manual/5.1/manual.html#5.6>. The following is an extract of the latter reference.

The math library is an interface to the standard C math library.

<b>math.abs</b> ( $x$ )	Returns the absolute value of $x$
<b>math.acos</b> ( $x$ )	Returns the arc cosine of $x$ (in radians)
<b>math.asin</b> ( $x$ )	Returns the arc sine of $x$ (in radians)
<b>math.atan</b> ( $x$ )	Returns the arc tangent of $x$ (in radians)
<b>math.atan2</b> ( $y, x$ )	Returns the arc tangent of $y/x$ (in radians), but uses the signs of both parameters to find the quadrant of the result (It also handles correctly the case of $x$ being zero)
<b>math.ceil</b> ( $x$ )	Returns the smallest integer larger than or equal to $x$
<b>math.cos</b> ( $x$ )	Returns the cosine of $x$ (assumed to be in radians)
<b>math.cosh</b> ( $x$ )	Returns the hyperbolic cosine of $x$
<b>math.deg</b> ( $x$ )	Returns the angle $x$ (given in radians) in degrees
<b>math.exp</b> ( $x$ )	Returns the value $e^x$
<b>math.floor</b> ( $x$ )	Returns the largest integer smaller than or equal to $x$
<b>math.fmod</b> ( $x, y$ )	Returns the remainder of the division of $x$ by $y$
<b>math.frexp</b> ( $x$ )	Returns $m$ and $e$ such that $x = m2^e$ , $e$ is an integer and the absolute value of $m$ is in the range $[0.5, 1)$ (or zero when $x$ is zero)
<b>math.huge</b>	The value HUGE_VAL, a value larger than or equal to any other numerical value
<b>math.ldexp</b> ( $m, e$ )	Returns $m2^e$ ( $e$ should be an integer)
<b>math.log</b> ( $x$ )	Returns the natural logarithm of $x$
<b>math.log10</b> ( $x$ )	Returns the base-10 logarithm of $x$
<b>math.max</b> ( $x, \dots$ )	Returns the maximum value among its arguments
<b>math.min</b> ( $x, \dots$ )	Returns the minimum value among its arguments
<b>math.modf</b> ( $x$ )	Returns two numbers, the integral part of $x$ and the fractional part of $x$
<b>math.pi</b>	The value of $\pi$
<b>math.pow</b> ( $x, y$ )	Returns $x^y$ (You can also use the expression $x^y$ to compute this value)
<b>math.rad</b> ( $x$ )	Returns the angle $x$ (given in degrees) in radians
<b>math.random</b> ( $[m [, n]]$ )	This function is an interface to the simple pseudo-random generator function rand provided by ANSI C (No guarantees can be given for its statistical properties). When called without arguments, it returns a uniform pseudo-random real number in the range $[0, 1)$ . When called with an integer number $m$ , math.random returns a uniform pseudo-random integer in the range $[1, m]$ . When called with two integer numbers $m$ and $n$ , math.random returns a uniform pseudo-random integer in the range $[m, n]$
<b>math.randomseed</b> ( $n$ )	Sets $n$ , where $n$ is an integer in the range $-2147483648$ to $2147483647$ as the “seed” for the pseudo-random generator: equal seeds produce equal sequences of random numbers
<b>math.sin</b> ( $x$ )	Returns the sine of $x$ (assumed to be in radians)
<b>math.sinh</b> ( $x$ )	Returns the hyperbolic sine of $x$
<b>math.sqrt</b> ( $x$ )	Returns the square root of $x$ (You can also use the expression $x^{0.5}$ to compute this value)
<b>math.tan</b> ( $x$ )	Returns the tangent of $x$ (assumed to be in radians)
<b>math.tanh</b> ( $x$ )	Returns the hyperbolic tangent of $x$