A model of the reserve asset

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45 min

Motivation

- US Treasury bonds have been the world reserve asset for a long time
  - Safe asset portfolios tilted towards US Treasury bonds
  - "Convenience yield" on US Treasury bonds; Higher premium in bad states ("negative β")
  - Despite increasing size of US debt, reserve asset status has persisted
  - Despite deteriorating US fiscal position, US interest rates remain low

- German bund occupies a similar position in the Euro area

- History
  - Gold;
  - UK consol bond pre-WWI;
  - Joint reserve asset: UK and US in interwar period
  - Pre 2009 in Europe: German, French, Italian bonds are all negative β

- Since crisis, many proposals to create a Euro-wide government bond, to serve as a Euro reserve asset
  - But no models of a reserve asset, so no framework to analyze different Eurobond designs

Literature

- International finance, economic history literature on reserve currency
  - Eichengreen (many), Krugman (1984), Frankel (1992)
  - **Store of value**, medium of exchange, unity of account, multiple equilibria
- Shortages of store of value
  - Safe assets literature (Holmstrom and Tirole, 1998, Caballero, Farhi, Gourinchas 2008, Caballero and Krishnamurthy 2009, Krishnamurthy and Vissing-Jorgensen 2012)
  - Multiplicity: Samuelson (1958) on money, Diamond (1965) on govt debt
  - No formal models of endogenous determination of which asset is chosen as store of value
- Global games and sovereign debt rollover risk
  - Morris and Shin (1998), Angeletos, Hellwig and Pavan (2006), Calvo (1988), Cole and Kehoe (2000)

Model Setup

Investors (j)**:**

- Measure $1 + f$ of investors with one unit of funds each
- Risk neutral, each investor **must** invest his funds in sovereign debt

Countries/debt (i)**:**

- Two countries, debt size $s_1 = 1$ and size $s_2 = s < 1$
- Fundamental ("surplus") $s_i\theta_i$
  - Foreign denominated debt: true surplus plus foreign reserves
  - Domestic denominated debt: true surplus plus any resources CB is willing to provide to forestall a rollover crisis
- Debt of face value of s_i (exogenous) issued at endogenous price p_i
- Default if surplus plus bond proceeds insufficient for obligations

$$\underbrace{s_i\theta_i + s_i p_i}_{\text{total funds available}} \quad < \quad \underbrace{s_i}_{\text{debt obligations}}$$

$\Rightarrow$ Given price p_i, default decision depends on θ_i

- Recovery in default $= 0$

Multiple equilibria in a special case

- No default if,

$$s_i p_i \geq s_i(1 - \theta_i)$$

- Suppose sufficient funding for both countries:

$$\underbrace{1 + f}_{\text{total funds available}} \geq \underbrace{(1 - \theta_1) + s_2(1 - \theta_2)}_{\text{funding needs}} \tag{1}$$

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Possible equilibria

1. Country 1 is safe (=reserve asset), country 2 defaults:

$$p_1 = 1 + f, p_2 = 0 \qquad \text{Investor return} = \frac{1}{1 + f}$$

2. Country 2 is safe (=reserve asset), country 1 defaults:

$$p_1 = 0, p_2 = 1 + f \qquad \text{Investor return} = \frac{s}{1 + f}$$

3. Both countries safe, two reserve assets... but unstable

Multiple equilibria

- No default if,

$$s_i p_i \geq s_i(1-\theta_i)$$

- Suppose sufficient funding for both countries:

$$\underbrace{1+f}_{\text{total funds available}} \geq \underbrace{(1-\theta_1)+s_2(1-\theta_2)}_{\text{funding needs}}$$

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3. Both countries safe, two reserve assets... but unstable
4. **If $s_2 = 0$, country 1 is safe (Japan?)**

Full characterization as function of s, f and θ

Global games technique:

- Unobserved relative fundamentals (higher $\tilde{\delta}$ means country 1 is stronger):

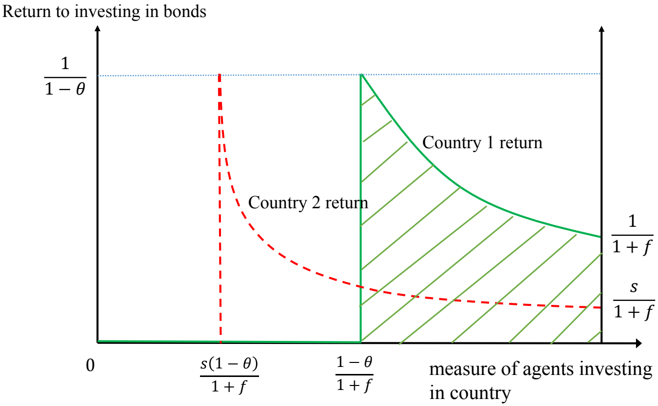
$$\begin{aligned} 1-\theta_1 &= (1-\theta)\exp\left(-\tilde{\delta}\right); \\ 1-\theta_2 &= (1-\theta)\exp\left(+\tilde{\delta}\right). \end{aligned}$$

- Each investor receives a noisy private signal before investing

$$\delta_j = \tilde{\delta} + \epsilon_j$$

- Take $\tilde{\delta} \in \left[-\bar{\delta},\bar{\delta}\right]$ (any cdf, but wide support) and noise $\epsilon_j \sim U\left[-\sigma,\sigma\right]$
  - We will (mostly) look at $\sigma \to 0$: fundamental uncertainty vanishes, but strategic uncertainty remains
- Timing assumptions:
  - Investors place *market orders* to buy debt
  - Country default decision after orders are submitted

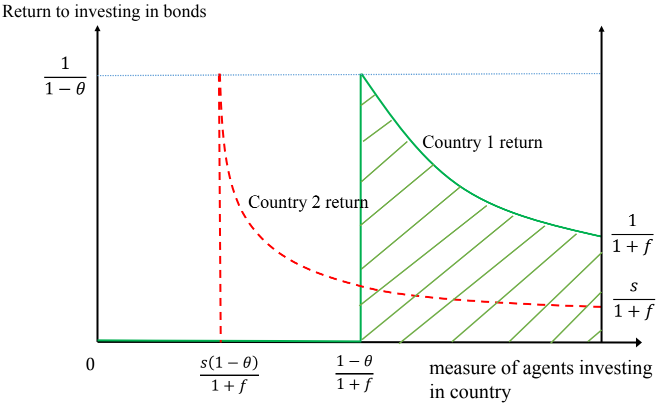
Returns for $\tilde{\delta} = 0$ (Rollover risk)



- Given proportion x investing in country 1, no default if:

$$x \geq \frac{1-\theta_1\left(\tilde{\delta}\right)}{1+f} \text{ (country 1)} \qquad 1-x \geq \frac{s\left(1-\theta_2\left(\tilde{\delta}\right)\right)}{1+f} \text{ (country 2)}$$

Returns for $\tilde{\delta} = 0$ (Liquidity)



- Liquidity/market depth: country 2 price-rises/return-falls faster
- Given proportion x investing in country 1, conditional returns are

$$\frac{1}{p_1} = \frac{1}{(1+f)\,x} \quad \text{and} \quad \frac{1}{p_2} = \frac{s}{(1+f)\,(1-x)}$$

Strategy space

Threshold Equilibrium:

- Let $\phi\left(\delta_j\right)$ be investment in country 1 of agent with signal δ_j
- Consider threshold strategies

  If $\delta_j > \delta^*$ invest in country 1 i.e. ϕ=1; otherwise country 2 i.e. ϕ=0

- The equilibrium cutoff δ^* is determined by indifference of marginal investor with signal $\delta_j = \delta^*$
  - Must be indifferent between investing in country 1 versus 2

- *We can prove uniqueness of the equilibrium, at least for some part of the parameter space, among monotone functions $\phi\left(\delta_j\right)$.*

Expected returns

- Marginal investor $\delta_j = \delta^*$ does not know other investors' signals
  - He asks, suppose fraction $x \in [0,1]$ of investors have signals$> \delta_j$
  - Marginal agent backs out true $\tilde{\delta}$ for given x as follows

$$\tilde{\delta} = \delta^* + (2x - 1)\sigma$$

  - Take $\sigma \to 0$ so *only strategic uncertainty* remains....
  - Global games result: $x \sim U[0,1]$ from the view of marginal investor, for any prior of $\tilde{\delta}$

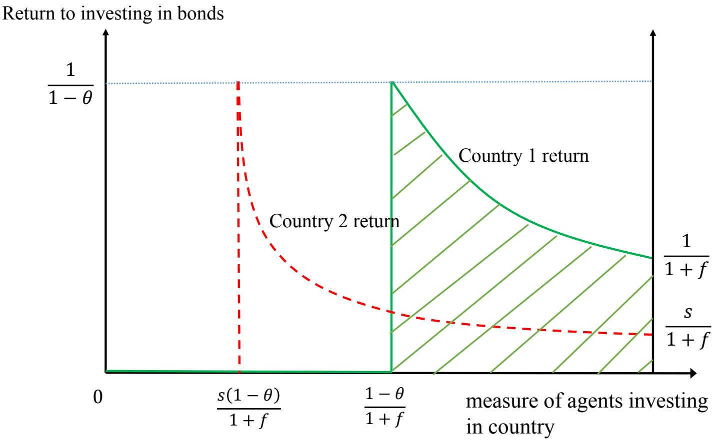
- Integrating over possible values of $x \sim U[0,1]$ gives expected profits

$$\Pi_1 = \int_{\frac{(1-\theta)e^{-\delta^*}}{1+f}}^{1} \frac{1}{(1+f)\,x} dx = \frac{1}{1+f}\left(\ln \frac{1+f}{1-\theta} + \delta^*\right)$$

and

$$\Pi_2 = \int_0^{\frac{1+f-s(1-\theta)e^{\delta^*}}{1+f}} \frac{s}{(1+f)(1-x)} dx = \frac{1}{1+f} s\left(-\ln s + \ln \frac{1+f}{1-\theta} - \delta^*\right)$$

Expected returns



- For marginal agent, proportion of investors in country 1 is x:
  - Π_1 = Integral under green curve
  - Π_2 = Integral under red curve

Threshold

- Threshold δ^* is determined by the indifference condition

$$\Pi_1(\delta^*) = \Pi_2(\delta^*)$$

- Solving for δ^* (recall that $s \in (0, 1]$)

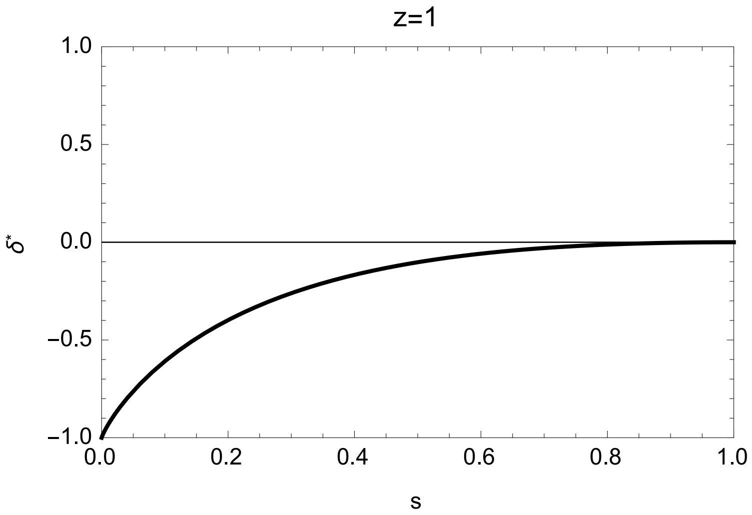
$$\delta^* = \underbrace{-\frac{1-s}{1+s}}_{\text{negative, liquidity}} z + \underbrace{\frac{-s \ln s}{1+s}}_{\text{positive, rollover}} \quad (2)$$

where we define "aggregate funding conditions"

$$z \equiv \ln \frac{1+f}{1-\theta} > 0 \quad (3)$$

- High z means high savings ("savings glut"), good average fundamentals, low average interest rates
  - Low z is opposite
- δ^*: lowest value of $\tilde{\delta}$ so that country 1's bonds are reserve asset

Graphically δ^* as function of country 2 size



Country 1 is reserve asset if fundamental $\tilde{\delta} > \delta^*$

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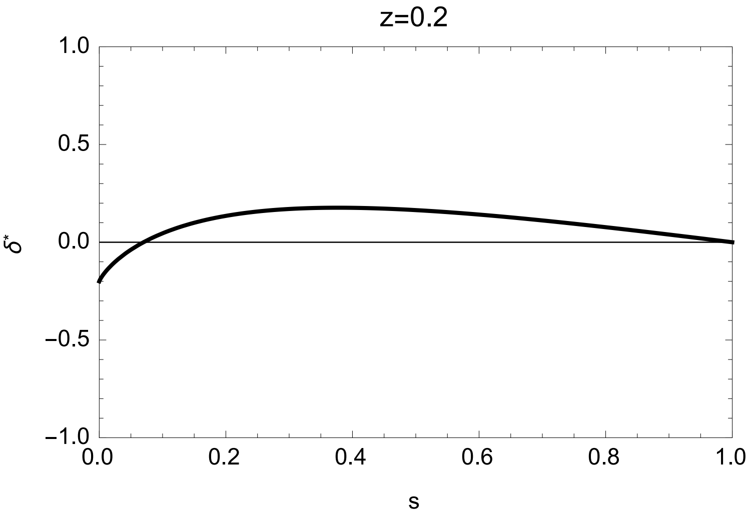

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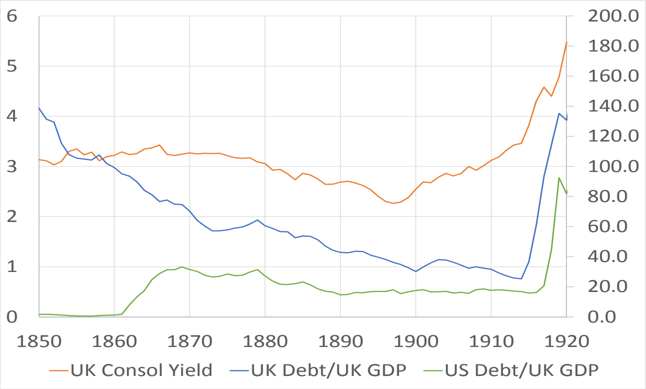
When will world switch?

- In high z world (savings glut)
  - US Treasury size: Debt = \$12.7tn, (CB money $\approx$ \$4.6tn) : maximum liquidity for the world
  - Even if US fiscal position is worse than others (i.e. $\delta^* < 0$)
  - ... Switch not on the horizon

- Unless macro moves to low z world
  - US Treasury size becomes a concern – can the country rollover such a large debt?
  - Investors coordinate on smaller debt country
  - Germany? Debt = \$1.5tn

Era of UK consol bond



- UK government debt was reserve asset until sometime after WWI
  - US GDP exceeds UK GDP by 1870
  - In 1890, UK Govt Debt $\approx 3 \times$ US Govt Debt
  - UK Debt/GDP $= 0.43$, US Debt/GDP $= 0.10$

Relative fundamentals

- Relative fundamentals/GE in safe assets is central to our model
  - Take model with no coordination, where repayment is equal to surplus (θ) and world interest rate is normalized to one.

$$p_1 = E[\theta_1], p_2 = E[\theta_2]$$

  - Our model (for $\delta^* = 0$)

$$\begin{array}{lcl} \theta_1 > \theta_2 & \Rightarrow & p_1 = 1 + f, \quad p_2 = 0 \\ \theta_1 < \theta_2 & \Rightarrow & p_2 = 1 + f, \quad p_1 = 0 \end{array}$$

- US fiscal position is weaker now than before, but still better than everyone else
- Same for Germany within Eurozone

Negative β

Take an extreme case where country 1 is a.s. reserve asset, $\tilde{\delta} >> \delta^*$

- Suppose there is some recovery even in default $L_i = \theta_i$
- Country 1 bond price and return (R)

$$p_1 = 1 + f - sp_2 \qquad\qquad R = \frac{1}{1+f-sp_2}$$

  Country 2 bond price $p_2 = \frac{\theta_2}{R}$ (no arbitrage)
- Solving:

$$p_1 = \frac{1+f}{1+s\theta_2} \qquad \text{and} \qquad p_2 = \frac{1+f}{1+s\theta_2}\theta_2$$

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- Small shock to $\theta_1 \downarrow$ has no effect on p_1, but $\theta_2 \downarrow \Rightarrow p_1 \uparrow$
- Reduce world average fundamentals θ_1, θ_2 equally:
  - Reduces p_2, increases p_1
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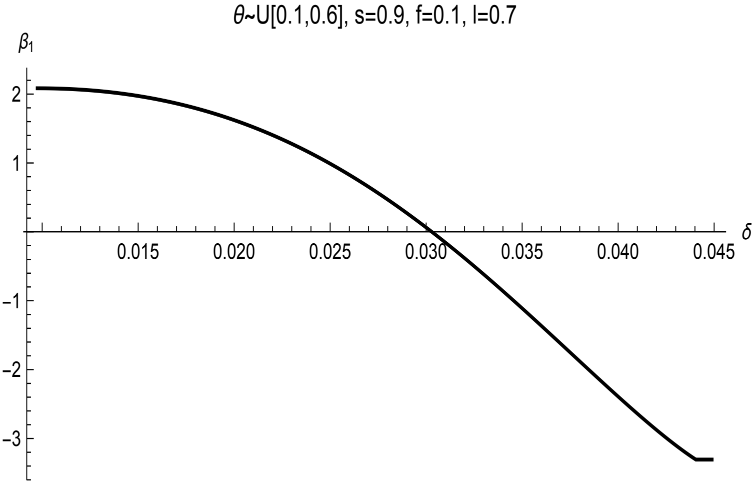
- Lehman shock: Negative shock to US and world fundamentals
  - Treasury yields fall (alternatives rise)

Negative β



Country 1 $\beta_1 = \frac{Cov(p_1,\theta_1)}{Var(\theta_1)}$, as function of relative fundamental δ.

Switzerland?

- What if there were "full-commitment" reserve assets available to investors?
  - Switzerland: Debt $=$ \$127bn, (CB money $\approx$ \$500bn)
  - Denmark: Debt $=$ \$155bn
- US: Debt $=$ \$12.7tn, (CB money $\approx$ \$4.6tn)
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- Define

  $$\hat{f} = f - p_s \underline{s}$$

  where $\underline{s}$ is quantity of "full-commitment" assets
  - In equilibrium p_s is set based on expected return from investing in country 1/country 2.
  - Otherwise, model is as analyzed based on total demand of $\hat{f}$

Two reserve assets ("joint safety")

- Monotone threshold strategies, only one reserve asset
  - $\phi(\delta_j) = 1$ if $\delta_j > \delta^*$, o.w. 0; where ϕ is investment in country 1

Two reserve assets ("joint safety")

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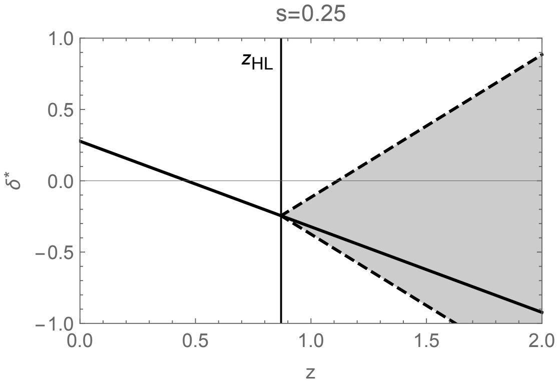
  - $\phi(\delta_j) = 1$ if $\delta_j > \delta^*$, o.w. 0; where ϕ is investment in country 1
- If we allow for non-monotone "oscillating" strategies:
  - $\phi(\delta_j)$: 1,0,1,0,1,0... in a non-monotone fashion (not quite "mixing," but similar)
  - Then, for high $z > z_{HL}$, joint safety for values of $\tilde{\delta}$ in GRAY



Sovereign choices

- Debt size (s), fundamentals (θ), are choice variables
  - Externalities in model
  - Role for coordination

- **Security design as coordination**

Eurobonds and coordination

- Policy proposals to create a Euro-area reserve asset
  - Proceeds to all countries, so all countries get some seignorage
  - Flight to quality is a flight to all, rather than just German Bund
- We study: Countries issue two bonds:
  - A common bond in α share
  - An individual bond in $(1-\alpha)$ share
  - Common bond is pooled bond (essentially a "bundle"), for which each country is responsible for paying its respective share of the obligation
  - No cross-default provisions (structure is closest to "ESBIES")
- We set aside moral hazard considerations which are likely first-order

Common bond and individual bonds

- Two-stage game
  - **Stage 1:**
    - Countries issue common bonds: α (large) and αs (small)
    - Investors pay $f - \hat{f}$, so common bond price

$$p_c = \frac{f - \hat{f}}{\alpha (1 + s)}$$

    - Split proceeds $\alpha p_c \frac{s_i}{s_i + s_{-i}}$
  - **Stage 2:**
    - Investor gets signal δ_j
    - Individual country bonds issued at prices p_1 and p_2
    - Investors invest remainder of funds $1 + \hat{f}$ into individual (non-bundled) bonds

Common bond and individual bonds

- If country$-i$ defaults, it does so on both individual and portion of common bond
- New no-default condition:

$$
\begin{aligned}
(1-\alpha)p_1 + \theta_1 + \overbrace{\alpha p_c}^{\text{Common bond proceeds}} &\geq 1 \\
(1-\alpha)p_2 + \theta_2 + \underbrace{\alpha p_c}_{\text{Common bond proceeds}} &\geq 1
\end{aligned}
$$

- Importantly, common bond proceeds are allocated in a *state-independent way* across the two countries
  - Contrast this with the "winner takes all" funding provided by the individual bonds; this is a state-dependent allocation

Why might this work?

- In basic model ($\alpha = 0$) no default if,

$$s_i p_i \geq s_i(1 - \theta_i)$$

- Suppose global funds exceeds funding need:

$$\underbrace{1 + f}_{\text{total funds available}} \quad \geq \quad \underbrace{(1 - \theta_1) + s_2(1 - \theta_2)}_{\text{sum of individual funding needs}}$$

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- When $\alpha = 1$, neither country defaults if,

$$\underbrace{1 + f}_{\text{total funds available}} \geq \underbrace{(1 - \theta_1) + s_2(1 - \theta_2)}_{\text{funding need of common bond}}$$

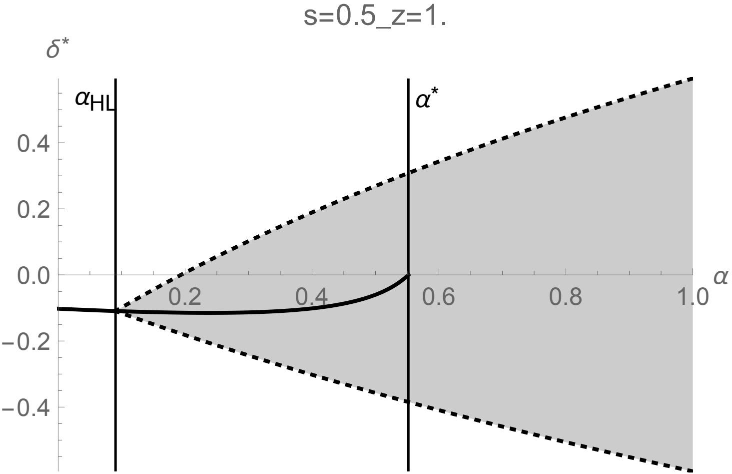
- Security design coordinates investor actions
  - Flight to the reserve asset generates stable funding for both countries

Common bond equilibrium

- Stage 2 game: investors with $\hat{f}$
  - Default conditions for each country, and individual bond prices p_i
  - Almost same as previous analyses
- Stage 1 game sets investment in common bond $f - \hat{f}$ based on:

$$E[R_c] = E[R_{stage2}]$$

Equilibrium as function of α



- High $\alpha > \alpha^* \Rightarrow$ joint safety equilibrium always
- Low $\alpha < \alpha_{HL} \Rightarrow$ single reserve asset, threshold equilibrium
- For $\alpha \in [\alpha_{HL}, \alpha^*]$ both equilibria are possible

Debt size and fundamentals:

- Suppose country i can choose size, S_i
  - Debt float is S_i
  - Surplus is adjusted to $\theta_i S_i$ - i.e. keep tax revenues to debt constant
- Suppose country i can separately choose to increase surplus by δ_i

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- Easy to solve the model:

$$\delta^* = \frac{S_2 - S_1}{S_1 + S_2} z + \frac{S_1 \ln S_1 - S_2 \ln S_2}{S_1 + S_2} - \delta_1 \frac{S_1}{S_1 + S_2} + \delta_2 \frac{S_2}{S_1 + S_2}.$$

- One obvious effect:
  - Increasing surplus always increases reserve asset status
  - E.g., higher δ_1 reduces δ^*because country 1 become safer
- Less obvious, effect of changing S_i

Crowding out/contagion

- Take,
$$\delta^*(S_1, S_2) = \frac{S_2 - S_1}{S_1 + S_2} z + \frac{S_1 \ln S_1 - S_2 \ln S_2}{S_1 + S_2}.$$
- Effect of increasing S_1 on δ^*:
$$h(S_1, S_2; z) \equiv \frac{\partial \delta^*(S_1, S_2)}{\partial S_1} = \frac{1}{(S_1 + S_2)^2} (S_1 + S_2(\ln S_1 + \ln S_2 + 1 - 2z))$$
    - Decreasing in z, negative for large z;
- Expanding US debt can increase US reserve asset status
    - Decreases other country's position
- Conjecture: Expansion of US debt/QE 2007-2009 precipated European debt crisis

Endogenous choices:

- Suppose country 1,2 have "natural" debt size (s_1^*, s_2^*) and choose size:

$$\max_{S_1} -\delta^* (S_1, S_2) - c(S_1 - s_1^*).$$

  - Reduce default probability subject to adjustment costs

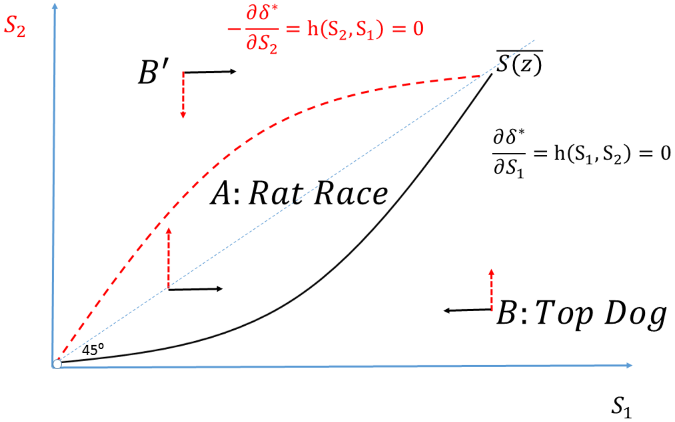
$$\max_{S_2} +\delta^* (S_1, S_2) - c(S_1 - s_1^*).$$

- Equilibrium:

$$h(S_1, S_2; z) = c'(S_1 - s_1^*) \quad \text{and,} \quad h(S_2, S_1; z) = c'(S_2 - s_2^*).$$

Equilibrium via a phase diagram

- High z case; $\delta^* = 0$ along diagonal



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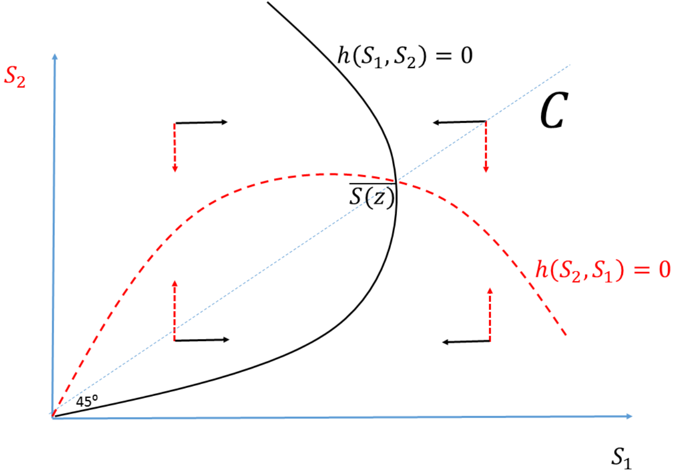

Conclusion

- US government debt is reserve asset because:
  - Good relative fundamentals
  - Debt size is large, and world is in high demand for reserve asset (savings glut)
    - Nowhere else to go
- Economics of reserve asset suggest that there can be gains from coordination
  - Eurobonds as coordinated security-design