A model of the reserve asset

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Amsterdam “Safe asset” conference
45 min
Motivation

- US Treasury bonds have been the world reserve asset for a long time
  - Safe asset portfolios tilted towards US Treasury bonds
  - “Convenience yield” on US Treasury bonds; Higher premium in bad states (“negative $\beta$”)
  - Despite increasing size of US debt, reserve asset status has persisted
  - Despite deteriorating US fiscal position, US interest rates remain low

- German bund occupies a similar position in the Euro area

- History
  - Gold;
  - UK consol bond pre-WWI;
  - Joint reserve asset: UK and US in interwar period
  - Pre 2009 in Europe: German, French, Italian bonds are all negative $\beta$

- Since crisis, many proposals to create a Euro-wide government bond, to serve as a Euro reserve asset
  - But no models of a reserve asset, so no framework to analyze different Eurobond designs
Literature

- International finance, economic history literature on reserve currency
  - Eichengreen (many), Krugman (1984), Frankel (1992)
  - **Store of value**, medium of exchange, unity of account, multiple equilibria

- Shortages of store of value
  - Multiplicity: Samuelson (1958) on money, Diamond (1965) on govt debt
  - No formal models of endogenous determination of which asset is chosen as store of value

- Global games and sovereign debt rollover risk
Model Setup

**Investors** \((j)\):
- Measure \(1 + f\) of investors with one unit of funds each
- Risk neutral, each investor **must** invest his funds in sovereign debt

**Countries/debt** \((i)\):
- Two countries, debt size \(s_1 = 1\) and size \(s_2 = s < 1\)
- Fundamental ("surplus") \(s_i\theta_i\)
  - Foreign denominated debt: true surplus plus foreign reserves
  - Domestic denominated debt: true surplus plus any resources CB is willing to provide to forestall a rollover crisis
- Debt of face value of \(s_i\) (exogenous) issued at endogenous price \(p_i\)
- Default if surplus plus bond proceeds insufficient for obligations

\[
\underbrace{s_i\theta_i + s_i p_i}_\text{total funds available} < \underbrace{s_i}_\text{debt obligations}
\]

\(\Rightarrow\) Given price \(p_i\), default decision depends on \(\theta_i\)
- Recovery in default = 0
Multiple equilibria in a special case

- No default if,
  \[ s_i p_i \geq s_i (1 - \theta_i) \]

- Suppose sufficient funding for both countries:
  \[
  \underbrace{1 + f}_{\text{total funds available}} \geq \underbrace{(1 - \theta_1) + s_2 (1 - \theta_2)}_{\text{funding needs}}
  \] (1)
Multiple equilibria in a special case

- No default if,
  \[ s_i p_i \geq s_i (1 - \theta_i) \]

- Suppose sufficient funding for both countries:
  \[ \frac{1 + f}{\text{total funds available}} \geq (1 - \theta_1) + s_2 (1 - \theta_2) \]  
  \[ (1) \]

Possible equilibria

1. Country 1 is safe (=reserve asset), country 2 defaults:
   \[ p_1 = 1 + f, \quad p_2 = 0 \]
   Investor return = \[ \frac{1}{1 + f} \]

2. Country 2 is safe (=reserve asset), country 1 defaults:
   \[ p_1 = 0, \quad p_2 = 1 + f \]
   Investor return = \[ \frac{s}{1 + f} \]

3. Both countries safe, two reserve assets... but unstable
Multiple equilibria

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  \[ s_i p_i \geq s_i (1 - \theta_i) \]

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Possible equilibria

1. Country 1 is safe (=reserve asset), country 2 defaults:
   \[ p_1 = 1 + f, \quad p_2 = 0 \]
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2. Country 2 is safe (=reserve asset), country 1 defaults:
   \[ p_1 = 0, \quad p_2 = 1 + f \]
   Investors return = \( \frac{s}{1 + f} \)

3. Both countries safe, two reserve assets... but unstable

4. If \( s_2 = 0 \), country 1 is safe (Japan?)
Full characterization as function of $s, f$ and $\theta$

**Global games technique:**

- Unobserved relative fundamentals (higher $\tilde{\delta}$ means country 1 is stronger):
  
  \[
  1 - \theta_1 = (1 - \theta) \exp (-\tilde{\delta}) ; \\
  1 - \theta_2 = (1 - \theta) \exp (+\tilde{\delta}) .
  \]

- Each investor receives a noisy private signal before investing
  
  \[
  \delta_j = \tilde{\delta} + \epsilon_j
  \]

- Take $\tilde{\delta} \in [-\bar{\delta}, \bar{\delta}]$ (any cdf, but wide support) and noise
  \[
  \epsilon_j \sim U [-\sigma, \sigma]
  \]
  
  - We will (mostly) look at $\sigma \to 0$: fundamental uncertainty vanishes, but strategic uncertainty remains

- **Timing assumptions:**
  
  - Investors place *market orders* to buy debt
  - Country default decision after orders are submitted
Returns for $\tilde{\delta} = 0$ (Rollover risk)

Given proportion $x$ investing in country 1, no default if:

$$x \geq \frac{1 - \theta_1 (\tilde{\delta})}{1 + f} \quad \text{(country 1)}$$

$$1 - x \geq \frac{s (1 - \theta_2 (\tilde{\delta}))}{1 + f} \quad \text{(country 2)}$$
Returns for $\tilde{\delta} = 0$ (Liquidity)

Liquidity/market depth: country 2 price-rises/return-falls faster

Given proportion $x$ investing in country 1, conditional returns are

$$\frac{1}{p_1} = \frac{1}{(1 + f)x} \quad \text{and} \quad \frac{1}{p_2} = \frac{s}{(1 + f)(1 - x)}$$
Strategy space

Threshold Equilibrium:

- Let $\phi(\delta_j)$ be investment in country 1 of agent with signal $\delta_j$
- Consider threshold strategies

  If $\delta_j > \delta^*$ invest in country 1 i.e. $\phi=1$; otherwise country 2 i.e. $\phi=0$

- The equilibrium cutoff $\delta^*$ is determined by indifference of marginal investor with signal $\delta_j = \delta^*$
  - Must be indifferent between investing in country 1 versus 2

- We can prove uniqueness of the equilibrium, at least for some part of the parameter space, among monotone functions $\phi(\delta_j)$. 
Expected returns

- Marginal investor $\delta_j = \delta^*$ does not know other investors’ signals
  - He asks, suppose fraction $x \in [0, 1]$ of investors have signals $> \delta_j$
  - Marginal agent backs out true $\tilde{\delta}$ for given $x$ as follows
    $$\tilde{\delta} = \delta^* + (2x - 1)\sigma$$
  - Take $\sigma \to 0$ so only strategic uncertainty remains....
  - Global games result: $x \sim U[0, 1]$ from the view of marginal investor, for any prior of $\tilde{\delta}$

- Integrating over possible values of $x \sim U[0, 1]$ gives expected profits
  $$\Pi_1 = \int_{\frac{1}{1+\theta}}^{1} \frac{1}{(1+\theta)e^{-\delta^*}} \frac{1}{(1+f)x} dx = \frac{1}{1+f} \left( \ln \frac{1+f}{1-\theta} + \delta^* \right)$$
  and
  $$\Pi_2 = \int_{0}^{\frac{1+f-s(1-\theta)e^{\delta^*}}{1+f}} \frac{s}{(1+f)(1-x)} dx = \frac{1}{1+f} s \left( -\ln s + \ln \frac{1+f}{1-\theta} - \delta^* \right)$$
Expected returns

For marginal agent, proportion of investors in country 1 is \( x \):

- \( \Pi_1 = \) Integral under green curve
- \( \Pi_2 = \) Integral under red curve
Threshold

- Threshold $\delta^*$ is determined by the indifference condition
  \[ \Pi_1 (\delta^*) = \Pi_2 (\delta^*) \]

- Solving for $\delta^*$ (recall that $s \in (0, 1]$)
  \[ \delta^* = -\frac{1-s}{1+s} z + \frac{-s \ln s}{1+s} \]
  where we define “aggregate funding conditions”
  \[ z \equiv \ln \frac{1+f}{1-\theta} > 0 \]

- High $z$ means high savings (“savings glut”), good average fundamentals, low average interest rates
  - Low $z$ is opposite
- $\tilde{\delta}^*$: lowest value of $\tilde{\delta}$ so that country 1's bonds are reserve asset
Graphically $\delta^*$ as function of country 2 size

Country 1 is reserve asset if fundamental $\tilde{\delta} > \delta^*$
Graphically $\delta^*$ as function of country 2 size

Country 1 is reserve asset if fundamental $\tilde{\delta} > \delta^*$
When will world switch?

- In high $z$ world (savings glut)
  - US Treasury size: Debt = $12.7tn, (CB money $\approx$ $4.6tn): maximum liquidity for the world
  - Even if US fiscal position is worse than others (i.e. $\delta^* < 0$)
  - ... Switch not on the horizon

- Unless macro moves to low $z$ world
  - US Treasury size becomes a concern – can the country rollover such a large debt?
  - Investors coordinate on smaller debt country
  - Germany? Debt = $1.5tn
Era of UK consol bond

- UK government debt was reserve asset until sometime after WWI
  - US GDP exceeds UK GDP by 1870
  - In 1890, UK Govt Debt ≈ 3 × US Govt Debt
  - UK Debt/GDP = 0.43, US Debt/GDP = 0.10
Relative fundamentals

- Relative fundamentals/GE in safe assets is central to our model
  - Take model with no coordination, where repayment is equal to surplus ($\theta$) and world interest rate is normalized to one.

\[
p_1 = E[\theta_1], \quad p_2 = E[\theta_2]
\]

- Our model (for $\delta^* = 0$)

\[
\begin{align*}
\theta_1 > \theta_2 & \implies p_1 = 1 + f, \quad p_2 = 0 \\
\theta_1 < \theta_2 & \implies p_2 = 1 + f, \quad p_1 = 0
\end{align*}
\]

- US fiscal position is weaker now than before, but still better than everyone else

- Same for Germany within Eurozone
Negative $\beta$

Take an extreme case where country 1 is a.s. reserve asset, $\tilde{\delta} > \delta^*$

- Suppose there is some recovery even in default $L_i = \theta_i$
- Country 1 bond price and return ($R$)

\[
p_1 = 1 + f - sp_2 \quad \quad \quad R = \frac{1}{1+f-sp_2}
\]

Country 2 bond price $p_2 = \frac{\theta_2}{R}$ (no arbitrage)

- Solving:

\[
p_1 = \frac{1 + f}{1 + s\theta_2} \quad \quad \text{and} \quad \quad p_2 = \frac{1+f}{1+s\theta_2} \theta_2
\]
Negative $\beta$

Take an extreme case where country 1 is a.s. reserve asset, $\tilde{\delta} \gg \delta^*$

- Suppose there is some recovery even in default $L_i = \theta_i$
- Country 1 bond price and return ($R$)

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p_1 = \frac{1 + f}{1 + s\theta_2} \quad \text{and} \quad p_2 = \frac{1+f}{1+s\theta_2} \theta_2
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- Small shock to $\theta_1 \downarrow$ has no effect on $p_1$, but $\theta_2 \downarrow \Rightarrow p_1 \uparrow$
- Reduce world average fundamentals $\theta_1, \theta_2$ equally:
  - Reduces $p_2$, increases $p_1$
  - Reserve asset has “negative $\beta$”
Negative $\beta$

Take an extreme case where country 1 is a.s. reserve asset, $\tilde{\delta} >> \delta^*$

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- Solving:

$$p_1 = \frac{1 + f}{1 + s\theta_2} \quad \text{and} \quad p_2 = \frac{1 + f}{1 + s\theta_2} \theta_2$$

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- Reduce world average fundamentals $\theta_1, \theta_2$ equally:
  - Reduces $p_2$, increases $p_1$
  - Reserve asset has “negative $\beta$”
- Lehman shock: Negative shock to US and world fundamentals
  - Treasury yields fall (alternatives rise)
Country 1 $\beta_1 = \frac{\text{Cov}(p_1, \theta_1)}{\text{Var}(\theta_1)}$, as function of relative fundamental $\delta$. 

$\theta \sim U[0.1, 0.6], s=0.9, f=0.1, l=0.7$
Switzerland?

- What if there were “full-commitment” reserve assets available to investors?
  - Switzerland: Debt = $127bn, (CB money ≈ $500bn)
  - Denmark: Debt = $155bn
  - US: Debt = $12.7tn, (CB money ≈ $4.6tn)

- Implicit assumption in our analysis is that substantially all of reserve asset demand is satisfied by debt subject to rollover risk
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- US: Debt = $12.7tn, (CB money ≈ $4.6tn)
- Implicit assumption in our analysis is that substantially all of reserve asset demand is satisfied by debt subject to rollover risk
- Define
  \[ \hat{f} = f - p_s s \]
  where \( s \) is quantity of “full-commitment” assets
  - In equilibrium \( p_s \) is set based on expected return from investing in country 1/country 2.
  - Otherwise, model is as analyzed based on total demand of \( \hat{f} \)
Two reserve assets ("joint safety")

- Monotone threshold strategies, only one reserve asset
  - \( \phi(\delta_j) = 1 \) if \( \delta_j > \delta^* \), o.w. 0; where \( \phi \) is investment in country 1
Two reserve assets ("joint safety")

- Monotone threshold strategies, only one reserve asset
  - $\phi(\delta_j) = 1$ if $\delta_j > \delta^*$, o.w. 0; where $\phi$ is investment in country 1
- If we allow for non-monotone "oscillating" strategies:
  - $\phi(\delta_j)$: 1,0,1,0,1,0... in a non-monotone fashion (not quite "mixing," but similar)
  - Then, for high $z > z_{HL}$, joint safety for values of $\tilde{\delta}$ in GRAY

\begin{align*}
  s &= 0.25 \\
  z_{HL} & \quad \text{gray}
\end{align*}
Sovereign choices

- Debt size ($s$), fundamentals ($\theta$), are choice variables
  - Externalities in model
  - Role for coordination

- Security design as coordination
Eurobonds and coordination

- Policy proposals to create a Euro-area reserve asset
  - Proceeds to all countries, so all countries get some seignorage
  - Flight to quality is a flight to all, rather than just German Bund

- We study: Countries issue two bonds:
  - A common bond in $\alpha$ share
  - An individual bond in $(1 - \alpha)$ share
  - Common bond is pooled bond (essentially a “bundle”), for which each country is responsible for paying its respective share of the obligation
  - No cross-default provisions (structure is closest to “ESBIES”)

- We set aside moral hazard considerations which are likely first-order
Common bond and individual bonds

- **Two-stage game**
  - **Stage 1:**
    - Countries issue common bonds: $\alpha$ (large) and $\alpha s$ (small)
    - Investors pay $f - \hat{f}$, so common bond price
      \[
p_c = \frac{f - \hat{f}}{\alpha (1 + s)}
      \]
    - Split proceeds $\alpha p_c \frac{s_i}{s_i + s_{-i}}$
  - **Stage 2:**
    - Investor gets signal $\delta_j$
    - Individual country bonds issued at prices $p_1$ and $p_2$
    - Investors invest remainder of funds $1 + \hat{f}$ into individual (non-bundled) bonds
If country—$i$ defaults, it does so on both individual and portion of common bond

New no-default condition:

$$
(1 - \alpha)p_1 + \theta_1 + \alpha p_c \geq 1
$$

$$
(1 - \alpha)p_2 + \theta_2 + \alpha p_c \geq 1
$$

Importantly, common bond proceeds are allocated in a state-independent way across the two countries

Contrast this with the “winner takes all” funding provided by the individual bonds; this is a state-dependent allocation
Why might this work?

- In basic model ($\alpha = 0$) no default if,
  \[ s_ip_i \geq s_i(1 - \theta_i) \]

- Suppose global funds exceeds funding need:
  \[ 1 + f \geq (1 - \theta_1) + s_2(1 - \theta_2) \]
  
  total funds available \hspace{1cm} sum of individual funding needs

- Multiple equilibria....
Why might this work?

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$$s_i p_i \geq s_i (1 - \theta_i)$$

- Suppose global funds exceeds funding need:

$$\underbrace{1 + f}_{\text{total funds available}} \geq \underbrace{(1 - \theta_1) + s_2 (1 - \theta_2)}_{\text{sum of individual funding needs}}$$

- Multiple equilibria....

- When $\alpha = 1$, neither country defaults if,

$$\underbrace{1 + f}_{\text{total funds available}} \geq \underbrace{(1 - \theta_1) + s_2 (1 - \theta_2)}_{\text{funding need of common bond}}$$

- Security design coordinates investor actions
  - Flight to the reserve asset generates stable funding for both countries
Common bond equilibrium

- Stage 2 game: investors with $\hat{f}$
  - Default conditions for each country, and individual bond prices $p_i$
  - Almost same as previous analyses
- Stage 1 game sets investment in common bond $f - \hat{f}$ based on:

\[
E[R_c] = E[R_{stage2}]
\]
Equilibrium as function of $\alpha$

$s=0.5 \_ z=1.$

- High $\alpha > \alpha^*$ ⇒ joint safety equilibrium always
- Low $\alpha < \alpha_{HL}$ ⇒ single reserve asset, threshold equilibrium
- For $\alpha \in [\alpha_{HL}, \alpha^*]$ both equilibria are possible
Debt size and fundamentals:

- Suppose country $i$ can choose size, $S_i$
  - Debt float is $S_i$
  - Surplus is adjusted to $\theta_i S_i$ - i.e. keep tax revenues to debt constant
- Suppose country $i$ can separately choose to increase surplus by $\delta_i$

$$1 - \theta_1 = (1 - \theta) \exp (-\tilde{\delta} - \delta_1)$$
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- Suppose country $i$ can choose size, $S_i$
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$$1 - \theta_1 = (1 - \theta) \exp \left( -\tilde{\delta} - \delta_1 \right)$$

- Easy to solve the model:

$$\delta^* = \frac{S_2 - S_1}{S_1 + S_2} z + \frac{S_1 \ln S_1 - S_2 \ln S_2}{S_1 + S_2} - \delta_1 \frac{S_1}{S_1 + S_2} + \delta_2 \frac{S_2}{S_1 + S_2}.$$ 

- One obvious effect:
  - Increasing surplus always increases reserve asset status
  - E.g., higher $\delta_1$ reduces $\delta^*$ because country 1 become safer
- Less obvious, effect of changing $S_i$
Crowding out/contagion

▶ Take,
\[ \delta^* (S_1, S_2) = \frac{S_2 - S_1}{S_1 + S_2} z + \frac{S_1 \ln S_1 - S_2 \ln S_2}{S_1 + S_2}. \]

▶ Effect of increasing \( S_1 \) on \( \delta^* \):
\[ h(S_1, S_2; z) \equiv \frac{\partial \delta^* (S_1, S_2)}{\partial S_1} = \frac{1}{(S_1 + S_2)^2} (S_1 + S_2(\ln S_1 + \ln S_2 + 1 - 2z)). \]

▶ Decreasing in \( z \), negative for large \( z \);
▶ Expanding US debt can increase US reserve asset status
  ▶ Decreases other country's position
Endogenous choices:

- Suppose country 1, 2 have “natural” debt size \((s_1^*, s_2^*)\) and choose size:
  \[
  \max_{S_1} -\delta^* (S_1, S_2) - c(S_1 - s_1^*).
  \]

- Reduce default probability subject to adjustment costs
  \[
  \max_{S_2} +\delta^* (S_1, S_2) - c(S_1 - s_1^*).
  \]

- Equilibrium:
  \[
  h(S_1, S_2; z) = c'(S_1 - s_1^*) \quad \text{and,} \quad h(S_2, S_1; z) = c'(S_2 - s_2^*).
  \]
Equilibrium via a phase diagram

- High $z$ case; $\delta^* = 0$ along diagonal

\[
\frac{\partial \delta^*}{\partial S_2} = h(S_2, S_1) = 0
\]

\[
\frac{\partial \delta^*}{\partial S_1} = h(S_1, S_2) = 0
\]
Equilibrium via a phase diagram

- $\delta^* = 0$ along diagonal

\[ h(S_1, S_2) = 0 \]

\[ h(S_2, S_1) = 0 \]
Conclusion

- US government debt is reserve asset because:
  - Good relative fundamentals
  - Debt size is large, and world is in high demand for reserve asset (savings glut)
    - Nowhere else to go

- Economics of reserve asset suggest that there can be gains from coordination
  - Eurobonds as coordinated security-design