Exchange market pressure in interest rate rules

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Abstract

Many central banks pursue some kind of exchange rate objective. We derive what variables the central bank should look at when setting the interest rate to implement a given objective. Exchange market pressure (EMP), the tendency of the exchange rate to change, emerges as the key variable. This yields a policy rule for the interest rate where EMP is added to, say, a Taylor rule. The coefficient for EMP depends on two structural parameters, namely the effectiveness of the interest rate to ward off depreciation, and the degree of exchange rate management. The rule can implement many regimes, from floating to intermediate to fixed rates. It can be applied to many models, and we illustrate it in a New Keynesian model for a small open economy.

Key words: exchange market pressure, exchange rate regime, fixed exchange rate, monetary policy, open economy Taylor rule.
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1 Introduction

Most central banks engage in some form of exchange rate management, particularly in small open and emerging economies; IMF (2022). Recent experience has again illustrated that emerging economies tighten policy to stabilize forex markets, as World Bank (2019) and BIS General Manager Carstens (2022) argue. Our aim is to find the variable(s) the central bank should look at when setting its policy instrument to achieve a given exchange rate objective. We provide a novel derivation for that. It reveals an important role for exchange market pressure, EMP, a variable introduced by Girton and Roper (1977). Loosely speaking, EMP is the tendency of the exchange rate to change, where positive (negative) EMP means depreciation (appreciation) pressure.[1] EMP is a function of other variables, such as the foreign interest rate and home minus foreign labor productivity, determined by the theoretical model at hand, as we will illustrate. So, EMP is a key channel through which such variables matter for policy.

We derive this in a setting where we aim for generality. That is, by exploiting the information on exchange rate determination that is already in the model, we try to limit additional assumptions. Moreover, the exchange rate objective can be a fixed, some intermediate exchange rate regime, or the (perfectly free) float, regarding the level or the change of the exchange rate. Our finding applies to several policy instruments, such as the interest rate and official forex intervention. This paper focuses on the interest rate, as it is a key policy instrument of many central banks and it provides the simplest framework for introducing the idea. We thus conclude that EMP results from a derivation that is applicable to many settings and argue that EMP is a key variable for central banking.

We take the exchange rate objective as given, focusing on what central banks actually pursue. So, we do not derive the degree of exchange rate management they should pursue. The latter would depend on the imposed economic structure,[2] while we prefer avoiding a specific structure to broaden the relevance of our insights. For simplicity

[1] The formal definition of EMP follows in Section 2.3.1 and is taken from the EMP literature, in particular Weymark (1995). Interesting applications include the Financial Stress Index of IMF (2009), Frankel and Xie (2010), Aizenman et al. (2017), and Pinter and Pourroy (2023).

[2] Engel (2014) concludes that welfare-based optimal monetary policy analysis in open-economy models is still in the early stages, but that the analysis to date suggests a role for exchange rates in an optimal rule. Their importance depends on the economic structure. Schmitt-Grohé and Uribe (2016) show that in case of downward nominal wage rigidity optimal exchange rate policy calls for large devaluations during crises to ensure full employment. Davis et al. (2018) show that as central bank credibility falls and thereby the ability to commit to future policy, a highly open economy will quickly find it optimal to set the interest rate to peg the nominal exchange rate as the single mandate. Buffie et al. (2018) analyze less developed countries that pursue inflation targeting. In a float, currency substitution causes a high risk of indeterminacy (multiple equilibria), as well as escalation of inflation shocks. Both problems disappear by tight management of the nominal exchange rate.

and following the related literature, we assume that the regime is fully credible.

What do central banks actually do? Some are explicit and consider what they call "pressure." For example, Danmarks Nationalbank (2023) writes "in situations with upward or downward pressure on the krone, Danmarks Nationalbank unilaterally changes its interest rates in order to stabilise the krone." Likewise, the Hong Kong Monetary Authority (2009) describes its "automatic interest rate adjustment ... against downward pressure on the exchange rate."[3] The idea is that high selling pressure requires a high interest rate. Indeed, Carstens (2022) reports that in emerging markets the impact of pressure has been contained recently, at the cost of very high interest rates.[4]

Despite the relevance of pressure in actual policy, there is no formalization yet. We show that EMP is close to what central bankers mean by pressure, and we derive that it is natural to have EMP in a policy rule. A model can then reveal the determinants of EMP. This can help policymakers to develop new indicators of pressure, which they can then use in policy making. So, our derivation provides theoretical and practical support for actual policy. The reverse is also true: actual policy confirms the realism of our approach.

Our derivation leads to a new interest rate rule that contains EMP and implements the exchange rate objective exactly. The rule has three main novelties. First, it extends a domestically-oriented rule, such as the Taylor rule, by adding EMP in deviation from the exchange rate change that is acceptable according to the objective. Excess pressure implies a high interest rate, in line with actual policy. This paper thus connects two strands of the literature, that on interest rate rules and EMP. It shows that EMP, which is often proxied in applied work, also matters for theory and policy.

The second novelty is that the coefficient for EMP depends on the interest rate effectiveness to ward off depreciation: the larger the effectiveness, the less intensively the policymaker should use the interest rate for given EMP. This is a natural property but still a novel feature of our rule. The effectiveness is no additional parameter, as it is determined by existing model parameters. Our rule thus automatically accommodates changes in those parameters, ensuring that the objective remains implemented.

[3]Hong Kong has a currency board system based on an automatic interest rate adjustment mechanism. In case of downward pressure, the central bank purchases Hong Kong dollars from banks so as to increase market interest rates and thereby capital inflows and achieve exchange rate stability. Because of its focus on the interest rate, we use Hong Kong as an example in this paper. An alternative would be to model policy as unsterilized intervention, but then the main idea relevant for us would be similar: the central bank responds to pressure and exploits the interest rate.

[4]Calvo and Reinhart (2002) provide further examples. He et al. (2011) from the HKMA write that they monitor "foreign exchange market pressure" in their daily work. Mohanty (2013) reports that in a BIS survey among emerging market central bank intervention almost 80% said that curbing speculative pressures on the exchange rate was the most important priority.

The third novelty of our rule is that it can handle many exchange rate objectives. For example, we introduce a "weighted fixed-floating regime", a weighted combination of the fixed and floating exchange rate regimes, governed by the degree of exchange rate management. The higher this degree, the stronger the interest rate responds to a given EMP. This degree is a new structural parameter, chosen by the policymaker. It facilitates a clean analysis of the impact of exchange rate management, making our rule attractive to use in theoretical analyses.

The traditional way of modeling the interest rate in case of an exchange rate objective is to add the exchange rate gap (realized minus target value) to a Taylor rule, as in Monacelli (2004), or to the foreign interest rate, as in Benigno et al. (2007). This has contributed to valuable insights, in other papers as well.[5] However, we cannot use such an approach of adding a preselected variable to answer our research question. After all, we want to know what variables to include, and how. That is why we *derive* our rule.

Even though our approach has a different focus than traditional rules, and the scope of this paper is not to recommend a new interest rate rule, one can link our rule to them, as follows. The traditional rules rely on the realized exchange rate. If we can use that too, our rule can replicate both traditional rules. Hence, our derivation provides support for them. The bonus is that our derivation unlocks the two structural parameters underlying the exchange rate gap parameter — the interest rate effectiveness and the degree of exchange rate management. We have discussed the benefits of this before, in terms of accommodating structural change and facilitating clean theoretical analyses.

Without using the realized exchange rate, our rule continues to apply. But then EMP is no longer observed. In theoretical analyses that is no problem. In practice, the unobservability can create a difficulty. This, however, matches reality, as central bankers encounter unobservable pressure in policy making. They use indicators to monitor pressure. As EMP is a function of other variables, our framework provides the opportunity to find other variables that policy makers can look at.

Finally, our rule helps to derive a new simple rule, one that only contains observables. The simple rule has the lagged exchange rate instead of the contemporaneous one used in traditional rules. That improves implementability, in line with the McCallum (1997) advice. Moreover, Spange and Wagner Toftdahl (2014) document that, after a persistent weakening of the Danish krone, the Danish central bank may increase the

[5] The former rule has also been used by Engel and West (2005), Corsetti and Müller (2015), and Galí and Monacelli (2016), for example. The latter rule has been applied by Benigno (2004) and Born et al. (2013), among others. These papers focus on the nominal exchange rate. Instead, some authors add the real rate to an interest rate rule, as in Clarida et al. (1998) and Mimir and Sunel (2019). Oskolkov (2023) studies changes of nominal and real rates. All these papers add preselected variables, and we let Monacelli (2004) and Benigno et al. (2007) represent this approach.

interest rate to strengthen the currency. Our simple rule reflects that sequence.

The structure of the rest of the paper is as follows. In Section 2 we derive the interest rate rule and show the relevance of EMP. Section 3 discusses the characteristics of the rule and introduces the simple rule. In Section 4 we set out a New Keynesian DSGE model for a small open economy to illustrate our method and derive the interest rate rule and EMP for that model. Section 5 illustrates their characteristics using a simulation study. Section 6 concludes.

2 Interest rate rule

For a given exchange rate regime, the goal is to derive the variables the monetary authorities should look at when setting the interest rate to implement that regime. We do so by deriving an interest rate rule in a two-country setting. The domestic monetary authority, being the central bank throughout this paper, pursues some degree of exchange rate management as one of the policy goals (the float is a valid special case). The foreign authority does not try to control the exchange rate.

2.1 Simple setting: UIP

Only in this section do we assume that the uncovered interest parity (UIP) holds. This is purely for expositional simplicity, to set out the main ideas of our derivation. Those ideas hold irrespective of the validity of UIP. Section 2.2 generalizes the framework.

2.1.1 Exchange rate function if UIP holds

Let s_t be the (logarithm of the nominal) exchange rate at time t, which is the domestic currency price of one unit of foreign currency. $\mathbb{E}_t\left\{s_{t+1}\right\}$ is the expected next-period exchange rate conditional on information available in period t. Finally, i_t and i_t^* denote the one-period domestic and exogenous foreign interest rates, respectively, both at time t. Hence, if UIP holds (in log-linearized form), the exchange rate satisfies

$$s_t = -i_t + i_t^* + \mathbb{E}_t\left\{s_{t+1}\right\}. \tag{1}$$

2.1.2 First try

If the central bank pursues a fixed exchange rate target s^t, a researcher can incorporate that by adding $s_t = s^t$ to the model (for all t). This determines the regime exogenously. That can suffice for some fixed rate analyses, but it will not tell us what variables the central bank should study to implement the target, the focus of this paper.

A rule that delivers the same regime endogenously is $i_t = i_t^* + \mathbb{E}_t\{s_{t+1}\} - s^t$, as setting this rate implies $s_t = s^t$, by construction.[6] It shows that i_t^* and $\mathbb{E}_t\{s_{t+1}\}$ are important variables for the central bank to implement the regime.

Our approach, presented below, improves on this. First, it provides additional insights into what variables matter for exchange rate policy. Next, our approach also applies to settings where UIP does not hold, so that we can obtain a more complete picture of the relevant variables. Third, we will show how to deal with the impact of i_t on $\mathbb{E}_t\{s_{t+1}\}$. Moreover, from the above rule it is not clear how to implement other exchange rate regimes, while our rule can deliver many regimes, including the float, a weighted combination of fixed and float, and leaning against the wind. Finally, many traditional rules have a domestically-oriented part to which an exchange rate part is appended. Our rule has that form as well, which facilitates comparison.

2.1.3 Derivation of the rule if UIP holds

Our method consists of three steps. The first two steps just rewrite a relation that is already in the model at hand, while the third step brings in a new piece of information.

The first step is to ensure that the interest rate is consistent with the exchange rate generated by the model. In the current highly-simplified setting, this is accomplished by UIP (1), which we rewrite as

$$i_t = i_t^* + \mathbb{E}_t\{s_{t+1}\} - s_t. \tag{2}$$

The second step recognizes that i_t reflects not only forex policy (exchange rate management) but also domestically-oriented policy, for example to keep inflation under control. There is a large literature on rules for the latter part. We leave the choice free and let i_t^d denote the outcome of the chosen rule. We call the rule for i_t^d the domestically-oriented part of the rule for i_t (for brevity, we will often leave out the words "the rule for"). An example is a Taylor rule like $i_t^d = 1.5\pi_t$, where π_t is inflation and the inflation target is zero, for simplicity.[7]

The other part of i_t is the forex-policy part. For the float, this part is zero. For exchange rate management, the current section intends to derive what this part should be. To already provide an example, consider Monacelli (2004) and an exchange rate

[6] This rule is similar to the price level targeting rule for a closed economy in Adão et al. (2011), who use a Fisher equation with the current price level substituted by the target.

[7] Being domestically oriented does not necessarily exclude implicit reactions of the interest rate to exchange rate fluctuations. For instance, reacting to CPI inflation implies reacting to the exchange rate change. The point is that in i_t^d the exchange rate is no objective by itself.

target of zero. His rule then sets the forex-policy part to some multiple of s_t, say $4s_t$. As in the Taylor rule, the larger this factor, the stronger the exchange rate management.

As in the literature, such as Monacelli (2004), both the domestically-oriented and forex-policy parts depend on the actual economic situation, and thus on i_t, say. So, it is not that having an explicit split between the two parts introduces a different policy or so; the parts just group the interest rate drivers.

We make the domestically-oriented and forex-policy parts explicit in (2) by adding and subtracting i_t^d, so that

$$i_t = i_t^d + s_t^d - s_t, \tag{3}$$

where we define

$$s_t^d = -i_t^d + i_t^* + \mathbb{E}_t \left\{ s_{t+1} \right\}. \tag{4}$$

Hence, (3) shows that $s_t^d - s_t$ reflects forex policy. Definition (4) shows that s_t^d is the exchange rate resulting from UIP when substituting i_t by the domestically-oriented interest rate i_t^d. Importantly, the substitution does not affect $\mathbb{E}_t \{s_{t+1}\}$: this remains *actual* expectations, not expectations based on i_t^d or so, because there is no policy change.[8] This also implies that s_t^d is just a combination of variables that already exist in the model at hand; it does not add information to the model.

The intuition of s_t^d is as follows. As usual, an increase in the foreign interest rate i_t^* or investors' expectations $\mathbb{E}_t \{s_{t+1}\}$ yields excess supply of the home currency, and thus a weaker home currency; the higher s_t^d represents that. The relevance of i_t^d is similar, though we have to account for the fact there is no actual interest rate change, so that setting i_t^d does not affect $\mathbb{E}_t \{s_{t+1}\}$. Specifically, if the central bank lowered the interest rate from i_t to i_t^d to increase inflation, say, excess supply would weaken the currency, and this is represented by s_t^d being higher than s_t.

The presence of i_t^d instead of i_t in (4) together with actual expectations $\mathbb{E}_t \{s_{t+1}\}$ shows that the contemporaneous impact of exchange rate policy is removed, that is, the impact not involving expectations. Hence, s_t^d is the notional exchange rate that prevails when taking out the contemporaneous impact of exchange rate policy. As s_t^d combines an alternative interest rate with actual values of the other determinants, we call s_t^d the intermedial exchange rate. It is just a convenient combination of existing variables, not a necessary variable in the model.

[8] Because — as always in the paper — i_t^d and expectations are both based on the actual interest rate i_t, s_t^d is *not* the counterfactual exchange rate based on i_t^d and expectations and variables consistent with i_t^d. The latter would boil down to the exchange rate under a float, but that is not the regime in place, as we do not study a policy change. So, s_t^d is not the exchange rate in another general equilibrium.

The third and final step of our derivation proposes the rule

$$i_t = i_t^d + s_t^d - s_t^o, \tag{5}$$

where s_t^o denotes the exchange rate objective. The forex-policy part of the rule is $s_t^d - s_t^o$. The intuition is that a high s_t^d reflects that, abstracting from the contemporaneous impact of policy, investors intend to sell the currency, and to the extent that it exceeds the objective s_t^o, the central bank has to set a high interest rate. Because in the float the central bank sets $i_t = i_t^d$, here the excess value $s_t^d - s_t^o$ is zero, implying that here s_t^o is specified as s_t^d. For the fixed rate, s_t^o is the target s^t. Other regimes are handled by other choices for s_t^o, as described in Section 2.4.

It is clear that adding rule (5) to the model is equivalent to adding $s_t = s_t^o$. Hence, the rule implements the exchange rate objective exactly, by construction. The reverse is also true: to implement the objective, the central bank should use rule (5).

We have thus derived what the forex-policy part of the rule should be, $s_t^d - s_t^o$, and that this adds to the domestically-oriented part, i_t^d, giving rule (5). For example, if $i_t^d = 1.5\pi_t$, the float results from $i_t = 1.5\pi_t$, the fixed rate where $s^t = 0$ results from the rule $i_t = 1.5\pi_t + s_t^d$, and to implement an intermediate exchange rate regime around $s^t = 0$, one could use $i_t = 1.5\pi_t + 0.8s_t^d$. Comparing our examples to the Monacelli (2004) example given above, $i_t = 1.5\pi_t + 4s_t$, shows that deriving the rule has resulted in a new driver of the forex-policy part, s_t^d instead of s_t. Section 3 analyzes this difference.

In the special case of the fixed exchange rate $s_t^o = s^t$, our rule yields the same outcome as the try in Section 2.1.2. Still, our approach reveals another variable of interest for the central bank in its exchange rate policy, which is i_t^d, entering via s_t^d in (4). For example, a drop in i_t^d reflects that, say, from an inflation perspective the central bank would like to lower the interest rate, which would then increase excess supply of its currency and thus s_t^d, which the central bank has to offset by exchange rate policy. Actually, here the central bank fully offsets the initial drop, in line with the incompatible trinity. The relevance of $-i_t^d + i_t^*$ for exchange rate policy is also in line with the symmetry expected for a relative variable such as the exchange rate.

2.2 More general setting, not assuming UIP

Because of the potential relevance of imperfect capital mobility and risk premia, as of here we no longer assume that the exchange rate depends on UIP. Instead, in Section 2.2.1 we deliberately try to abstain from strong restrictions on the exchange rate function, so as to obtain a framework that can accommodate many features of the forex market. The generalized derivation of the interest rate rule itself is in Section 2.2.2.

2.2.1 Exchange rate function

We assume that it is possible to solve the exchange rate s_t from the particular model at hand as a function of the exchange rate determinants. The interest rate i_t is one determinant, and we cluster its effects on s_t in two groups. First, i_t operates via contemporaneous channels. For example, a high i_t attracts capital and thus lowers s_t (appreciation). Or a high i_t weakens current consumption, reducing the home price level, increasing foreign demand for home goods, and appreciating the home currency, all in period t.

The second impact of i_t on s_t goes via expectations. For example, a high i_t may increase the currently expected interest rate next period, i_{t+1}, which then weakens expected consumption at $t+1$, and similar to the causal chain above leads to expected home appreciation at $t+1$, appreciating the home currency at t.

We now write the exchange rate function s in the form

$$s_t = s\left(i_t, E_t\right), \tag{6}$$

where the i_t argument represents the contemporaneous channels, and the vector E_t consists of expectations $\mathbb{E}_t\left\{.\right\}$ and all other exchange rate determinants. The (i_t, E_t)-separation will be convenient in the next section. It is not restrictive — it just splits the full impact of i_t on s_t into the two groups. For example, in the UIP Section 2.1 the function is (1), so that $E_t = [i_t^*, \mathbb{E}_t\left\{s_{t+1}\right\}]'$. Section 4.2 derives the (i_t, E_t)-form in our New Keynesian model, in particular formula (39), where (41) specifies E_t.

The variables in E_t depend on expectations, predetermined, and contemporaneous variables, but the separation implies that the latter no longer include the impact of i_t. So, contemporaneous variables such as goods prices, interest rates concerning other maturities than the one underlying i_t, national income, and fiscal policy are first cleaned for i_t by moving the i_t dependencies to the i_t argument, and then the remainder enters E_t. For example, consider goods prices. The second example of contemporaneous channels above, that a high i_t lowers goods prices and appreciates the currency, is captured by the i_t argument. What remains in the E_t argument is, for example, that lower expected future income weakens current consumption, causing lower prices and appreciation, and that exogenous technological progress via lower prices causes appreciation.

For simplicity, we impose that the s-function is linear in its first argument, that is,

$$s_t = -wi_t + s\left(0, E_t\right), \tag{7}$$

where the scalar

$$w = -\frac{\partial s}{\partial i}(., E_t) \neq 0 \tag{8}$$

reflects the effectiveness of the interest rate to counteract depreciation via all contemporaneous channels.[9] It follows from the parameters in the s-function, so w does not extend the number of free parameters. We leave out the time subscript from w for notational simplicity. Linearity holds in the UIP section, where s-function (1) has $w = 1$ and $s(0, E_t) = i_t^* + \mathbb{E}_t\{s_{t+1}\}$. Linearity also holds in our New Keynesian model, where w is a positive constant and $s(0, E_t) = v'E_t$ follows from (41).

2.2.2 Derivation of the rule

The derivation resembles the three-step structure of the UIP-based derivation in Section 2.1.3, and the intuitions provided there also apply here. The first step realizes that the model at hand determines how the interest rate i_t affects the exchange rate s_t, represented by s-function (7). To ensure that our rule implements the objective, we take that function and rewrite it as

$$i_t = \frac{1}{w}(s(0, E_t) - s_t). \tag{9}$$

The second step introduces the domestically-oriented interest rate i_t^d, as motivated in the UIP section, by adding and subtracting i_t^d. That yields

$$i_t = i_t^d + \frac{1}{w}\left(s_t^d - s_t\right), \tag{10}$$

where

$$s_t^d = -wi_t^d + s(0, E_t) = s\left(i_t^d, E_t\right). \tag{11}$$

The interpretation of both equations is the same as given below (4).

Third, we propose the rule

$$i_t = i_t^d + \frac{1}{w}\left(s_t^d - s_t^o\right), \tag{12}$$

where s_t^o is again the exchange rate objective. If UIP holds, $w = 1$, so that the rule becomes (5), and the intuition there also applies here. As another example, objective (16) implies rule (17), which in the New Keynesian illustration becomes (44).

As the model at hand delivers exchange rate function (7), and the latter is equiv-

[9] One usually considers w to be positive, that is, an interest rate increase appreciates the currency. In intuitive explanations below we will do as if w is positive, but we do not impose it in the derivation.

alent to (10), adding rule (12) to the model is again equivalent to adding $s_t = s_t^o$. Hence, the rule implements the exchange rate objective exactly at every t. Conversely, to implement the objective, the central bank should use rule (12). This is also a consequence of the fact that our approach exploits information that is already in the model, equation (7). An additional insight compared to the UIP-based rule (5) is that the interest-rate effectiveness w matters, and thus the structure of the economy: the more effective the instrument, the smaller its required use, as expected.

2.3 EMP as the key determinant

Exchange market pressure (EMP) is a concept introduced by Girton and Roper (1977). Intuitively, it represents the reluctance of investors to hold the domestic currency at the forex market. This reluctance tends to affect the exchange rate, and that may trigger the central bank to act. This resembles the idea of our interest rate rule. Indeed, we will show that the rule implies a prominent role for EMP in policy.

2.3.1 EMP definition

The idea of the EMP concept is to split the actual (relative) depreciation of the home currency, resulting from the interplay of investors and authorities, into a part reflecting the reluctance of investors to hold the currency, called EMP, and the policy-based part, which usually intends to counteract EMP. EMP applies to any exchange rate regime and can be positive as well as negative, where the latter means there is pressure on the currency to appreciate. One example is a fixed exchange rate that is under attack by speculators and where the attack is successfully offset by policy. Then EMP is positive, the policy-based counteracting depreciation is negative and offsets EMP exactly.

More formally, EMP_t is defined as the relative depreciation of the home currency at time t in the absence of exchange rate policy, while keeping expectations at the levels determined by actual policy. This is the standard definition in the EMP literature, due to Weymark (1995). The definition is unique and does not depend on a model. We here apply it to the setting where the interest rate is the policy instrument.[10]

[10]Klaassen and Jager (2011) set out how the interest rate appears in the EMP definition and provide empirical support for that. Our approach is also in line with that.

Apart from the unique definition of EMP_t, the EMP literature also offers two ways to quantify EMP_t. First, one can take a theoretical model and derive EMP_t as a function of fundamentals. This function varies across models, and Weymark (1995) and (13) with (43) in our paper provide examples. So, these are model-dependent quantifications of EMP, while the definition of the EMP concept itself is model independent.

Second, the EMP literature offers practical ways to measure EMP_t. In some papers, the term "EMP" not only refers to EMP itself, but also to the *measure* of EMP. EMP measures vary across papers and include the current value of the policy instrument, such as i_t. In contrast, in our paper

One key element in the definition is the absence of exchange rate policy. The interest rate rule in this situation is i_t^d, the domestically-oriented rule introduced in Section 2.1.3. So, EMP does not directly depend on the actual interest rate i_t.

Without the second key element in the EMP definition, the condition on expectations, the use of i_t^d would make EMP like the depreciation under a floating exchange rate regime. But that is not what EMP intends to capture; EMP is about the reluctance of investors to hold the currency in the *actual* regime. So, EMP uses the same values for the expectations as E_t defined earlier.

We can thus express the standard EMP definition in our notation as

$$EMP_t = s_t^d - s_{t-1}. \tag{13}$$

Therefore, just as s_t^d, also EMP_t adds no information to the model at hand. It is a function of existing variables.

2.3.2 EMP in the rule

Rewriting rule (12) using (13) yields

$$i_t = i_t^d + \frac{1}{w}\left(EMP_t - (s_t^o - s_{t-1})\right). \tag{14}$$

Given the derivation of the rule, we conclude that EMP emerges naturally as the key determinant of the forex-policy part of the interest rate. In this sense, our derivation reveals the insight that central banks should look at EMP. So, our rule guides policy. Because EMP_t is a function of other variables, EMP_t is the channel through which those variables matter for forex policy. That is, insofar they create pressure, the central bank should set the interest rate.

The rule says that the central bank has to set $i_t > i_t^d$ to ward off EMP_t insofar pressure exceeds the target depreciation $s_t^o - s_{t-1}$. The magnitude of $i_t - i_t^d$ is the amount of excess pressure converted into interest rate units by dividing by the effectiveness w of the interest rate instrument.

It is contemporaneous pressure that matters, not expected future pressure. This marks a difference with the inclusion of, say, expected inflation in some Taylor rules. The latter are typically used to model central bank policy to control inflation between today and a year ahead, say. Such a focus on the future is not what matters in exchange

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"EMP" always means EMP itself, not the measure, and we use EMP to explain the current value of the policy instrument. Note that EMP measures often include the change in foreign reserves divided by money supply. Klaassen and Jager (2011) explain why that is in line with the EMP definition, for a central bank using forex intervention as instrument.

rate management. The obvious example concerns the fixed rate: if today's interest rate does not offset the pressure to move away from the target today, there will be an immediate breakdown of the peg, irrespective of expected future developments. Hence, today's EMP_t is what matters for i_t.

2.3.3 Justifying and being supported by actual policy

As argued in the Introduction, central bankers consider what they call "pressure" on their currency at the forex market when implementing exchange rate management. A high s_t^d reflects that investors intend to sell the currency. That mimics what central bankers mean by pressure. The EMP variable relates s_t^d to the lagged rate s_{t-1} and the EMP literature calls this pressure. That is in line with the phrase "downward pressure" used by Danmarks Nationalbank (2023) and Hong Kong Monetary Authority (2009), and with He et al. (2011) from the HKMA, who write that they monitor "foreign exchange market pressure." This advocates a formalization of the word "pressure" by

$$\text{pressure} = EMP_t. \tag{15}$$

Because pressure matters in actual policy, our rule not only guides policy, but it also justifies and is supported by actual policy. As (14) makes the relevance of pressure explicit, we view that formula as the preferred representation of our rule. Still, we will use equivalent representations, such as (12), when convenient.

2.4 The rule for specific exchange rate regimes

Our rule can be combined with many exchange rate objectives s_t^o. The current section applies it to six regimes. Five are inspired by practice, namely the float, fixed rate, crawling peg without band, peg with possibly time-varying band, and a policy that moderates the rate of change (called "leaning against the wind"). IMF (2022) shows that these regimes cover the majority of the countries. Examples include the United States, Bulgaria, Nicaragua, China, and Brazil, respectively, albeit that we examine only one type of policy to implement the regime, that is, interest rate policy. The other regime is a weighted combination of the fixed and floating exchange rate regimes, which we introduce because it will be convenient in theoretical analyses and we will focus on it after this section to simplify the exposition.

In the float the central bank does not try to affect the exchange rate, so any tendency for the rate to move to a particular value is ignored by its interest rate policy. More formally, $s_t^o = s_t^d$. Our rule, represented by (12), then sets $i_t = i_t^d$, as expected.

For the fixed exchange rate the question at hand is what i_t the central bank should choose to ensure the exchange rate equals the target s^t. Substituting $s_t^o = s^t$ into (12) gives the interest rate that hits this target.

The weighted fixed-floating exchange rate is a weighted average of the fixed and floating rate regimes, where the given weight $\mu \in [0, 1]$ denotes the degree of exchange rate management, constant over time. The regime and the rule implementing it are

$$\text{Policy objective: } s_t^o = (1 - \mu)\, s_t^d + \mu s^t \tag{16}$$

$$\text{Interest rate rule: } i_t = i_t^d + \frac{1}{w}\mu \left(s_t^d - s^t\right). \tag{17}$$

For $\mu = 0$ this confirms the rule for the float. The higher μ, the more i_t responds to a given $s_t^d - s^t$, meaning tighter exchange rate management. For $\mu = 1$ the system represents the fixed rate. Here, on balance i_t^d is irrelevant for i_t, because a one percentage point lower i_t^d by itself motivates an equally lower i_t, but implementing that would cause a w %-points higher s_t, which would have to be offset by a one %-point higher i_t to maintain the peg (in (17), the latter increase in i_t comes from the w %-points higher s_t^d). Finally, for general μ, (17) and (10) imply $w\left(i_t - i_t^d\right) = \mu\left(s_t^d - s^t\right)$ and $s_t - s^t = (1 - \mu)\left(s_t^d - s^t\right)$, so that μ captures how much of $s_t^d - s^t$ the central bank offsets by policy, and $1 - \mu$ shows how much ends up in the actual exchange rate gap.

The crawling peg generalizes the fixed rate by having a time-varying target: $s_t^o = s_t^t$.

In the peg with band the exchange rate must lie in a band $[\underline{s}_t, \overline{s}_t]$. One example, inspired by Krugman (1991), is where $s_t^o = s_t^d$ if $s_t^d \in [\underline{s}_t, \overline{s}_t]$, but once the exchange rate tends to leave the band, the central bank uses the interest rate to make sure that the exchange rate settles at the nearest boundary, so $s_t^o = \underline{s}_t$ if $s_t^d < \underline{s}_t$, and $s_t^o = \overline{s}_t$ if $s_t^d > \overline{s}_t$. A special case is the one-sided band, as applied in Switzerland until 2015 and in the Czech Republic until 2017, where $\underline{s}_t$ restricts appreciation but $\overline{s}_t$ is infinite.

In the "leaning against the wind" regime the central bank aims at mitigating the change in the exchange rate. So, it counteracts the wind $s_t^d - s_{t-1}$, that is, EMP_t. This regime follows from the weighted fixed-floating regime by using s_{t-1} instead of s^t. Despite the practical relevance of leaning against the wind, the rest of this paper simply focuses on the exchange rate level. Substituting s^t below by s_{t-1} gives the features of our rule for the change.

3 Characteristics of the rule and relation to the literature

We have derived a new interest rate rule, not to recommend a new rule, but because our research question of finding relevant variables for exchange rate management differs from what existing rules focus on. Still, existing rules have been applied successfully in several studies, as indicated in the Introduction. In this section we thus not only show the properties of our rule, but also analyze how it can provide additional insights.

3.1 The rules to be discussed, and the regimes they imply

Traditional rules rely on the gap between the realized exchange rate and its target. The Monacelli (2004) rule adds this exchange rate gap to a Taylor rule, formalized by

$$i_t = i_t^d + \varphi_s \left(s_t - s^t\right), \tag{18}$$

where $\varphi_s \geq 0$. Substituting this into the exchange rate function (10) yields the implied exchange rate regime. That is, the resulting s_t is a weighted average of s_t^d and s^t, with weight $\frac{w\varphi_s}{1+w\varphi_s}$ on the latter. Hence, the rule implements the weighted fixed-floating regime, and the weight is our μ, where $0 \leq \mu < 1$. Monacelli does not intend to implement a specific regime. We show that computing w from (8) reveals μ and thereby the implied regime. The fixed exchange rate, $\mu = 1$, is approached by letting $\varphi_s \to \infty$.

Benigno et al. (2007) assume UIP.[11] They add the gap to the foreign interest rate,

$$i_t = i_t^* + \varphi_s^{BBG} \left(s_t - s^t\right), \tag{19}$$

where $\varphi_s^{BBG} > 0$.[12] This rule says that the central bank commits to raising i_t above i_t^* if s_t tends to exceed s^t. The authors prove that in equilibrium $s_t = s^t$, that is, the fixed exchange rate regime $\mu = 1$. The specific value of φ_s^{BBG} is irrelevant for this outcome.

Our rule is (17), given that we focus on the weighted fixed-floating regime. So, we have $\frac{1}{w}\mu$, a new variable s_t^d (and thus EMP_t), and we do not impose UIP. Our rule implements the weighted fixed-floating regime for all μ, that is, for $0 \leq \mu \leq 1$.

3.2 Using the realized exchange rate s_t

The traditional rules use the realized exchange rate s_t to determine the interest rate i_t. Instead, our rule has the intermedial rate s_t^d. The latter is not observed directly. Still, what matters is whether i_t can be computed when *all* available information is used, not just the rule. For one thing, the s-function (7) and s_t^d definition (11) add equations that may help. This section shows how using s_t and such equations indeed makes s_t^d calculable in the settings of Monacelli (2004) and Benigno et al. (2007), so that i_t can

[11] Monacelli (2004) also uses UIP in his model, but his rule does not hinge on it.

[12] They also present a similar non-linear version of (19), and a rule where the central bank reacts to the exchange rate change instead of level, which we do not discuss separately.

be computed. Hence, using s_t, there is no difference in computability between our and the traditional approaches. Nevertheless, even in this situation our approach can yield additional insights, as this section sets out.

3.2.1 Calculating s_t^d from s_t, and replicating both traditional rules

First, consider $\mu < 1$, the Monacelli (2004) setting. Substituting our rule (17) for $i_t - i_t^d$ in (10) shows that for the weighted fixed-floating regime the actual exchange rate is linked to the intermedial rate by $s_t - s^t = (1-\mu)\left(s_t^d - s^t\right)$. This indicates that only $1-\mu$ part of $s_t^d - s^t$ enters the actual rate; the rest is offset by policy. Hence, treating s_t as known, as the traditional rules do, implies that also s_t^d can be treated as known. Substituting the solution for s_t^d in our rule then expresses i_t in terms of s_t. In formula,

$$s_t^d = s^t + \frac{1}{1-\mu}\left(s_t - s^t\right) \tag{20}$$

$$i_t = i_t^d + \frac{1}{w}\frac{\mu}{1-\mu}\left(s_t - s^t\right). \tag{21}$$

Therefore, our rule replicates the Monacelli rule by choosing $\mu = \frac{w\varphi_s}{1+w\varphi_s}$.

Next, take $\mu = 1$, so the fixed rate, and assume UIP. This is the Benigno et al. (2007) setting. Consider s_t^d definition (4). As the rule is also adopted in $t+1$, the fixed rate is also in place there, so $\mathbb{E}_t\left\{s_{t+1}\right\} = s_t$. This yields s_t^d, which we then substitute into our rule to compute i_t:

$$s_t^d = i_t^* - i_t^d + s_t \tag{22}$$

$$i_t = i_t^* + s_t - s^t. \tag{23}$$

Hence, in the Benigno et al. setting our rule boils down to their rule, realizing that the specific value of $\varphi_s^{BBG} > 0$, here unity, does not matter.

In summary, the present section has demonstrated that in the weighted fixed-floating regime (and imposing UIP for the fixed rate) the realized s_t makes s_t^d calculable. Section 2 has shown that s_t^d drives i_t. Combining these implies that s_t delivers i_t. This explains how our approach can replicate both traditional rules. The replication strengthens the foundation and supports the usefulness of those rules in specific cases.

3.2.2 Structural parameters w and μ

In our rule (17) the relevance of s_t^d for the interest rate is $\frac{1}{w}\mu$. This ratio discloses the two structural parts that matter, namely the model-determined interest rate effectiveness w and the degree of exchange rate management μ chosen by the policymaker. So, our rule disentangles a Taylor-rule type of coefficient into two structural parameters.

One can use this insight also to disentangle the response parameter in the Monacelli rule, as (21) shows that

$$\varphi_s = \frac{1}{w}\frac{\mu}{1-\mu}. \tag{24}$$

Our approach thus reveals that φ_s is more than a policy-choice parameter: it is a reduced-form coefficient combining the policy-choice parameter μ with the interest rate effectiveness w. As w is determined by existing parameters only, disentangling φ_s does not increase the number of model parameters; we have simply used information already present in the exchange rate function.

3.2.3 Adapting to changes in the interest rate effectiveness w

How do the rules handle changes to the economic structure? Consider a change in financial openness that makes the interest rate more effective for exchange rate purposes (higher w). In our rule, the impact of a given pressure on i_t depends on the effectiveness w, and the larger w weakens the required interest rate reaction, as expected. So, the policy recommendation is that central bankers should account for the effectiveness of their instrument when determining its use. This may be straightforward, but it is convenient that our rule automatically incorporates this.

To study the consequence of the larger w when using the Monacelli (2004) rule, we use (24). If one fixes φ_s, the structural change due to the increase in w implies that the actual regime becomes one of tighter exchange rate management.

Finally, consider the Benigno et al. (2007) rule. Keeping φ_s^{BBG} constant means that the central bank responds in the same way as before the structural change. Still, the exchange rate regime does not change, because the regime outcome is the same for all positive values of φ_s^{BBG}. So, here the regime is robust.

3.2.4 Estimating the de facto regime μ

Instead of using our rule for determining the actual interest rate i_t for a given degree of exchange rate management μ, this section explains how one can apply the rule to estimate the de facto μ using time series data of i_t.[13] This offers a simple check of the realism of our approach. The option to estimate μ also adds to the traditional rules.

Representation (21) of our rule for $\mu < 1$ implies

$$\frac{1}{w}\frac{\mu}{1-\mu} = \frac{\text{stdev}\left\{i_t - i_t^d\right\}}{\text{stdev}\left\{s_t\right\}}. \tag{25}$$

[13] The large literature on de facto regime estimation includes Frankel and Xie (2010) and IMF (2022).

We can use data on $i_t - i_t^d$ and s_t to estimate the left-hand side by the ratio of sample standard deviations and then, for a given w, estimate μ.

To operationalize this, the current section restricts i_t^d to be a linear function of domestic producer price inflation π_{Ht} with coefficient 1.5, following Monacelli (2004). One way to obtain a value of w is by specifying a model and computing w from the model parameters. We will do so in Sections 4 and 5. Given the illustrative purpose of the current exercise, we simply take the value computed there, that is, $w = 1.62$.

We examine five countries, Australia, Canada, New Zealand, Denmark, and Hong Kong, the countries taken from Section 5 and the Introduction. The first three have an official inflation targeting policy, while the latter two pursue an exchange rate target. We use 20 years of quarterly data, from 2000 through 2019. This sample is for illustration only, and we leave a broad empirical study for future research.[14]

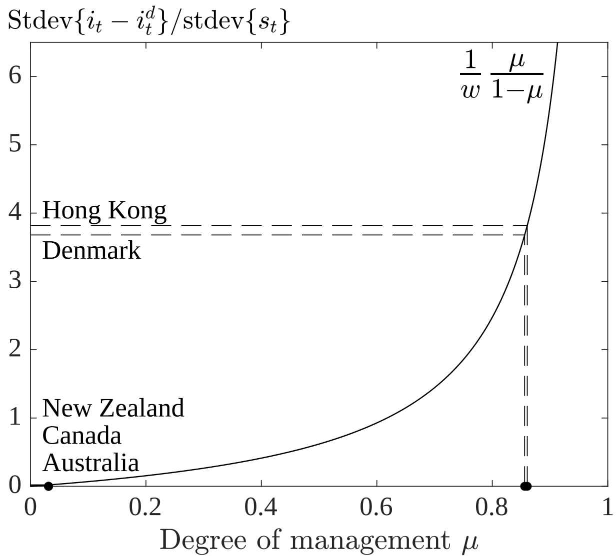

Figure 1: Estimating the degree of exchange rate management.

The estimates of stdev$\{i_t - i_t^d\}$ /stdev$\{s_t\}$ are 0.02, 0.02, 0.02, 3.68, and 3.82 for the respective countries. Figure 1 illustrates the implied μ. For Australia and Canada the estimated μ is 0.03, for New Zealand 0.04. For Denmark and Hong Kong we obtain 0.86, meaning that their regimes can be characterized as an about 10% float and 90% fixed

[14]The variables for quarter t are measured as follows. For i_t we take the three-month interbank interest rate, calculated as the period average of the daily rates in the quarter. Given period-average PPI values, we use year-on-year inflation for π_{Ht} and thus i_t^d. Then we express i_t and i_t^d at a quarterly basis; all interest rates in the paper are at this basis, so not annualized. The rate s_t is the log of the average daily price of one dollar (euro for Denmark). All data have been obtained from Datastream.

exchange rate regime.[15] All five are in line with the IMF (2022) de facto classification. We conclude from this simple analysis that our interest rate rule can deliver useful insights into structural parameters such as μ.

3.3 Not using the realized exchange rate s_t

The previous section discussed the rules assuming that s_t can be used to set the interest rate i_t. That enabled the computation of i_t for all three rules. Hence, our rule is not at a disadvantage in terms of practical applicability.

In the current section we no longer use s_t. This can be relevant if one views s_t as the outcome of policy and is hesitant to use s_t as input in a rule describing that policy. We study the usefulness of having s_t^d and thus pressure EMP_t, even though that is not observed. The insights from the previous section, that the structural parameters w and μ matter for policy and that our rule can handle changes in w, still apply here, but pressure learns us more. We study the insights from having s_t^d from a theoretical perspective and, in the next section, we use s_t^d to develop a simple rule depending on the lag s_{t-1} instead of s_t, thereby facilitating implementation in practice.

3.3.1 Tinbergen Rule

This paper focuses on one policy instrument, i_t, for simplicity. Still, our rule (17) typically addresses two targets, the exchange rate target, s^t, and the target inside i_t^d, say an inflation target. To show that this does not violate the Tinbergen Rule, we rewrite our rule somewhat.

Define $s_t^* = s\left(i_t^*, E_t\right)$. This is a similar type of exchange rate as s_t^d, albeit with the foreign interest rate i_t^* instead of the domestically-oriented i_t^d, that is, $s_t^* = s_t^d + w\left(i_t^d - i_t^*\right)$. For example, $s_t^* = \mathbb{E}_t\left\{s_{t+1}\right\}$ if UIP holds. We bring s_t^* into our rule by substituting out s_t^d, yielding

$$i_t = (1-\mu)\, i_t^d + \mu\left(i_t^* + \frac{1}{w}\left(s_t^* - s^t\right)\right). \tag{26}$$

Hence, the interest rate is a weighted average of a rate that focuses on the inflation

[15] The value of w that underlies the μ estimates is based on the core parameter values in Table 1. These are estimates. Moreover, in reality, countries may have different w. To quantify the reliability of the μ values, we first consider the $w = 1$ that applies under UIP. Then the resulting μ are 0.02 for Australia, Canada, and New Zealand, and 0.79 for Denmark and Hong Kong, which are similar to the fractions in the main text. Next, we use the information on the posterior distributions of the core parameters, as reported by Justiniano and Preston (2010), to estimate the posterior distribution of w. The resulting 95% credible interval for w is [1.39, 2.19]. The implied intervals for μ are [0.03, 0.04] for Australia, Canada, and New Zealand, and [0.84, 0.89] for Denmark and Hong Kong. These are narrow, so that we simply focus on the point estimates.

target and a rate that concerns the exchange rate target. The trade-off between the targets is driven by μ. In a float ($\mu = 0$), only the inflation target matters. In intermediate regimes ($0 < \mu < 1$), both targets matter, in a restricted manner. For a fixed exchange rate ($\mu = 1$), only s^t matters.

We thus have one instrument and a one-dimensional (combined) target.[16] Hence, our approach automatically accounts for the Tinbergen Rule. Monacelli (2004) recognizes the dependence between φ_s and the inflation parameter in i_t^d, and he tests whether his results are sensitive to their choices. The Benigno et al. (2007) rule automatically accounts for the Rule, because it concerns one target, s^t.

3.3.2 Covering the full range of μ

With s_t^d we have a single rule for the full range of μ, from zero up to and including one. There is no separation between $\mu < 1$ and $\mu = 1$, but a gradual transition. This graduality is also reflected in (26), which shows that for increasing μ, the relevance of i_t^d is gradually taken over by i_t^*, until i_t^d disappears upon arrival at the fixed rate.

3.3.3 Observability of s_t^d: theory versus practice

If s_t^d is not observed, that is no problem in theoretical models. There, one can compute s_t^d and thus all elements needed for our rule, as we will illustrate in a New Keynesian model later. Hence, one can fully exploit the properties of s_t^d. In particular, the consistency between our rule and the exchange rate function of the model, with the structural parameters w and μ, can be convenient for clean theoretical analyses. Moreover, our rule implements the preferred regime exactly, so that it can serve as a benchmark for other rules.

What about practice? Here, an unobservable s_t^d can create a difficulty. This, however, matches reality. After all, central bankers encounter unobservable pressure when deciding on policy. They use indicators to monitor pressure. For example, the Hong Kong Monetary Authority uses forward exchange rates, prices of currency options, balance of payment statistics on capital flows, and market surveys; see He et al. (2011). As s_t^d is the channel through which variables can matter for the forex-policy part of the interest rate, having s_t^d offers the possibility to study what variables policy makers should look at. Section 4.3 illustrates how our approach can give inspiration for new indicators and thus assist policy.

[16] This simplicity makes the interest rate an attractive instrument for introducing our idea. In contrast, taking official forex intervention as instrument would practically require i_t as a second instrument, to account for the inflation target. Though feasible, that would unnecessarily complicate this paper.

3.4 Simple rule, using the lag s_{t-1} instead of s_t

In the literature on monetary policy rules in general, authors have introduced "simple" rules, that is, rules that are functions of observable variables only. Now that we have a rule that exactly implements the exchange rate objective, we have the possibility to learn what parts of the rule are essential for that, and from there get inspiration for a new simple rule. Thus, we here give up some advantages of having pressure in the rule and, in return, improve the ease of application.

Start from the fixed exchange rate regime, $s_t^o = s^t$. Our rule is (14), and we make the relevance of i_t^* explicit by rewriting it as $i_t = i_t^* + \frac{1}{w}\left(EMP_t + w\left(i_t^d - i_t^*\right) + s_{t-1} - s^t\right)$. It is well known that using just the first part, $i_t = i_t^*$, fails to implement $s_t = s^t$; see Benigno et al. (2007). For one thing, the level of the exchange rate is not determined. Adding $EMP_t + w\left(i_t^d - i_t^*\right)$ does not help, because this is in depreciation units. Because the complete rule does implement the fixed rate, we conclude that the last part, $s_{t-1} - s^t$, is crucial for that. This suggests $i_t = i_t^* + s_{t-1} - s^t$ as a simple rule, where we leave out the $\frac{1}{w}$-factor for $s_{t-1} - s^t$, for simplicity. This simple rule can indeed deliver the fixed rate, as it does so in the setting of Benigno et al. (2007) and in our New-Keynesian model of Section 4.[17]

To also cover other regimes, we introduce the simple rule

$$i_t = (1 - \tilde{\mu})\, i_t^d + \tilde{\mu}\left(i_t^* + s_{t-1} - s^t\right), \tag{27}$$

where $0 \leq \tilde{\mu} \leq 1$. The rule only depends on variables that are observable. It is a weighted average of the rule for the floating exchange rate and our simple rule for the fixed rate. If $\tilde{\mu}$ increases, the role of i_t^d shrinks, while i_t^* gets more prominent. Under UIP, $w = 1$, so ignoring $\frac{1}{w}$ in front of $s_{t-1} - s^t$ may be reasonable. However, if $w > 1$, then $\tilde{\mu} > \tilde{\mu}\frac{1}{w}$, so that the simple rule implies stronger exchange rate management.

The simple rule has a number of attractive features, which make it a worthwhile alternative to traditional rules. Our simple rule fulfills the Tinbergen Rule, and it covers the full spectrum from the float to the fixed rate. Having the lag also facilitates implementation, an advantage stressed by McCallum (1997), and it comforts those who are hesitant to use the outcome s_t as input in the rule. Finally, the rule also matches some aspects of actual policy. As Spange and Wagner Toftdahl (2014) document, the Danish central bank typically responds to ECB interest rate changes by similar changes

[17] In Benigno et al. (2007) UIP holds. Following their idea, we combine their rule (19) with UIP (2) to obtain $\mathbb{E}_t\left\{s_{t+1}\right\} = 2s_t - s^t$ for $\varphi_s^{BBG} = 1$. This system is explosive, so (locally) the unique solution is the fixed rate $s_t = s^t$ for all t. Similarly, combining the suggested simple rule with UIP yields $\mathbb{E}_t\left\{s_{t+1}\right\} = s_t + s_{t-1} - s^t$. Also this system is explosive, implying the fixed rate s^t as unique solution.

on the same day. This supports the presence of the contemporaneous i_t^* in the rule. Moreover, they write that after a persistent weakening of the Danish krone, the bank may unilaterally increase the interest rate to boost demand for its currency. Our simple rule reflects that sequence by having the lag s_{t-1} instead of the actual rate s_t. We now put aside the simple rule and return to the baseline version (17).

4 Illustration in a log-linearized DSGE model

In Section 2 we have derived the key role for EMP in forex policy and this has led to a new interest rate rule. The resulting insights are valid in many models. The current section presents one specific model to illustrate the computation of EMP in a model, the rule, and some of their properties. Hence, we keep the model simple. Various extensions are possible to make the model more realistic, such as a more complete description of the role of financial markets including financial frictions in exchange rate determination, for example based on Gabaix and Maggiori (2015). These are left for future study but do not affect the key role for EMP in forex policy, our focus.[18]

We take a two-country rational expectations New Keynesian model where the home country is a small open economy, in the spirit of De Paoli (2009). Section 4.1 presents its non-policy block, where many elements and derivations will be standard, as described by Galí (2008). The two sections thereafter contain our innovations.

4.1 The model: non-policy block

Web Appendix A, available from our homepages, specifies the non-policy part of the model, and derives the zero-inflation and zero-depreciation, symmetric and efficient steady state. We log-linearize the equations around that steady state and use the log-linearized version from now on. The relevant equations are (28)-(38), derived in Web Appendix B, and the foreign equivalents of (28)-(32). Lowercase Latin letters denote the logarithm of variables, except for the interest rate, and an asterisk refers to the foreign country or currency. Table 1 defines all parameters, gives their ranges, and shows the values used in simulation Section 5.

$$\text{Labor supply} \quad : \quad \gamma \ell_t + \sigma c_t = \mathrm{w}_t - p_t \tag{28}$$

$$\text{Consumption Euler} \quad : \quad c_t = \mathbb{E}_t \{c_{t+1}\} - 1/\sigma \left(i_t - \mathbb{E}_t \{\pi_{t+1}\} - \delta\right) \tag{29}$$

[18] Several model assumptions we will make, such as producer currency pricing, are relevant for the optimality of the exchange rate regime. However, recall that we take the regime as given, so from this point of view those model assumptions are not restrictive. In fact, our rule has been derived in the general setup of Section 2, so we can safely impose some assumptions here to simplify the exposition.

$$
\begin{array}{rll}
\text{Real marginal cost} & : & mc_{Ht} = \log(1-1/\theta) + \mathrm{w}_t - a_t - p_{Ht} \qquad (30) \\
\text{Calvo-based pricing} & : & \pi_{Ht} = \beta \mathbb{E}_t \{\pi_{H,t+1}\} + \kappa_{mc}(mc_{Ht} - \log(1-1/\theta)) \qquad (31) \\
\text{Labor market equilibrium} & : & \ell_t = y_t - a_t \qquad (32) \\
\text{International risk sharing} & : & \sigma(c_t - c_t^*) = s_t + p_t^* - p_t \qquad (33) \\
\text{Law of one price} & : & p_{Ft} = p_{Ft}^* + s_t \qquad (34) \\
\text{Goods market equilibrium} & : & y_t = \nu c_t + (1-\nu) c_t^* + \left(1-\nu^2\right) \eta (p_{Ft} - p_{Ht}) \qquad (35) \\
\text{Goods market eq. abroad} & : & y_t^* = c_t^* \qquad (36) \\
\text{CPI} & : & p_t = \nu p_{Ht} + (1-\nu) p_{Ft} \qquad (37) \\
\text{CPI abroad} & : & p_t^* = p_{Ft}^*. \qquad (38)
\end{array}
$$

The world is populated with a continuum of households, where the population in the home country H lies in the segment $[0, n)$, while that of the rest of the world F is in $[n, 1]$. Households live forever and have identical preferences, both within and across countries. They derive utility from the consumption of domestic and foreign goods, with home bias in preferences, and disutility from supplying labor to firms. They live in cashless economies. For simplicity, capital markets are complete, both domestically and internationally, with frictionless trade in assets.

Households maximize expected lifetime utility. Optimization yields labor supply equation (28) and consumption Euler equation (29), where ℓ_t is labor supply in period t, c_t is consumption, w_t is the wage rate, p_t is the consumer price index (CPI), and $\pi_t = p_t - p_{t-1}$ is CPI inflation.

Firms specialize in the production of one firm-specific good. Domestic firms produce the varieties in $[0, n)$ and foreign firms those in $[n, 1]$. Each firm uses labor supplied by the households and a linear technology, where a_t is (the log of) labor productivity, which is common across home firms and evolves exogenously according to some stationary stochastic process. The firm receives an employment subsidy that renders the steady state efficient. Real marginal cost mc_{Ht}, expressed in terms of the producer price index (PPI) p_{Ht}, thus depends on the product wage $\mathrm{w}_t - p_{Ht}$ by (30).

The firm sells its good under monopolistic competition. It sells at home and abroad without trade frictions. Prices are set in the producer's currency, and they are sticky a la Calvo (1983). Hence, p_{Ht} depends on its lag and the price chosen by firms that are allowed to reset the price. Profit maximization then yields PPI inflation $\pi_{Ht} = p_{Ht} - p_{H,t-1}$ based on (31), showing the importance of real marginal cost mc_{Ht}, which enters the formula in deviation from its steady-state value.

Equilibrium concerns three markets. First, labor market equilibrium is (32), where

Table 1: Model parameters

Par.	Range	Value	Description
Core parameters			
β	$(0, 1)$	0.99	subjective discount factor
γ	> 0	1.17	inverse of Frisch elasticity of labor supply
σ	> 0	1.20	inverse of elasticity of intertemporal substitution for consumption
η	> 0	0.68	elasticity of subst. between home & foreign goods
θ	> 1	8.00	elasticity of subst. between varieties produced within a country
ω	$(0, 1)$	0.72	Calvo fraction of firms not allowed to change prices (stickiness)
n	$(0, 1)$	$\to 0$	size of the home economy
ν	$(0, 1]$	0.75	home bias in preferences
φ_π	≥ 0	2.06	inflation impact on interest rate in Taylor rule
μ	$[0, 1]$	—	degree of exchange rate management
Additional parameters for simulation			
ρ_a	(-1, 1)	0.81	AR(1) coefficient in labor productivity process
σ_a	≥ 0	0.52	standard deviation of labor productivity shock (in %)
σ_i^*	≥ 0	0.12	standard deviation of foreign monetary policy shock (in %)
Derived parameters			
δ	$(0, 1)$	0.01	$= -\log(\beta)$: subjective discount rate
α	$[0, 1]$	$\to .75$	$= 1 - (1 - n)(1 - \nu)$: share of home goods in home consumption
α^*	$[0, 1]$	$\to 0$	$= n(1 - \nu)$: share of home goods in foreign consumption
τ	$[0, 1]$	0.13	$= 1 - \frac{\theta - 1}{\theta}$: employment subsidy
κ_{mc}	> 0	0.11	$= \frac{(1-\omega)(1-\omega\beta)}{\omega}$: marginal cost impact on PPI inflation in (31)
ϖ_c	> 0	2.08	$= \sigma + \gamma\nu$: consumption effect on product wage
ϖ_{tot}	≥ 0	0.60	$= 1 - \nu + \gamma\left(1 - \nu^2\right)\eta$: terms-of-trade effect on product wage
w	> 1	1.62	$= \frac{\kappa_{mc}\varpi_c}{\sigma} + \frac{1+\kappa_{mc}\varpi_{tot}}{\nu}$: effectiveness of i_t to counteract deprec.

Foreign parameters $\beta^*, \gamma^*, \sigma^*, \theta^*, \omega^*, \varphi_\pi^*, \rho_a^*$, and σ_a^* equal their home counterparts.
The values of the core and additional parameters for simulation have been taken from Justiniano and Preston (2010). The authors estimate a small open-economy model for three countries vis-à-vis the United States, namely for Australia, Canada, and New Zealand, using Bayesian techniques, though they calibrate the values for β, θ, and ν. We take the average of their three posterior medians.

y_t is domestic output. Second, the asset market is in equilibrium if the perfect international risk sharing relation (33) holds, given symmetric initial conditions, where $s_t + p_t^* - p_t$ is the real exchange rate. Third, for the goods market, free international trade implies the law of one price, so the import price index p_{Ft} follows from (34), where p_{Ft}^* is foreign PPI in foreign currency. The goods market also clears for all varieties.

To mimic that the domestic country is small, we now take the limit $n \to 0$. That gives goods market clearing at home (35) and abroad (36). The former captures that higher prices for imports relative to domestically produced goods (higher terms of trade $tot_t = p_{Ft} - p_{Ht}$) cause substitution towards domestic goods, stimulating domestic production. The limit also implies that home CPI in (37) follows from home PPI and the import price index, and that foreign CPI p_t^* is simply foreign PPI, as (38) shows.

4.2 Exchange rate function

Our interest rate rule requires the intermedial exchange rate s_t^d, so we first derive the exchange rate function (6) the model implies. Web Appendix B presents a streamlined derivation, starting from the fact that the exchange rate s_t clears the asset market and making sure the function is in the (i_t, E_t)-form defined in Section 2.2.1. This gives

$$s_t = -w i_t + v' E_t, \tag{39}$$

where

$$w = \frac{\kappa_{mc} \varpi_c}{\sigma} + \frac{1 + \kappa_{mc} \varpi_{tot}}{\nu} \tag{40}$$

and

$$v = \begin{bmatrix} w \\ w \\ w\sigma \\ 1 \\ -1 \\ \beta \\ -\kappa_{mc}(\gamma + 1) \end{bmatrix} \text{ and } E_t = \begin{bmatrix} i_t^* \\ \mathbb{E}_t \pi_{t+1} - \mathbb{E}_t \pi_{t+1}^* \\ \mathbb{E}_t c_{t+1} - \mathbb{E}_t c_{t+1}^* \\ s_{t-1} \\ tot_{t-1} \\ \mathbb{E}_t \pi_{H,t+1} - \mathbb{E}_t \pi_{F,t+1}^* \\ a_t - a_t^* \end{bmatrix}. \tag{41}$$

Formula (40) is the model-implied version of (8), the effectiveness of the interest rate to counteract depreciation while keeping E_t constant. It is fully determined by the structural parameters of the model. We get $w > 0$, so an interest rate increase strengthens the home currency.

Expression (41) for E_t discloses what else matters for the exchange rate according to the model. Most determinants occur in a simple relative form, an attractive consequence of our streamlined derivation of the s-function. Because s_{t-1} has a unit coefficient in v, one could also write (39) in terms of Δs_t and the adjusted E_t would then consist of stationary variables only. However, that does not imply that s_t is non-stationary, because to implement an exchange rate regime the interest rate i_t may depend on the exchange rate level so as to counteract deviations from target, and that can result in a stationary s_t, similar to an error-correction specification.

4.3 Interest rate rule

Suppose the domestic central bank pursues the weighted fixed-floating exchange rate regime (16). To implement this, the central bank should use interest rate rule (17). The ingredients of the rule are as follows. For the domestically-oriented rate we take

$$i_t^d = \delta + \varphi_\pi \pi_{Ht}, \tag{42}$$

though one could also use a Taylor rule with CPI inflation and output gap. The effectiveness w follows from (40), and the degree of exchange rate management μ and the target rate s^t are both taken as given.

The other ingredient, intermedial exchange rate s_t^d, follows directly from definition (11) and the model-implied s-function (39), so that

$$s_t^d = -w i_t^d + v' E_t. \tag{43}$$

Our rule (17) can now be expressed as

$$i_t = (1-\mu)\, i_t^d + \mu \frac{1}{w} \left(v' E_t - s^t \right). \tag{44}$$

So, the interest rate is a weighted average of i_t^d and $\frac{1}{w}\left(v'E_t - s^t\right)$. For example, in a float ($\mu = 0$), the interest rate is the domestically-oriented rule i_t^d, as usual. To implement the fixed rate ($\mu = 1$), i_t^d drops out, in line with the incompatible trinity. As expected, the rule then starts with i_t^*, given the top row in (41). Expression (44) is an example of (26), which highlights the role of i_t^*.

Rule (44) shows that π_{Ht}, via (42), and E_t in (41) are the variables the central bank should look at to implement a given exchange rate objective. The theory in Section 2.3.2 has shown that EMP_t is the key variable for the central bank to look at. Indeed, $EMP_t = s_t^d - s_{t-1}$, so that (43) shows π_{Ht} and E_t determine EMP_t in this economy. Hence, rule (44) supports our conclusion that EMP is the channel through which variables matter for exchange rate policy.

4.4 Exogenous processes

As of now, we assume the foreign central bank uses the following interest rate rule

$$i_t^* = \delta + \varphi_\pi \pi_t^* + \varepsilon_{it}^*, \tag{45}$$

where the monetary policy shock ε_{it}^* has mean zero and standard deviation σ_i^*. Home labor productivity a_t follows an AR(1) process with autoregressive coefficient ρ_a and mean-zero shock with standard deviation σ_a. The same holds for foreign productivity a_t^*. All shocks are normally distributed, independent from each other and over time.

5 Simulations from the model

To further illustrate EMP and our interest rate rule, we now simulate from the model just developed. The main insights from these simulations are not specific to the model, parameter values, or draws of the shocks.

5.1 Model calibration, solution, and simulation

One period in the model is one quarter. We set the target $s^t = 0$. All parameter values are based on Justiniano and Preston (2010). Table 1 presents them, in particular the interest rate effectiveness $w = 1.62$, and its note provides further motivation.

We solve the model numerically using the Sims (2002) algorithm. The solution can be cast as a reduced-form VAR model of the 20×1-vector with elements $c_t, \mathbb{E}_t c_{t+1}, c_t^*, \mathbb{E}_t c_{t+1}^*, \pi_{Ht}, \mathbb{E}_t \pi_{H,t+1}, \pi_t, \mathbb{E}_t \pi_{t+1}, \pi_t^*, \mathbb{E}_t \pi_{t+1}^*, y_t, y_t^*, i_t, i_t^*, s_t, \mathbb{E}_t s_{t+1}, tot_t, EMP_t, a_t, a_t^*$. We focus on unique stationary solutions, abstracting thus from sunspot equilibria.

The necessary and sufficient condition for equilibrium determinacy is as follows, given our parameter space and thus the symmetry assumption $\varphi_\pi = \varphi_\pi^*$. Under the float, the condition follows from Bullard and Mitra (2002), and it here reduces to satisfying the Taylor principle $\varphi_\pi > 1$. For the other regimes, from the managed float to the fixed rate, we verify determinacy for a grid of parameter values, using the Sims (2002) algorithm. Here the condition also turns out to be $\varphi_\pi > 1$. So, the same condition applies whatever the regime. It holds for all parameter values we study.

We set $s_0 = 0$ and initialize other variables at their steady-state values. We draw the three shocks for 60 periods (15 years), from which we compute the paths of the variables of interest. For ease of comparison, we keep the shocks the same across the plotted paths.

5.2 Implementing multiple regimes

Our rule (44) implements many exchange rate objectives and does so exactly, with a key role for EMP_t, determined by (43). To illustrate their properties, we simulate paths of the economy in three different regimes, the float ($\mu = 0$), an intermediate regime (say $\mu = 0.5$), and the peg ($\mu = 1$). A representative set of paths is depicted in Figure 2.

Under the float ($\mu = 0$), the interest rate i_t equals the domestically-oriented rate i_t^d, visualized by the horizontal line in the second panel. As i_t^d is driven by inflation, the interest rate does not stabilize the exchange rate s_t. That is, the exchange market pressures EMP_t indicated by the gray line in the top panel are not offset by policy and they equal the actual depreciations. This makes that the gray line for s_t in the bottom panel does not revert to zero.

The stronger the exchange rate management, the more the central bank has to account for exchange rate fundamentals. The dashed line in the bottom panel visualizes that $\mu = 0.5$ here already stabilizes the exchange rate considerably. The line also suggests that the weighted fixed-floating regime can be a practical linear approximation of various other exchange rate policies, such as the peg with band.

If the central bank pursues a fixed exchange rate ($\mu = 1$), then the black line in the middle panel visualizes that $i_t = i_t^*$ in equilibrium, and the bottom panel shows that the exchange rate stays on target continuously. This corroborates that the model contains UIP, by virtue of (33). Still, the top panel reveals that shocks cause periods of noticeable pressure EMP_t on the peg. There the central bank has to accept an interest rate that differs substantially from the domestically-oriented rate (second panel), which in practice may induce policymakers to give up the peg.

Next, consider our simple rule (27), with $\tilde{\mu}$ as the weight on the fixed rate. For $\tilde{\mu} = 0$ and 1, it gives the same paths as in the figure for $\mu = 0$ and 1. For $\tilde{\mu} = 0.5$ the exchange rate path (not included in the figure) looks like that in the figure for $\mu = 0.5$, but a bit closer to the target. The latter reflects that $w > 1$ in the model, so that the simple rule implies stronger exchange rate management, as set out in Section 3.4.

Finally, let us discuss the traditional rules. Section 3 gives reasons that make our rules a useful alternative. The advantages do not depend on specific simulations. Still, it may be instructive to compare the simulated paths. In this special case of the weighted fixed-floating regime, one can replicate our simulated paths of i_t and s_t for $\mu < 1$ by using the Monacelli (2004) rule and setting φ_s based on (24). However, this requires w (except for $\mu = 0$), which is not available in his approach. This makes that simulations from his and our rules generally differ. For $\mu = 1$, one can replicate our simulations with the Benigno et al. (2007) rule.

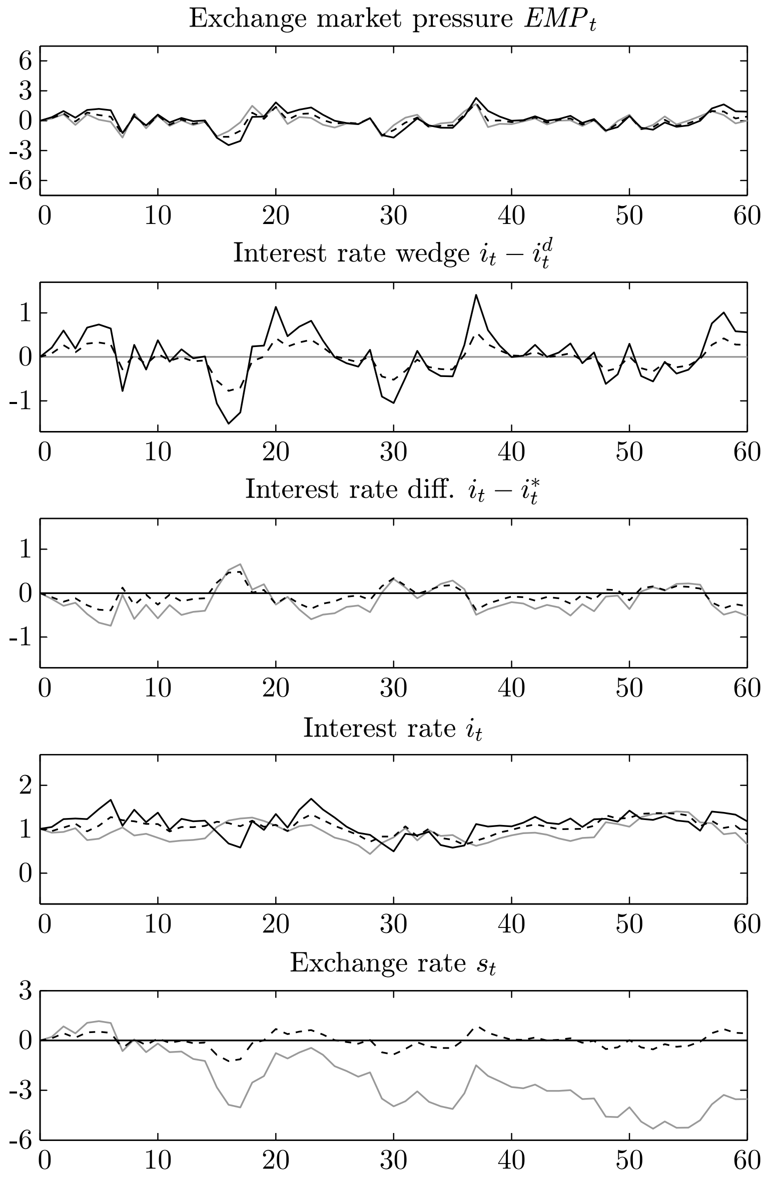

Figure 2: Paths implied by our rule (44) in various exchange rate regimes: from float ($\mu = 0$, gray) to intermediate ($\mu = 0.5$, dashed) to fixed ($\mu = 1$, black). The variables EMP_t, i_t, i_t^d, and i_t^* are in percentage terms, and s_t is 100 times the log exchange rate.

6 Conclusion

This paper has derived that EMP is a key variable for a central bank when setting the interest rate to implement a given exchange rate objective. This guides policy and, vice versa, actual policy confirms the relevance of EMP.

We have formalized this result in a new interest rate rule. It extends a domestically-oriented rule, such as the Taylor rule, by adding EMP in deviation from the exchange rate change that is acceptable according to the objective. Excess pressure implies a high interest rate, in line with actual policy. The rule implements the exchange rate objective exactly, and it does so for many regimes and models. The economic structure matters for the EMP coefficient in the rule: the more effective the interest rate, the less it should be used to offset a given pressure.

We have introduced the weighted fixed-floating regime, with weight μ on the fixed regime. Our rule can be conveniently combined with this regime. This leads to a coefficient of EMP in the rule that discloses two structural parts, namely the model-determined interest rate effectiveness w and the degree of exchange rate management μ chosen by the policymaker. We have thus disentangled a Taylor-rule type of coefficient into two underlying structural parameters.

Our approach has also suggested a simple variant of the rule, one that only contains observable variables. The simple rule depends on the contemporaneous foreign interest rate and the lagged exchange rate, both of which are in line with actual policy.

We have extended the EMP literature by refining the EMP formalization and computing EMP in a modern sticky-price model, as the EMP literature typically relies on some variant of the flexible-price monetary model. We have also formalized how EMP is an ingredient for policy, and how the sticky-price model helps the policymaker to learn the determinants of EMP. All this may stimulate further research on EMP.

The broad applicability of our rule and the inherent consistency with the regime and model can facilitate future research. Think of studies on the optimal degree of exchange rate management, further eased by our new structural parameter μ, and research on models with incomplete markets and risk premia. For example, in another project we apply our idea to analyze both interest rate and foreign exchange interventions by the central bank under capital controls. This could then facilitate studies on emerging markets where central banks use forex intervention to pursue leaning-against-the-wind exchange rate management. This is left for future research.

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Web appendix to
"Exchange market pressure in interest rate rules"

by Franc Klaassen & Kostas Mavromatis
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A DSGE model

A.1 Households

The world is populated with a continuum of households, where the population in the home country H lies in the segment $[0, n)$, while that of the rest of the world F is in $[n, 1]$. Domestic households maximize expected lifetime utility

$$\max \mathbb{E}_t \sum_{k=0}^{\infty} \beta^k \left(\frac{C_{t+k}^{1-\sigma}}{1-\sigma} - \frac{L_{t+k}^{1+\gamma}}{1+\gamma} \right), \tag{46}$$

subject to a budget constraint (specified later), by choosing a path $\{C_{t+k}, L_{t+k}\}_{k=0,\dots,\infty}$, where C_{t+k} is household consumption and L_{t+k} is labor supply at time $t+k$.

Consider period t.[19] Consumption enters the domestic household's utility as an index C_t, which is the CES aggregate of the indices of domestic consumption of home and foreign (imported) goods, C_{Ht} and C_{Ft}, respectively:

$$C_t = \left(\alpha^{\frac{1}{\eta}} C_{Ht}^{\frac{\eta-1}{\eta}} + (1-\alpha)^{\frac{1}{\eta}} C_{Ft}^{\frac{\eta-1}{\eta}} \right)^{\frac{\eta}{\eta-1}}. \tag{47}$$

The parameter α, determining the preference for home-produced goods, increases with the size of the home country, n, and with home bias ν. We model $1-\alpha = (1-n)(1-\nu)$. Hence, $\nu > 0$ means that domestic households consume fewer foreign-produced goods than the size of the foreign country implies, reflecting home bias.[20]

The index of domestic consumption of home goods, C_{Ht}, is the CES aggregate of the consumption of all varieties produced in country H. These are varieties $j \in [0, n)$. The index of domestic consumption of foreign goods, C_{Ft}, is a similar CES aggregate, but concerning all varieties produced in F, which are $j \in [n, 1]$. Domestic consumption

[19]Results for $t + k \geq t$ follow by substituting t by $t + k$, while keeping expectations conditional on t.

[20]Because foreign households have identical preferences, their consumption index C_t^* equals the right-hand side of (47) with α substituted by α^*, C_{Ht} by C_{Ht}^*, and C_{Ft} by C_{Ft}^*. Moreover, $\alpha^* = n(1-\nu)$.

of variety j is denoted by $C_t(j)$. In formula,

$$\begin{cases} C_{Ht} = & \left[\left(\frac{1}{n}\right)^{\frac{1}{\theta}} \int_0^n C_t(j)^{\frac{\theta-1}{\theta}} dj\right]^{\frac{\theta}{\theta-1}} \\ C_{Ft} = & \left[\left(\frac{1}{1-n}\right)^{\frac{1}{\theta}} \int_n^1 C_t(j)^{\frac{\theta-1}{\theta}} dj\right]^{\frac{\theta}{\theta-1}} . \end{cases} \tag{48}$$

So, θ concerns the substitutability between varieties produced within a country, whereas η in (47) is about the substitution between home and foreign goods.

As usual, utility maximization requires that within period t households maximize C_t for a given expenditure on home and foreign indices and they maximize C_{Ht} (C_{Ft}) for a given level of expenditure on home (foreign) varieties. Let $P_t(j)$ denote the price of variety j in domestic currency. The resulting demand function for each variety is

$$C_t(j) = \begin{cases} \alpha & \left(\frac{P_t(j)}{P_{Ht}}\right)^{-\theta} \left(\frac{P_{Ht}}{P_t}\right)^{-\eta} C_t, & \text{for home varieties } j \in [0, n) \\ (1-\alpha) & \left(\frac{P_t(j)}{P_{Ft}}\right)^{-\theta} \left(\frac{P_{Ft}}{P_t}\right)^{-\eta} C_t, & \text{for foreign varieties } j \in [n, 1], \end{cases} \tag{49}$$

where

$$\begin{cases} P_{Ht} = & \left[\frac{1}{n} \int_0^n P_t(j)^{1-\theta} dj\right]^{\frac{1}{1-\theta}} \\ P_{Ft} = & \left[\frac{1}{1-n} \int_n^1 P_t(j)^{1-\theta} dj\right]^{\frac{1}{1-\theta}} \end{cases} \tag{50}$$

are the home producer price index and the foreign producer price index expressed in domestic currency, respectively, and

$$P_t = \left(\alpha P_{Ht}^{1-\eta} + (1-\alpha) P_{Ft}^{1-\eta}\right)^{\frac{1}{1-\eta}} \tag{51}$$

is the consumer price index in the home country. This implies that total consumption expenditure by domestic households is $P_t C_t$.[21]

We can now specify the period budget constraint

$$P_t C_t + \mathbb{E}_t \left\{\Lambda_{t,t+1} S_{t+1} B_{t+1}\right\} \leq W_t L_t + S_t B_t + \Pi_t - T_t, \tag{52}$$

where we rule out Ponzi schemes. Here B_t is the value in foreign currency of a portfolio of a full set of state-contingent assets held at the beginning of period t, reflecting our

[21] Similar expressions hold for the foreign country, for both demand and prices. Foreign demand follows from (49) by substituting α, C, and the four P symbols by α^*, C^*, and P^*, respectively. The home producer price index in foreign currency P^*_{Ht} and the foreign producer price index (in foreign currency) P^*_{Ft} follow from the right-hand sides of (50) by substituting $P_t(j)$ by $P^*_t(j)$. The foreign consumer price index P^*_t (in foreign currency) equals the right-hand side of (51) with α substituted by α^*, P_{Ht} by P^*_{Ht}, and P_{Ft} by P^*_{Ft}.

complete markets assumption, $S_t = \exp(s_t)$ is the nominal exchange rate in level form, $\Lambda_{t,t+1}$ is the stochastic discount factor making $\mathbb{E}_t \{\Lambda_{t,t+1} S_{t+1} B_{t+1}\}$ the home-currency value at time t of the portfolio that yields a payoff in $t+1$, W_t is the nominal wage, Π_t is nominal firm profits transferred to households, and T_t is lump-sum taxes.

As usual, the first-order conditions consist of the optimality condition regarding the intratemporal consumption-leisure trade off

$$C_t^{\sigma} L_t^{\gamma} = \frac{W_t}{P_t} \tag{53}$$

and the intertemporal optimality relation linking the stochastic discount factor to the intertemporal marginal rate of substitution in Euler equation

$$\Lambda_{t,t+1} = \beta \left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \frac{P_t}{P_{t+1}}, \tag{54}$$

for all possible states of nature at times t and $t+1$. Note that $\mathbb{E}_t \Lambda_{t,t+1}$ is the value of a portfolio that yields one unit of the domestic currency in $t+1$ (mimicking a riskless domestic bond), so that the interest rate is $i_t = -\log(\mathbb{E}_t \{\Lambda_{t,t+1}\})$. Given i_t, prices, and the budget constraint, the (expectational) Euler equation determines C_t.

A.2 Firms

Firms use labor supplied by the households and a linear technology. Hence, output is

$$Y_t(j) = A_t L_t(j), \tag{55}$$

where A_t is exogenous labor productivity, which is common across firms (within a country). Because of a labor subsidy τ, financed by taxes T_t, marginal cost is

$$MC_t = (1-\tau) W_t / A_t, \tag{56}$$

which is independent of output and thus common across firms. The firm sells its good in a monopolistically competitive market with free international trade. Profits are

$$\Pi_t(j) = (P_t(j) - MC_t) Y_t(j). \tag{57}$$

The firm sets the price in a sticky fashion a la Calvo (1983). That is, each date with probability ω the firm is not allowed to change its price. When the firm is allowed to set a new price $P_t^{opt}(j)$, it will do so optimally, that is, by maximizing the current market value of the profits resulting while that price remains in place. Suppose the new price holds until $t+k \geq t$. Let $Y_{t+k|t}(j)$ denote total demand $C_{t+k}(j) + \frac{1-n}{n} C^*_{t+k}(j)$ evaluated at $P_t^{opt}(j)$. The firm's objective function is therefore

$$\max \sum_{k=0}^{\infty} \omega^k \mathbb{E}_t \left\{ \Lambda_{t,t+k} \left(P_t^{opt}(j) - MC_{t+k} \right) Y_{t+k|t}(j) \right\}. \tag{58}$$

To derive the first-order condition, first note that (49) and its foreign counterpart imply that $\partial Y_{t+k|t}(j) / \partial P_t^{opt}(j) = -\theta Y_{t+k|t}(j) / P_t^{opt}(j)$. Moreover, other home firms face the same optimization problem, so that all domestic firms will choose the same new price $P_{Ht}^{opt} = P_t^{opt}(j)$. The price can be solved from the first-order condition

$$\sum_{k=0}^{\infty} \omega^k \mathbb{E}_t \left\{ \Lambda_{t,t+k} Y_{t+k|t} \left[P_{Ht}^{opt} - \frac{\theta}{\theta - 1} MC_{t+k} \right] \right\} = 0. \tag{59}$$

A.3 Equilibrium

World equilibrium requires that labor, asset, and goods markets are in equilibrium.

A.3.1 Labor market

Labor market equilibrium at home and abroad requires

$$\begin{cases} L_t & = & \frac{1}{n} \int_0^n L_t(j)\, dj \\ L_t^* & = & \frac{1}{1-n} \int_n^1 L_t^*(j)\, dj. \end{cases} \tag{60}$$

A.3.2 Asset market

As for the home country, market completeness implies there is also a unique stochastic discount factor for foreign-currency payoffs, which is $\Lambda^*_{t,t+1} = \beta \left(C^*_{t+1} / C^*_t \right)^{-\sigma} P_t^* / P_{t+1}^*$ for all possible states of nature at times t and $t+1$. Given free international trade in assets, arbitrage yields the asset market equilibrium relation

$$\Lambda_{t,t+1} = \Lambda^*_{t,t+1} \frac{S_t}{S_{t+1}}, \tag{61}$$

which is a stochastic version of uncovered interest parity.

Substituting the expressions for $\Lambda_{t,t+1}$ and $\Lambda^*_{t,t+1}$ shows the model has the familiar perfect risk sharing relation between home and foreign households

$$C_t^{\sigma} = C_t^{*\sigma} Q_t, \tag{62}$$

assuming symmetric initial conditions, where $Q_t = S_t P_t^* / P_t$ is the real exchange rate.

A.3.3 Goods market

Goods market equilibrium consists of two parts. First, frictionless trade results in the law of one price. So, for each variety $j \in [0,1]$ the price set by the producer in its currency implies that the price in the other currency fulfills

$$P_t(j) = S_t P_t^*(j). \tag{63}$$

For the producer price indices this yields $P_{Ht} = P_{Ht}^* S_t$ and $P_{Ft} = P_{Ft}^* S_t$. Still, home bias implies $\alpha > \alpha^*$, so that in general for the consumer price index $P_t \neq P_t^* S_t$, meaning a deviation from purchasing power parity.

The second part of goods market equilibrium is the markets for all varieties clear:

$$\begin{cases} Y_t(j) & = & C_t(j) + \frac{1-n}{n} C_t^*(j), & \text{for home varieties} \\ Y_t^*(j) & = & \frac{n}{1-n} C_t(j) + C_t^*(j), & \text{for foreign varieties.} \end{cases} \tag{64}$$

For the home-varieties line, substitute the top demand function of (49) for $C_t(j)$ and its foreign counterpart (as explained in footnote 21) for $C_t^*(j)$. For the foreign-varieties line, we do the same, but now using the bottom demand function of (49). This yields

$$\begin{cases} Y_t(j) & = & \left(\frac{P_t(j)}{P_{Ht}}\right)^{-\theta} \left[\alpha \left(\frac{P_{Ht}}{P_t}\right)^{-\eta} C_t + \frac{1-n}{n} \alpha^* \left(\frac{P_{Ht}/S_t}{P_t^*}\right)^{-\eta} C_t^*\right], & \text{home v.} \\ Y_t^*(j) & = & \left(\frac{P_t^*(j)}{P_{Ft}^*}\right)^{-\theta} \left[\frac{n}{1-n}(1-\alpha) \left(\frac{P_{Ft}^* S_t}{P_t}\right)^{-\eta} C_t + (1-\alpha^*) \left(\frac{P_{Ft}^*}{P_t^*}\right)^{-\eta} C_t^*\right], & \text{foreign v.} \end{cases} \tag{65}$$

Substituting these into the definitions of aggregate output

$$\begin{cases} Y_t & = & \left[\frac{1}{n} \int_0^n Y_t(j)^{\frac{\theta-1}{\theta}} dj\right]^{\frac{\theta}{\theta-1}} \\ Y_t^* & = & \left[\frac{1}{1-n} \int_n^1 Y_t^*(j)^{\frac{\theta-1}{\theta}} dj\right]^{\frac{\theta}{\theta-1}} \end{cases} \tag{66}$$

gives

$$\begin{cases} Y_t & = & \alpha \left(\frac{P_{Ht}}{P_t}\right)^{-\eta} C_t + & \frac{1-n}{n} \alpha^* \left(\frac{P_{Ht}/S_t}{P_t^*}\right)^{-\eta} C_t^* \\ Y_t^* & = & \frac{n}{1-n}(1-\alpha) \left(\frac{P_{Ft}^* S_t}{P_t}\right)^{-\eta} C_t + & (1-\alpha^*) \left(\frac{P_{Ft}^*}{P_t^*}\right)^{-\eta} C_t^* \end{cases}. \tag{67}$$

A.4 Taking the limit $n \to 0$ to obtain the small economy

To mimic the small open economy we take the limit $n \to 0$. This implies $\alpha \to \nu$ and $\alpha^* \to 0$. The limiting CPIs resulting from (51) become

$$\begin{cases} P_t = \left(\nu P_{Ht}^{1-\eta} + (1-\nu) P_{Ft}^{1-\eta}\right)^{\frac{1}{1-\eta}} \\ P_t^* = P_{Ft}^*, \end{cases} \tag{68}$$

and the limiting values of aggregate output in (67) are

$$\begin{cases} Y_t = \nu \left(\frac{P_{Ht}}{P_t}\right)^{-\eta} C_t + (1-\nu)\left(\frac{P_{Ht}/S_t}{P_t^*}\right)^{-\eta} C_t^* \\ Y_t^* = C_t^*. \end{cases} \tag{69}$$

A.5 Steady state

Here we compute the symmetric zero-inflation and zero-depreciation efficient steady state of the model. All variables refer to the values in that steady state. Similar values apply to the foreign country, unless explicitly stated otherwise.

The constancy of P_H implies that firms choose $P_H^{opt} = P_H$. From (59) we obtain that MC is constant. Given that all shocks are set to zero, productivity is constant over time, denoted by A. Then (56) gives that the wage is constant W. Note the labor subsidy τ given in Table 1 implies $P_H = W/A$ and thus renders the steady state efficient, and real marginal cost $MC/P_H = (\theta - 1)/\theta$. Similarly, $P_F^* = W^*/A^*$. The constancy of S then gives $P_F = SP_F^*$. For simplicity, we assume $P_H = P_F$. So, $P = P_H$. As $P^* = P_F^*$, the real exchange rate $Q = 1$, so that PPP holds in the steady state.

Because all firms j charge the same price, (65) implies that output per firm is the same across varieties. First, consider the foreign country. Combining (65), (67) with the foreign version of (55), (60), (69), and the foreign version of (53), where $W^*/P^* = A^*$, implies that consumption is constant $C^* = A^{*(1+\gamma)/(\sigma+\gamma)}$. Similarly, combining the home equivalents of the formulas in the previous sentence and using the constancy of C^* shows that also home consumption is constant, where C is the unique solution from $C^\sigma \left(\left[\nu C + (1-\nu) C^*\right]/A\right)^\gamma = A$. Assuming $A = A^*$ and using the value of C^* yields as unique solution $C = C^*$. From (69) we obtain $Y = C$, and (65), (67), (55), and (60) then yield $L = Y/A$. Finally, (54) gives $\Lambda = \beta$.

B Derivations of equations (28)-(39)

Equations (28), (33), (34), (36), and (38) follow directly from (53), (62), (63), (69), and (68), respectively. This appendix derives the remaining equations.

- (29)

Start from (54). By definition, $\mathbb{E}_t \{\Lambda_{t,t+1}\} = \exp(-i_t)$ and $\beta = \exp(-\delta)$. Substitution into (54) gives

$$1 = \mathbb{E}_t \left\{ \exp\left(i_t - \delta - \sigma \left(c_{t+1} - c_t \right) - \pi_{t+1} \right) \right\}, \tag{70}$$

so that log-linearization yields (29):

$$1 = \mathbb{E}_t \left\{ 1 + i_t - \delta - \sigma \left(c_{t+1} - c_t \right) - \pi_{t+1} \right\}. \tag{71}$$

- (30)

This results from the log of real marginal cost (56) and employment subsidy $\tau = 1/\theta$.

- (31)

Producer prices are set by firms based on the Calvo structure, so that

$$P_{Ht} = \left[\omega P_{H,t-1}^{1-\theta} + (1-\omega) P_{Ht}^{opt1-\theta} \right]^{\frac{1}{1-\theta}}. \tag{72}$$

Log-linearization yields

$$\pi_{Ht} = (1-\omega) \left(p_{Ht}^{opt} - p_{H,t-1} \right). \tag{73}$$

The optimal price p_{Ht}^{opt} is the solution from the firm's first-order condition. First, rewrite first-order condition (59) as

$$\sum_{k=0}^{\infty} \omega^k \mathbb{E}_t \left\{ \Lambda_{t,t+k} Y_{t+k|t} \left[\frac{P_{Ht}^{opt}}{P_{H,t-1}} - \frac{\theta}{\theta-1} \frac{P_{H,t+k}}{P_{H,t-1}} MC_{H,t+k} \right] \right\} = 0, \tag{74}$$

where we have scaled some variables to obtain ratios that have a well-defined value in the steady state, and $MC_{H,t+k} = MC_{t+k}/P_{H,t+k}$ is the real marginal cost. Using that $\Lambda_{t,t+k} = \beta^k$ in the steady state, log-linearization yields

$$p_{Ht}^{opt} - p_{H,t-1} = (1-\omega\beta) \sum_{k=0}^{\infty} (\omega\beta)^k \mathbb{E}_t \left\{ p_{H,t+k} - p_{H,t-1} + \widehat{mc}_{H,t+k} \right\}, \tag{75}$$

where $\widehat{mc}_{H,t+k}$ denotes the deviation of log real marginal cost $mc_{t+k} - p_{H,t+k}$ from its steady state $\log\left(\frac{\theta-1}{\theta}\right)$. Writing this equation recursively yields

$$p_{Ht}^{opt} - p_{H,t-1} = \omega\beta \mathbb{E}_t \left\{ p_{H,t+1}^{opt} - p_{Ht} \right\} + \pi_{Ht} + (1-\omega\beta)\, \widehat{mc}_{Ht}. \tag{76}$$

Substitution into (73) gives (31), where κ_{mc} is defined in Table 1.

- (32)

Start from the domestic labor market equilibrium formula in (60) and substitute (55). Also substitute $Y_t(j) = \left(\frac{P_t(j)}{P_{Ht}}\right)^{-\theta} Y_t$, which is implied by (65) and (67), and use that $A_t(j) = A_t$. That gives

$$L_t = \frac{1}{n} \int_0^n \left(\frac{P_t(j)}{P_{Ht}}\right)^{-\theta} dj \frac{Y_t}{A_t}. \tag{77}$$

Finally, we take the logarithm and use Galí (2008, p.162) to motivate why the integral part is approximately zero.

- (35)

This results from log-linearizing home aggregate output in (69), which yields

$$y_t = \nu \left[c_t - \eta \left(p_{Ht} - p_t\right)\right] + (1-\nu) \left[c_t^* - \eta \left(p_{Ht} - s_t - p_t^*\right)\right], \tag{78}$$

and then substituting (37) and the law of one price (34).

- (37)

For $\eta \neq 1$, this follows by log-linearizing the top equation in (68). For $\eta = 1$, (68) becomes the Cobb-Douglas combination $P_t = P_{Ht}^{\nu} P_{Ft}^{1-\nu}$, which also yields (37).

- (39)

The exchange rate s_t clears the asset market, so we start from risk sharing (33). This is the core equation. After substitution of (37) and (38), it is

$$\sigma \left(c_t - c_t^*\right) = s_t + p_{Ft}^* - \left[\nu p_{Ht} + (1-\nu) p_{Ft}\right]. \tag{79}$$

This is not yet an s-function in the (i_t, E_t)-form defined in Section 2.2.1, where the i_t argument captures the interest rate impact on the exchange rate via all contemporaneous channels, and the vector E_t accounts for everything else. Therefore, we now substitute out contemporaneous variables affected by i_t. We do so in a streamlined manner, where first-order conditions and equilibrium relations are used only once, and they substitute out the choice and equilibrating variables based on the underlying economic mechanisms.

We first focus on p_{Ht}. Calvo pricing (31) and marginal cost (30) imply

$$p_{Ht} = p_{H,t-1} + \beta \mathbb{E}_t \left\{\pi_{H,t+1}\right\} + \kappa_{mc} \left(\mathrm{w}_t - a_t - p_{Ht}\right), \tag{80}$$

reflecting that p_{Ht} is driven by wage w_t. The latter is such that labor demand equals supply, reflected by labor market equilibrium (32). Households' labor supply ℓ_t satisfies

(28). Combining them and using (37) gives the equilibrium wage

$$\mathrm{w}_t = \nu p_{Ht} + (1-\nu)\, p_{Ft} + \gamma\,(y_t - a_t) + \sigma c_t, \tag{81}$$

so that w_t depends on output y_t. The latter equilibrates the goods market. Substituting goods market equilibrium (35) for y_t in the equilibrium wage yields product wage

$$\mathrm{w}_t - p_{Ht} = \varpi_{tot}\,(p_{Ft} - p_{Ht}) + \varpi_c c_t + \gamma\,[(1-\nu)\, c_t^* - a_t]\,, \tag{82}$$

where ϖ_c (ϖ_{tot}) is the full impact of c_t (tot_t) on the product wage for given terms of trade (consumption), as defined in Table 1. Substitution into (80) yields

$$p_{Ht} = \frac{1}{1+\kappa_{mc}\varpi_{tot}} \begin{bmatrix} p_{H,t-1} + \beta \mathbb{E}_t\{\pi_{H,t+1}\} \\ +\kappa_{mc}\,(\varpi_{tot} p_{Ft} + \varpi_c c_t + \gamma\,(1-\nu)\, c_t^* - (\gamma+1)\, a_t) \end{bmatrix}. \tag{83}$$

Substituting this for p_{Ht} and (34) for p_{Ft} in core equation (79) gives

$$\begin{aligned} \sigma\,(c_t - c_t^*) \quad &= \quad s_t + p_{Ft}^* - \nu\, \tfrac{1}{1+\kappa_{mc}\varpi_{tot}} \begin{bmatrix} p_{H,t-1} + \beta \mathbb{E}_t\{\pi_{H,t+1}\} \\ +\kappa_{mc}\,(\varpi_{tot}\,(p_{Ft}^* + s_t) + \varpi_c c_t) \\ +\kappa_{mc}\,(\gamma\,(1-\nu)\, c_t^* - (\gamma+1)\, a_t) \end{bmatrix} \\ &\quad - (1-\nu)\,(p_{Ft}^* + s_t)\,. \end{aligned} \tag{84}$$

Next, focus on the foreign price p_{Ft}^*. It follows similarly as p_{Ht}, using the foreign equivalents of (31), (30), (32), (28), and using (38) and (36) instead of (37) and (35). This gives

$$p_{Ft}^* = p_{F,t-1}^* + \beta \mathbb{E}_t\left\{\pi_{F,t+1}^*\right\} + \kappa_{mc}\,((\sigma+\gamma)\, c_t^* - (\gamma+1)\, a_t^*)\,. \tag{85}$$

Substituting (85) for p_{Ft}^* in (84) and then using the lag of (34) for $p_{F,t-1}^*$ and Euler equation (29) and its foreign equivalent to remove $c_t - c_t^*$ yields (39).

To understand that this is in (i_t, E_t)-form, realize that all predetermined, exogenous, and foreign variables are unaffected by i_t, so they have to be put in E_t. The recursive nature of (29) implies that $\mathbb{E}_t\{c_{t+1}\}$ is determined by expectations of future variables, so there is no contemporaneous effect of i_t, making $\mathbb{E}_t\{c_{t+1}\}$ part of E_t. Similarly, (31) implies that $\mathbb{E}_t\{\pi_{H,t+1}\}$ is part of E_t. For $\mathbb{E}_t\{\pi_{t+1}\}$ one should realize that households base their consumption decision on $\mathbb{E}_t\{\pi_{t+1}\}$ as a whole, not on just the p_t part within it. Hence, i_t can only affect c_t via $\mathbb{E}_t\{\pi_{t+1}\}$ if the latter as a whole changes, so that $\mathbb{E}_t\{\pi_{t+1}\}$ does not contain a contemporaneous channel and is thus part of E_t.