Pricing Mortality Swaps Using R
R in Insurance 2015

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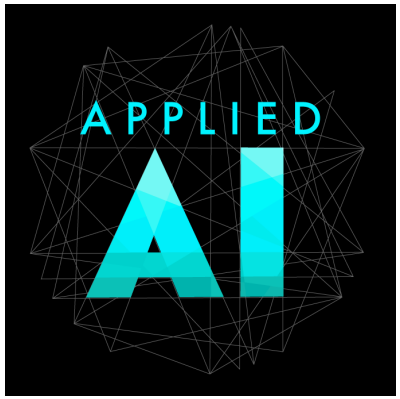
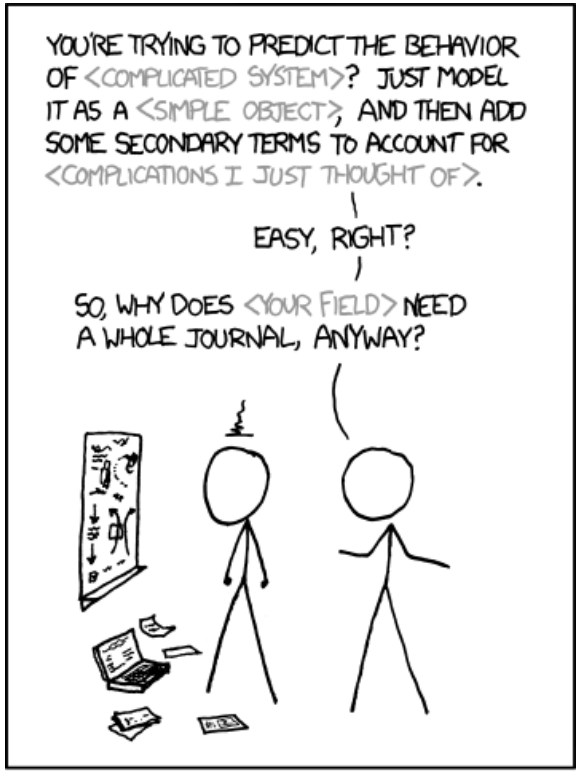


29 June 2015

Before I Begin...



https://xkcd.com/793/

What is a Mortality Swap?

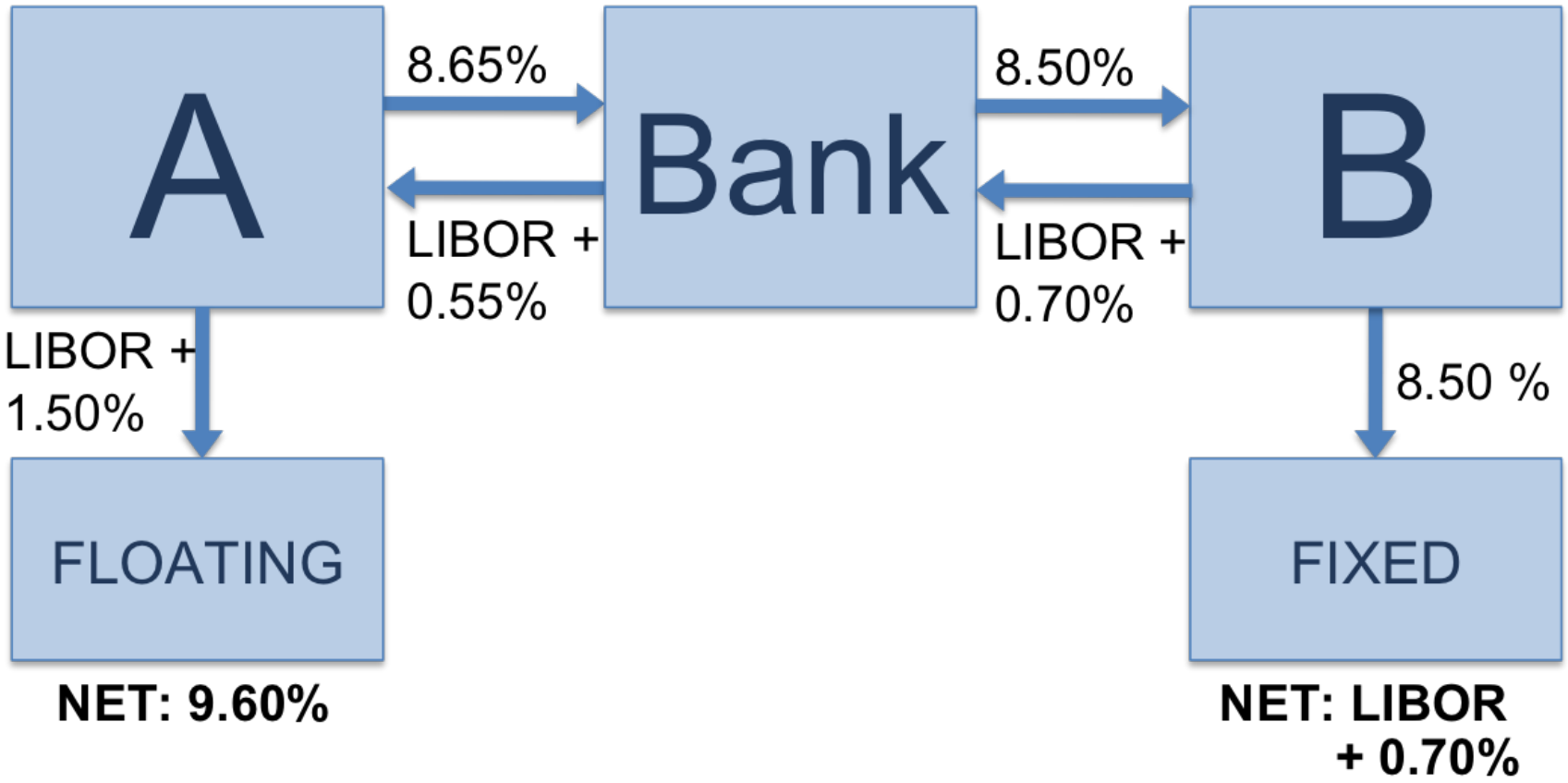
- Seller provides protection against mortality risk
- Portfolio of Life-Contingent Annuities
- Converts the portfolio to Guaranteed Annuities
- Only guaranteeing mortality-adjusted cashflows

Swap Annuity Portfolio

Random portfolio of annuities:

```
##       customerid age MUR   amount
##    1:  C97326460  39 350 $100,000
##    2:  C99971880  30 175  $80,000
##    3:  C65134119  34 225  $50,000
##    4:  C35313144  33 200  $90,000
##    5:  C17550793  45 200  $30,000
##   ---
## 196:  C37440555  40 200  $50,000
## 197:  C60347677  42 200  $60,000
## 198:  C93383952  45 200  $70,000
## 199:  C08447895  35 150  $60,000
## 200:  C64003360  41 325  $90,000
```

Interest Rate Swaps



Assumptions

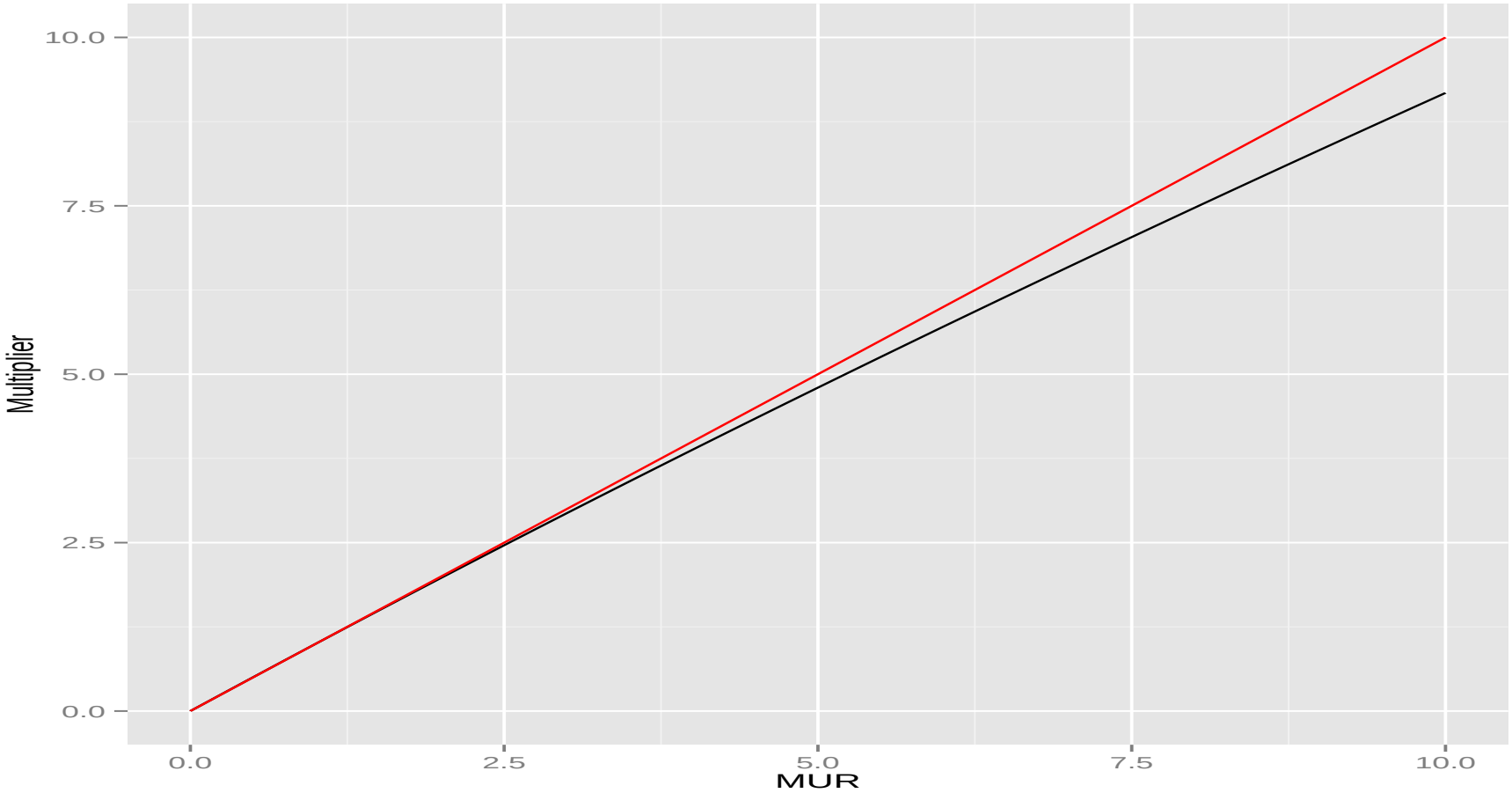
Starting point — some generalisable for flexibility

1. No consideration of credit risk
2. Swap has fixed lifetime
3. All annuities have annual payments received at same time
4. Fixed cost of capital over lifetime
5. All annuitants have undergone underwriting evaluation
6. APV calculations require a specific lifetable
7. All annuities have a lifetime at least as long as the swap lifetime

Calculating the MUR multiplier

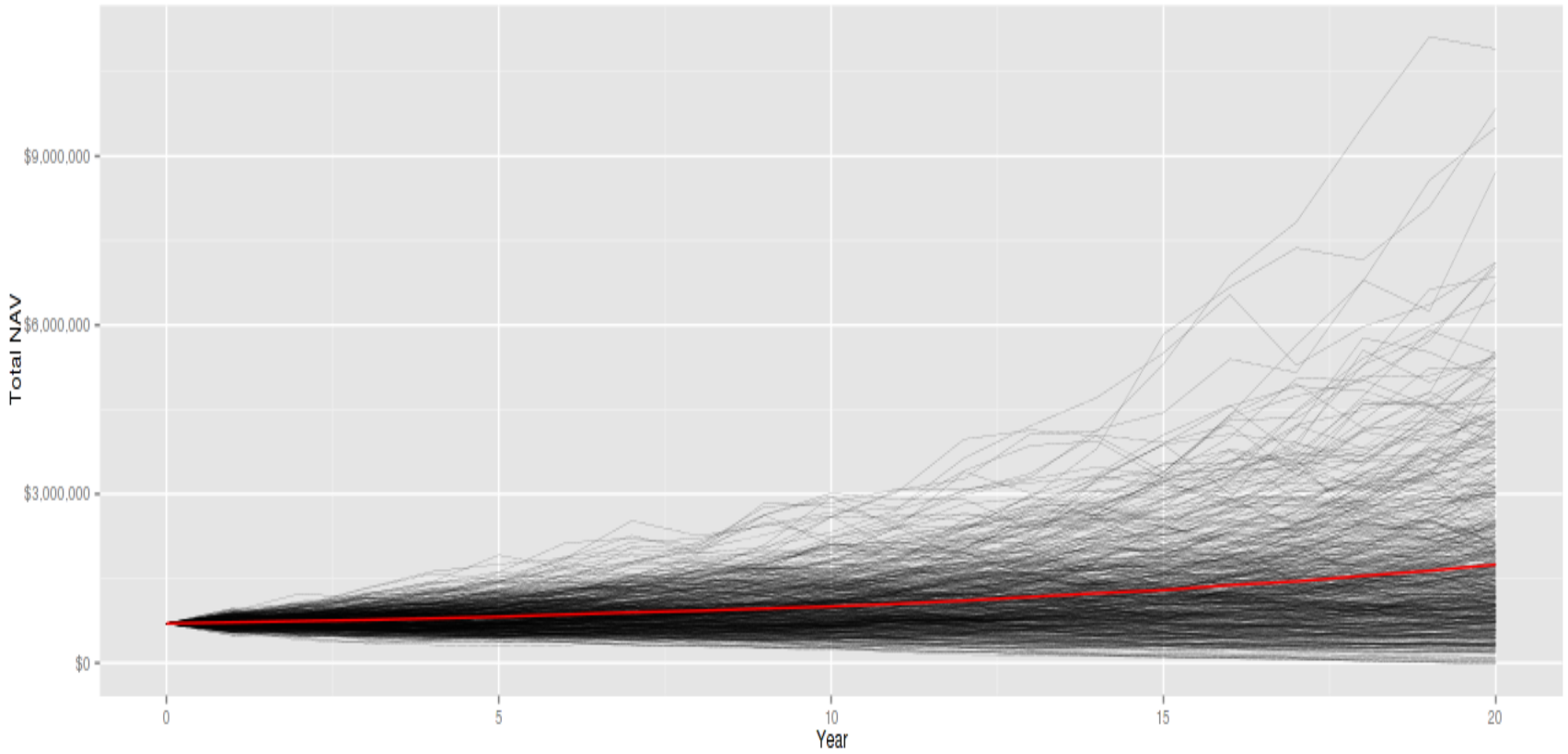
```
lifetable.dt <- fread("lifetable.csv");

A  <- 100000;
qx <- lifetable.dt[age >= 30][age < 50]$qx;
r  <- 0.05;

calculate.MUR.multiple.diff <- function(MUR, mult) {
    MUR * price.term.assurance(qx, A = A, r = r, P = 0) -
        price.term.assurance(mult * qx, A = A, r = r, P = 0);
}

MUR.values <- seq(0, 10, by = 0.25);


### Determine mortality multiplier that best matches the premium multiplier
MUR.mult <- sapply(MUR.values, function(MUR) {
    optimize(function(mult) abs(calculate.MUR.multiple.diff(MUR, mult)), c(0, 20))$minimum;
});
```

Calculating the MUR multiplier



MonteCarlo Methods

Calculating the NAV of a fund of correlated assets with a yearly drawdown:



Simulation Approach

Simulation example: 10 simulations of 5 years

1 – annuitant still alive
0 – annuitant has deceased

```
##       sim1 sim2 sim3 sim4 sim5 sim6 sim7 sim8 sim9 sim10
## year1    1    1    1    1    1    1    1    1    1     1
## year2    1    1    0    1    1    1    1    1    1     1
## year3    1    1    0    1    1    1    1    1    0     1
## year4    1    1    0    1    0    1    1    1    0     1
## year5    1    1    0    1    0    1    1    1    0     1
```

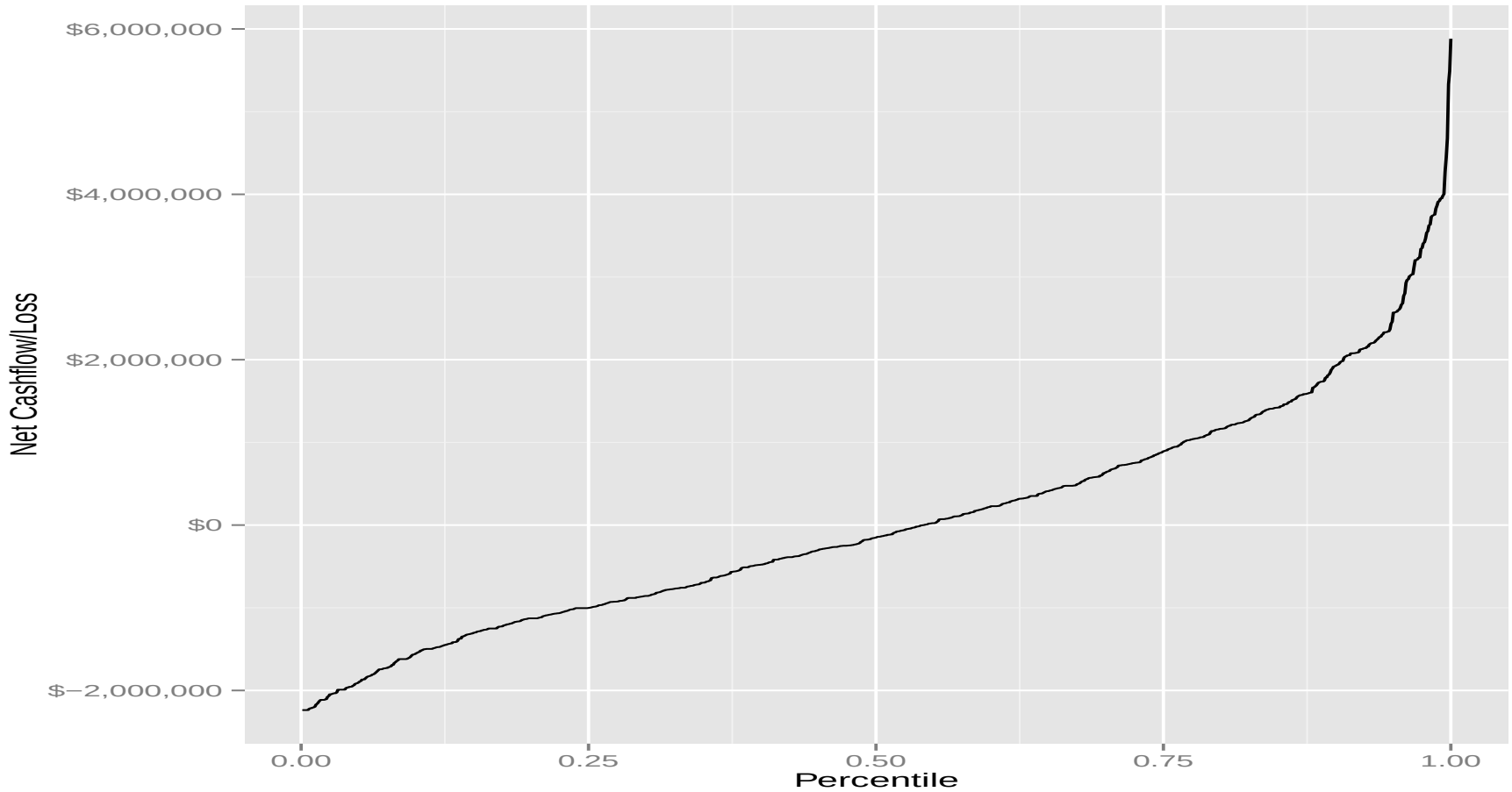
Running the Simulation

Run the calculation:

```
mortport.dt  <- fread("mortswap_portfolio.csv");
lifetable.dt <- fread("lifetable.csv");

n.sim <- 1000;

mortswap.value.sim <- calculate.mortality.swap.price(mortport.dt,
                                                     lifetable           = lifetable.dt,
                                                     hedge.apv.cashflows = TRUE,
                                                     interest.rate       = 0.05,
                                                     years.in.force      = 20,
                                                     n.sim               = n.sim,
                                                     return.all.data     = FALSE);

print(dollar_format(largest_with_cents = 1e8)(mortswap.value.sim[1:20]));


##  [1] "$748,064.31"    "$461,186.76"    "$2,274,787.66"  "$-1,251,047.55"
##  [5] "$1,954,488.45"  "$-1,251,047.55" "$-2,237,366.32" "$1,733,535.87"
##  [9] "$1,707,822.63"  "$-1,299,952.30" "$2,139,507.05"  "$-757,888.16"
## [13] "$-309,255.02"   "$-717,420.20"   "$1,927,893.22"  "$1,417,572.90"
## [17] "$1,571,751.87"  "$-739,440.89"   "$-1,990,786.63" "$475,010.30"
```

Viewing the Output

Simulated cashflows
(excludes initial premium):



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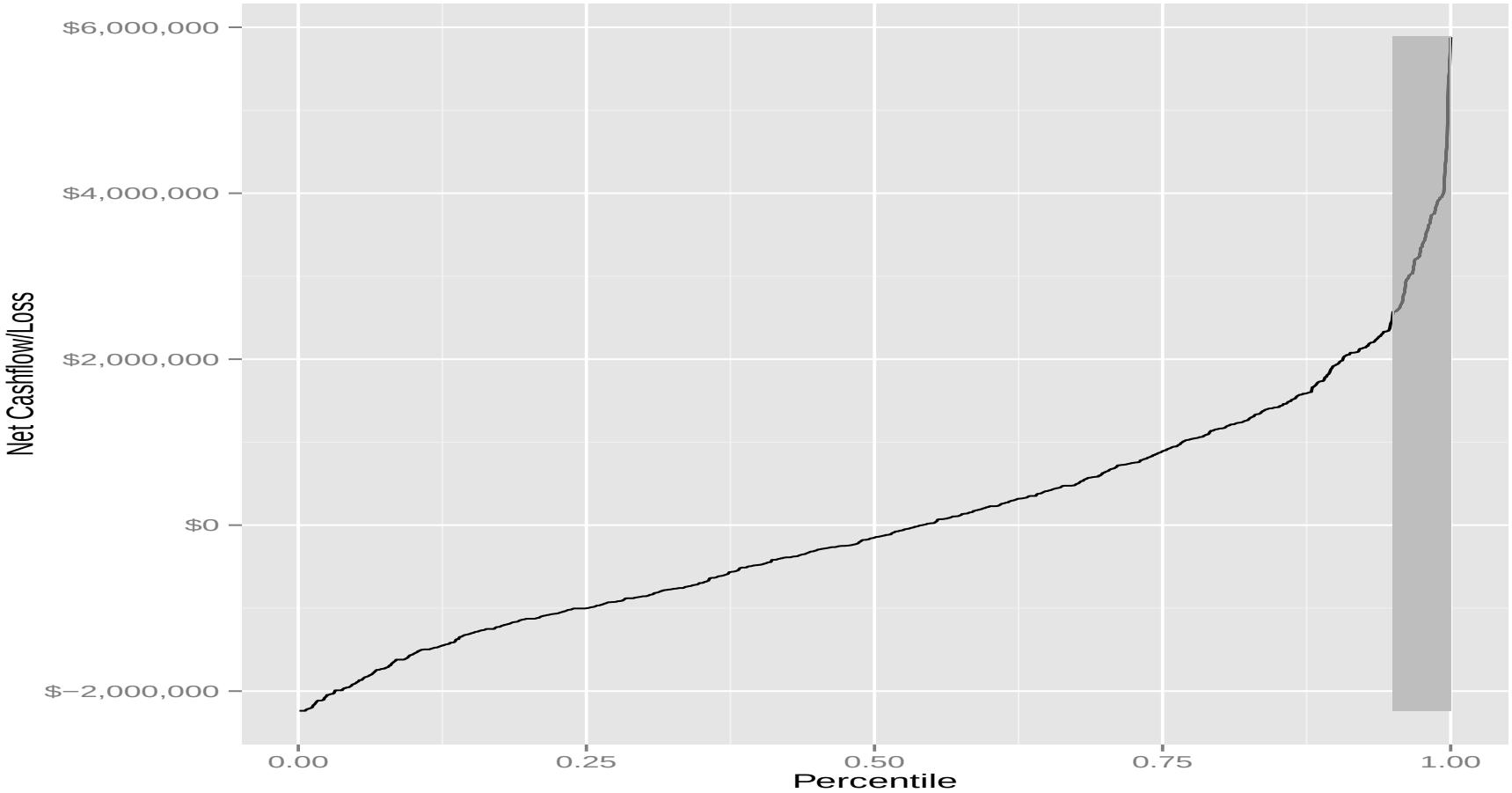


Pricing Tail Risk

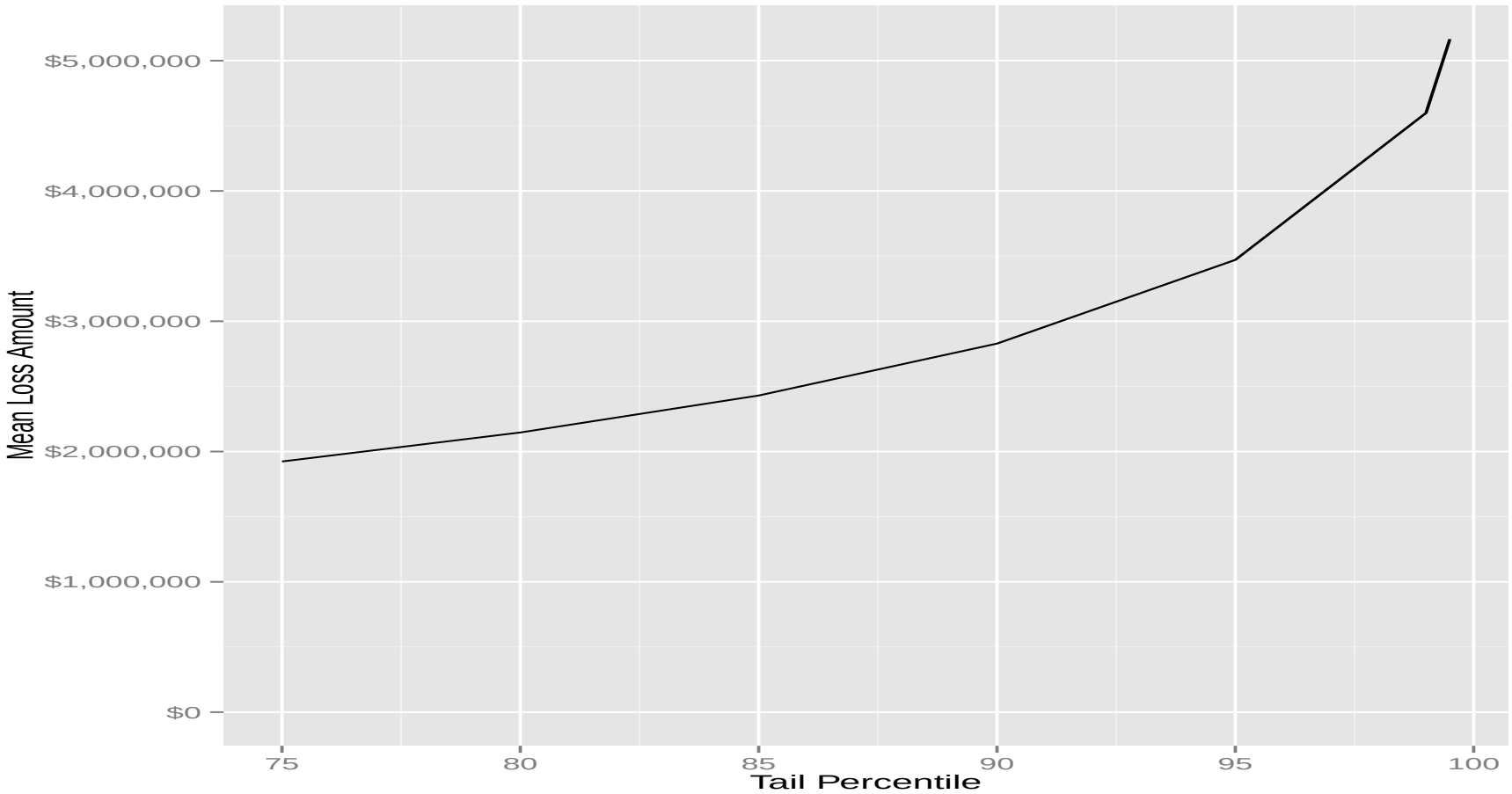
How to price the tail risk?

Michael Lewis *"In Nature's Casino"* – NYT Aug 2007
`http://www.nytimes.com/2007/08/26/magazine/26neworleans-t.html?pagewanted=all`

Consensus around 4$\times$ expected loss

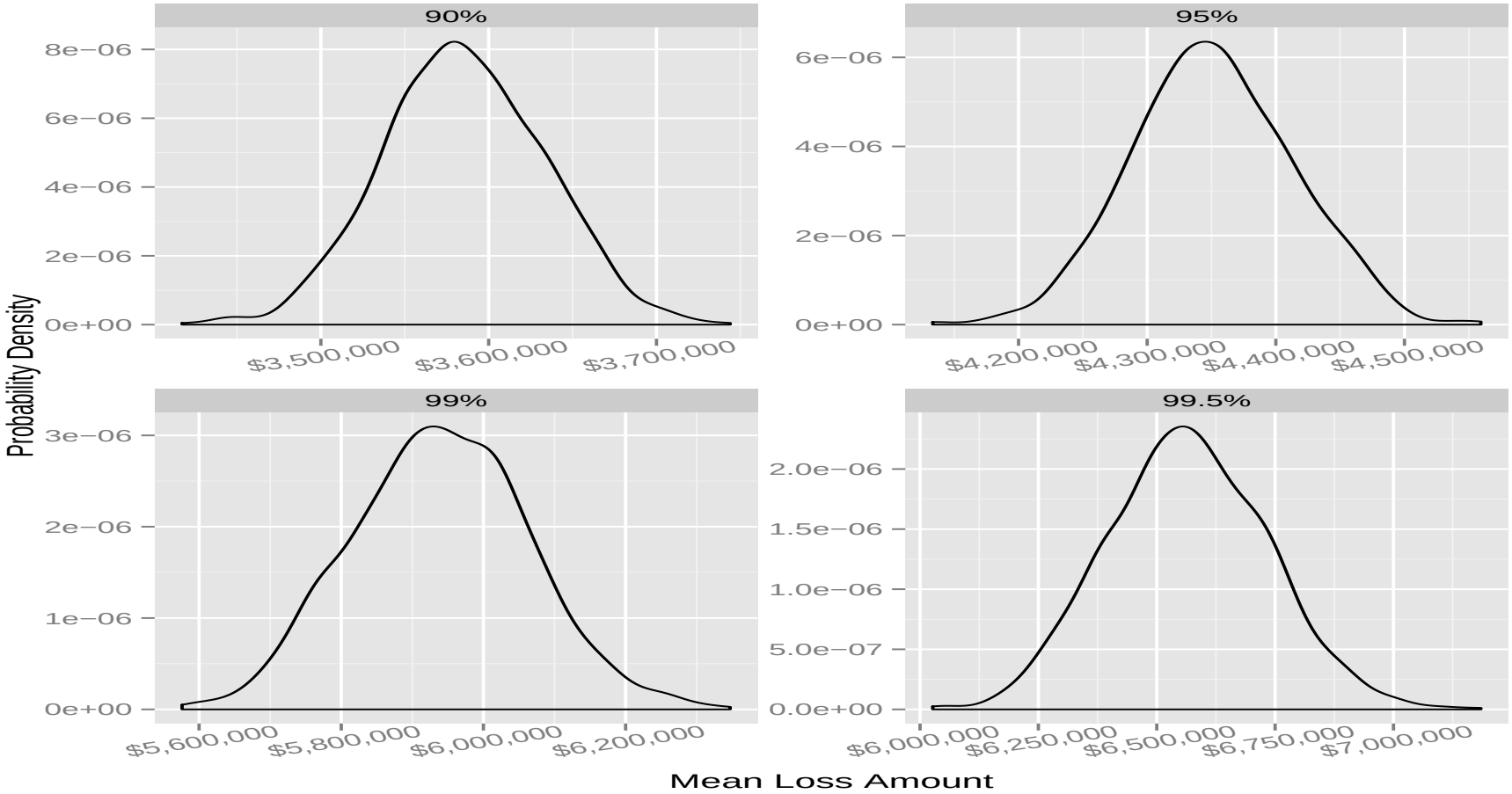
Need to calculate tail averages

Pricing Tail Risk



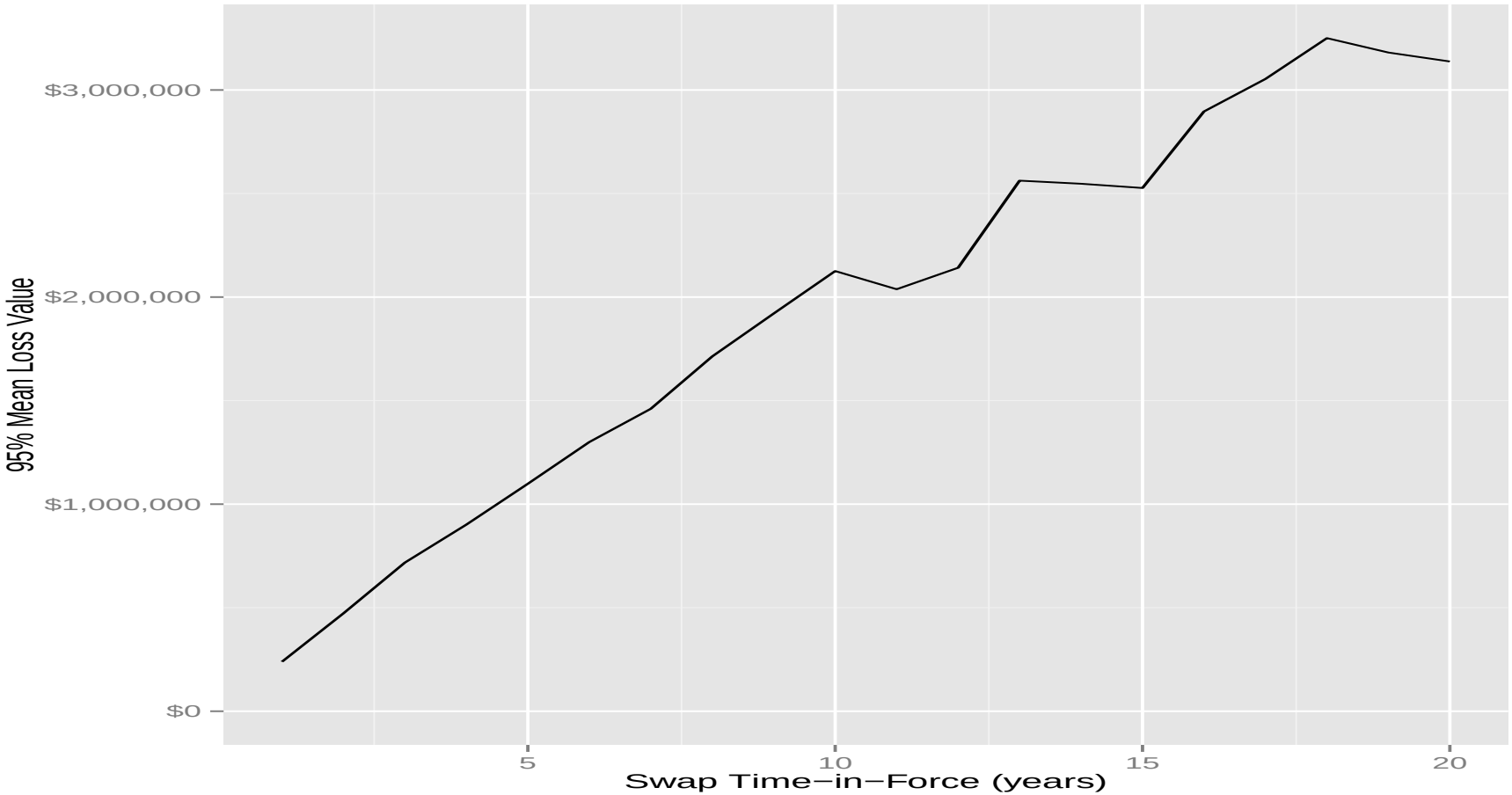
Pricing Tail Risk

Ensemble of 1,000 valuations of 10,000 iterations:



Time-in-Force Dependency

Scaling of 95% mean with time-in-force of swap:



Time-in-Force Dependency (Ensemble)

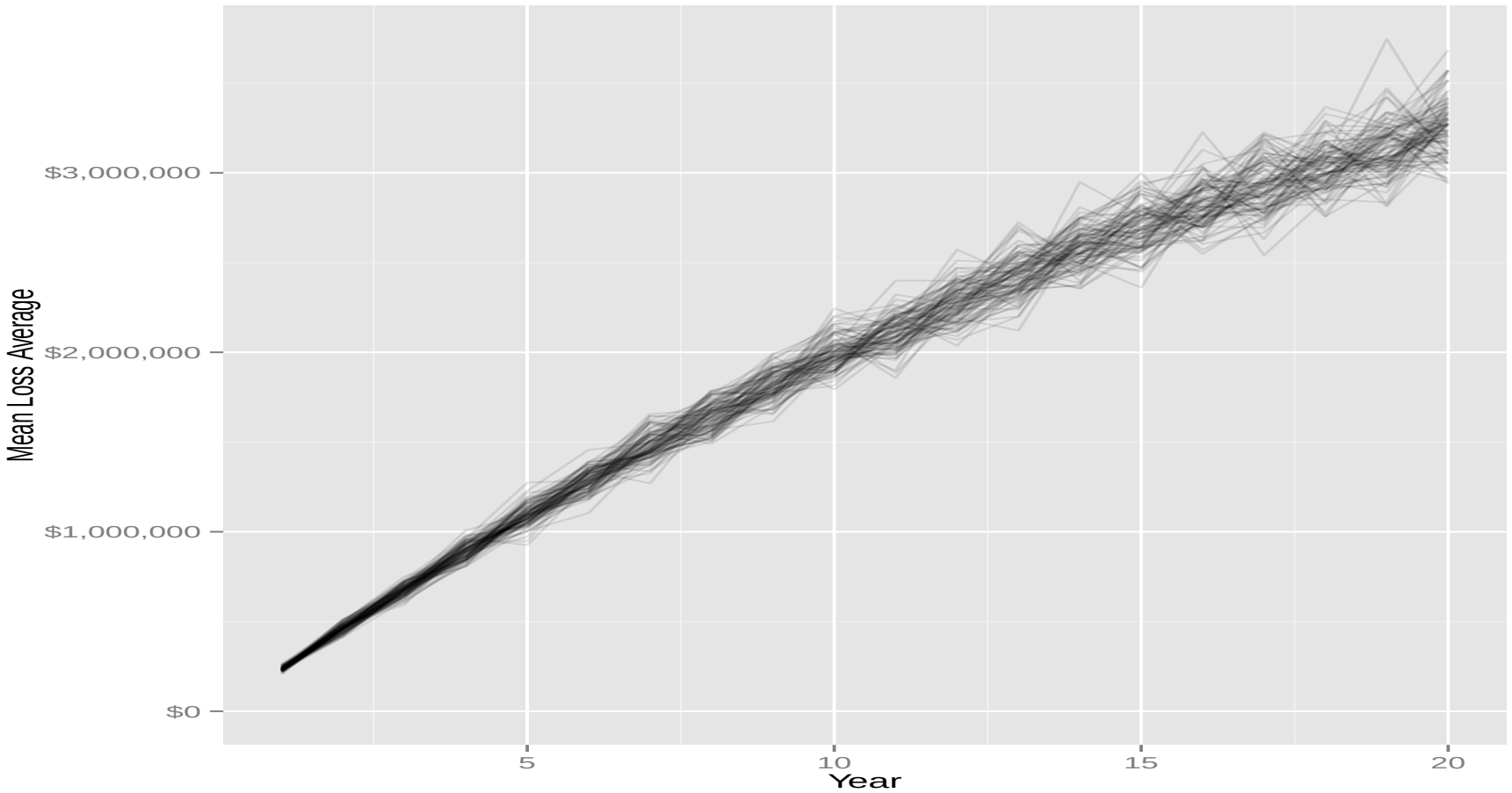
```
times.in.force <- 1:20;
n.sim          <- 1000;
n.ens          <- 10;

calculate.tif <- function(tif) {
    mortswap.value.sim <- calculate.mortality.swap.price(mortport.dt,
                                                         lifetable           = lifetable.dt,
                                                         hedge.apv.cashflows = TRUE,
                                                         interest.rate       = 0.05,
                                                         years.in.force      = tif,
                                                         n.sim               = n.sim,
                                                         return.all.data     = FALSE);


    iterval <- calculate.quantile.averages(mortswap.value.sim, 0.95);
}

tif.ensemble <- sapply(times.in.force, function(iter.tif) {
    replicate(n.ens, calculate.tif(iter.tif))
});
```

Time-in-Force Dependency (Ensemble)



Summary

R package: `mcmortswap`
`https://bitbucket.org/appliedai/mcmortswap`

Email: michael.cooney@applied.ai

Slides available on bitbucket:
`https://bitbucket.org/appliedai/r_in_insurance_2015`