Disaster risks, disaster strikes, and economic growth: the role of preferences*

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Abstract:

This paper studies the role of preferences on the link between disasters, growth, and welfare. An endogenous growth model with endogenous disasters is presented in which one can derive closed-form solutions with recursive preferences. The model distinguishes disaster risks and disaster strikes and highlights the numerous mechanisms through which they may affect growth. It is shown that separating aversion to risk from the elasticity of inter-temporal substitution bears critical implications that enable to better understand these mechanisms. In a calibration of the model based on empirical evidence about disaster impacts in the U.S., it is shown that precautionary savings are unlikely to be sufficient to generate a positive link between disasters and growth as sometimes encountered in the empirical literature. The paper also assesses the impact of disasters on welfare and highlights the large benefits that could be obtained by enhancing insurance coverage.

JEL classification: E21; O4; Q54

Keywords: Environmental disasters; Endogenous growth; Recursive utility; Precautionary savings

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1 Introduction

Risk is more than ever an essential concern for economic policies. The renewal of interest for the study of risk in macroeconomic models is not only the result of the 2008 economic and financial crisis but also reflects the growing concerns around environmental risks such as climate change and environmental disasters. The risk of rare catastrophic events bears critical welfare implications not only as disasters hurt when they strike but also as their anticipation may affect agents decisions. This view has been first introduced by Rietz (1988) in an attempt to explain the equity premium puzzle (Mehra & Prescott, 1985). Since, his idea that a low subjective probability of a catastrophic event may drive agents investment decisions has gained momentum with a development of new theoretical frameworks (e.g. Barro, 2006, 2009; Gabaix, 2012) supported by empirical evidences on the history of catastrophic events (e.g. Barro & Ursua, 2008). More recently, some authors have adopted similar frameworks to analyze the macroeconomic impacts of environmental disasters in endogenous growth frameworks (Ikefuji & Horii, 2012; Barro, 2015; Müller-Fürstenberger & Schumacher, 2015; Bakkensen & Barrage, 2016; Bretschger & Vinogradova, 2017; Akao & Sakamoto, 2018). As pointed out by Bakkensen & Barrage (2016), if these disasters reduce output or production means when they strike, they also affect consumption and savings decisions in an ambiguous way, resulting in potentially important long-term impacts.

In line with this literature, the objective of the present paper is to better understand the link between environmental disasters, economic growth, and welfare. To investigate the underlying mechanisms, I propose an endogenous growth model with endogenous disasters that can be fully solved analytically. The model builds on the frameworks proposed by Müller-Fürstenberger & Schumacher (2015) and Bretschger & Vinogradova (2017), and extends these earlier works by allowing for a more general representation of individuals' preferences. In particular, the model is solved for the class of utility functions proposed by Epstein & Zin (1989) and Weil (1990), building on Kreps & Porteus (1978) non-expected utility theory. As shown by a large literature in finance (see Bansal & Yaron, 2004), by distinguishing risk aversion from the inter-temporal elasticity of substitution, these utility functions enable to better explain individuals decision in front of risk. As the objective of the paper is to understand how disasters affect growth and welfare, allowing for this more general and flexible representation of preferences will prove critical. In particular, the paper shows analytically that the restrictions imposed by more standard utility functions — e.g. logarithmic or time-additive power utility — bias our understanding of the mechanisms that link disasters to growth, and welfare. In a calibration of the model that matches empirical evidence on environmental disasters, the paper also shows that these biases matter quantitatively.

In order to start from a simple benchmark, the model is first solved in the case of exogenous disasters. Several preliminary intuitions are derived in this situation. I then turn to the case of disasters whose probability can be mitigated through a policy. In the appendix the model is also solved for multiple types of disasters including catastrophes of endogenous intensity. It follows from the model that the optimal shares of output consumed, saved, and spent in risk-mitigation are all constant on the optimal path. The effects of the model's parameters are studied and in particular the role of the preference parameters are emphasized. While risk and risk aversion (RRA) drive the decision to mitigate risk, the inter-temporal elasticity of substitution (IES) plays no role in this decision. However, it appears to be critical in the risk sensitivity of the consumption/savings decision. When the risk of disasters increases, current consumption is partly transferred to the future through savings when the IES is below unity. Interestingly, if the *sign* of this effect solely depends on the IES, its *magnitude* depends on the RRA. While a low IES — i.e. high aversion to fluctuations — unambiguously leads to more precautionary savings, a high aversion to risk may increase either precautionary savings or precautionary consumption. This result shows that it is essential to depart from the standard time-additive utility function as aversion to risk and to fluctuations end up having very different effects on the optimal solution. A second result of importance is that, when introducing an instrument to mitigate disasters, an increase in risk also generates a transfer from savings to risk-mitigation spending. As a result, and contrary to what has been emphasized so far in the literature, an IES below unity is a necessary but insufficient condition to guarantee a net positive response of savings to risk.

From the law of capital accumulation, one can compute analytically the stochastic growth rate as well as the average long-run growth rate of the economy. Most interestingly, one can look at the effect of disasters on the latter. Following the terminology used by Bakkensen & Barrage (2016) I distinguish the impact of disaster *risks* from the one of disaster *strikes*. If damages from catastrophes (i.e. from strikes) reduce expected growth, their anticipation (i.e. risk) has an ambiguous effect through the sensitivity of capital accumulation to risk. For realistic parameter values — i.e. unless the crowding out of risk-mitigation spending over savings is too high — disasters foster average long-run growth if aversion to risk and to fluctuations are both large enough. Since the existence of disasters necessarily reduces welfare, there are therefore situations in which growth and welfare are inversely linked. To further examine the impact of disasters on welfare, I compute analytically the marginal rate of substitution between disaster parameters (i.e. frequency and intensity) and output, as well as a measure proposed by Lucas (1987, 2003) to assess the welfare benefits of the policy instrument relative to a business-as-usual scenario.

In order to illustrate quantitatively the analytic findings of the paper, the model is then calibrated so as to represent the U.S. — a country among the most impacted by environmental disasters (see Shi et al., 2015) — disaggregated at the county level. Disaster parameters are proxied from the most recent study on the impact of disasters in U.S. counties over the last 80 years by Boustan et al. (2017). From this exercise, we reach three important conclusions. First, if a positive impact of disasters on long-run growth is theoretically possible in this framework, such a positive relationship can occur only for extremely large disasters and (rather implausibly) high values of aversion towards risk and fluctuations. Second, the effects of disasters on welfare appear significant, even ignoring their impacts on human lives. For instance, reducing by only 10% the likelihood of disasters would be equivalent to an increase by 0.65% of GDP in our main scenario, even though yearly expected damages on GDP are as low as 0.13%. Interestingly, holding expected damages constant but increasing disaster intensity, the welfare effects become much larger. This result stresses the role of insurance as an adaptation strategy, as the welfare gains from trading-off disaster intensity against likelihood appear important. Third and last, the two previous results are sensitive to the calibration of preferences parameters. Thus, the constraints imposed by logarithmic or power utility functions do not only affect our qualitative understanding of the effects of disasters, but they also matter quantitatively. In particular, when using high values for the elasticity of the utility to capture risk aversion, one overestimates the importance of precautionary savings and may wrongly conclude that disasters positively affect growth. When using lower values to better match the IES, he instead underestimates the impact of disasters on welfare, and the level of optimal mitigation policies.

This paper contributes to two strands of the literature. First, it provides a novel framework to study the effect of environmental disasters on economic growth. Improving our understanding of the mechanisms underlying this link is critical not only from a theoretical point of view but also as to guide future empirical research on this issue. Indeed, the empirical literature on the link between disasters and growth points towards contrasted evidence. Skidmore & Toya (2002) conclude that higher frequencies of climatic disasters may foster growth, possibly through an effect on human capital accumulation and technology. While Cavallo et al. (2013) find no significant impact of disasters on short and long-run growth, Sawada et al. (2011) find significant negative effects in the short run, but positive effects in the longer term. Strobl (2011) studies hurricanes in the U.S. coastal counties and finds evidence of negative effects with very partial recovery, but the macroeconomic impact of these local catastrophes appears to be negligible. Noy (2009) also finds negative but heterogeneous impacts, with more developed countries being less exposed. More recently, Hsiang & Jina (2014) found a strong negative long-run effect of hurricanes on output and long-run growth with no evidence of a rebound effect in the twenty years following a catastrophe. Some previous theoretical works have recently attempted to understand these diverging empirical evidence. Ikefuji & Horii (2012) stress the role of human capital as a substitute for physical capital to sustain growth when physical capital pollutes. Bakkensen & Barrage (2016) try to reconcile the heterogeneous empirical findings by disentangling hurricanes *strikes* and hurricanes *risks*. They show that while the former may persistently reduce output, the second may foster growth through more accumulation due to precautionary savings. They argue that the contradictory results found in empirical studies might partly be explained by different methodologies that either capture the effect of disaster strikes or disaster risks. Akao & Sakamoto (2018) study exogenous disasters and discuss the role of human capital and technology. As Bakkensen & Barrage (2016), they emphasize the key role of the elasticity of the utility function for disaster risks to foster growth through precautionary savings. Although they do not focus directly on growth, Müller-Fürstenberger & Schumacher (2015) and Bretschger & Vinogradova (2017) both analyze the effect of risk on capital accumulation in a Ramsey type of model where risk can be mitigated through abatement activities. Their results also support the idea that disasters may accelerate capital accumulation depending on the elasticity of the utility function. By contrast, using a more satisfactory representation of preferences towards risk, calibrated so as to match empirical evidence of disaster impacts, this paper shows that precautionary savings are unlikely to be sufficient to generate a positive link between disasters and growth as sometimes found in the empirical literature. It remains an open question whether this empirical observation is robust, but if that is, future research will have to determine which other mechanisms could explain it.

Second, this paper adds to the theoretical literature on the optimal mitigation of environmental risks. In particular, it contributes to recent literature that incorporates recursive preferences into environmental models where risk matters. Previous studies have analyzed the effect of pollution (Soretz, 2007) or biodiversity losses (Augeraud-Véron et al., 2018) on fluctuations, and shown how optimal policies depended on preferences parameters. Considering larger shocks, Barro (2015) extends the previous disaster model of Barro (2009) to disentangle environmental disasters from other types of catastrophes. In a different set-up, Bansal & Ochoa (2011), Bansal et al. (2016), and Karydas & Xepapadeas (2019) examine the effect of temperature-driven disasters on market returns with non-expected utility. van der Ploeg & de Zeeuw (2017) study precautionary savings as a reaction to an endogenous climate tipping point. They characterize savings responses to the tipping depending on its impact delay and on the distance of the economy from its steady-state. However, the model does not provide closed-form solutions and does not enable to study repeated catastrophes. Other papers using numerical methods have introduced Epstein-Zin-Weil preferences in climate economy models, such as DSGE models (e.g. van den Bremer & van der Ploeg, 2018) and Integrated Assessment Models (see Crost & Traeger, 2014; Jensen & Traeger, 2014; Cai & Lontzek, 2018; Olijslagers & van Wijnbergen, 2019). To my knowledge, this paper is the first to present a framework to study analytically the relationship between endogenous growth and endogenous disasters in which agents display recursive preferences. Both through analytical results and a calibration consistent with observed impacts of disasters, the paper shows the importance of separating aversion towards risk and fluctuations, in order to better understand the effects of disasters on growth, welfare, and the implications for optimal policies.

The rest of the paper is organized as follows. Section 2 presents the general framework. Section 3 considers the case of exogenous disasters as a benchmark to highlight the first intuitions of the model. Section 4 turns to endogenous disasters whose probability can be reduced through a risk-mitigation policy. Section 5 provides a calibration of the model and a quantitative assessment of the link between disasters, growth and welfare, and the importance of using non-expected utility over more restrictive representations of preferences. Section 6 concludes. Computations are reported to the appendix, where the model is also extended to multiple types of disasters including of endogenous intensity.

2 General framework

The model features essentially two ingredients. One is the stochastic process driving catastrophes. The other is the representation of preferences. We assume utility is derived from the consumption of a unique good C. The central planner's preferences are defined recursively as first proposed by Epstein & Zin (1989) and Weil (1990), and extended to continuous time by Svensson (1989) and Duffie & Epstein

(1992). These preferences can be represented by the following utility function:

$$(1-\gamma)U_t = \left[C_t^{\frac{\epsilon-1}{\epsilon}} dt + e^{-\rho dt} \left((1-\gamma)\mathbb{E}U(t+dt) \right)^{\frac{\epsilon-1}{\epsilon(1-\gamma)}} \right]^{\frac{\epsilon(1-\gamma)}{\epsilon-1}} \tag{1}$$

where ρ is the pure rate of time preferences, γ the coefficient of relative risk aversion (RRA), and ϵ the inter-temporal elasticity of substitution (IES), so that $1/\epsilon$ can be understood as aversion towards inter-temporal fluctuations. In the specific case where $\gamma = 1/\epsilon$ we obtain the standard time-additive power utility function widely used in the literature. In the even more special case where this parameter tends to one, the power utility converges towards a logarithmic utility. The recursive form of the function defined in equation (1) yields the following Hamilton Jacobi Bellman (HJB) equation:

$$(1-\gamma)V(K_t) = \max \left[C_t^{\frac{\epsilon-1}{\epsilon}} dt + e^{-\rho dt} \left((1-\gamma)\mathbb{E}V(K_{t+dt}) \right)^{\frac{\epsilon-1}{\epsilon(1-\gamma)}} \right]^{\frac{\epsilon(1-\gamma)}{\epsilon-1}} \tag{2}$$

Now, let's consider an economy facing disasters, i.e. catastrophic events that may happen with small probability and destroy part of the capital stock. As Martin & Pindyck (2015), we consider multiple types of catastrophes and keep the specification general enough so that these events may include but are not limited to environmental disasters. Although they are rare events, their effect is long lasting: once capital is destroyed, it takes time to re-build. These disasters are assumed endogenous to some risk-mitigation activities, and are taken to be uninsurable. We denote τ the share of output spent to mitigate disasters. The central planner must therefore allocate production (Y) between consumption (C), risk-mitigation activities (τY) and savings (S). Assuming there are n types of disasters and m types of risk-mitigation technologies, the law of capital accumulation is defined as:

$$dK_t = [Y_t - \sum_{j=1}^{m} \tau_{j,t} Y_t - C_t]dt + \sigma_{w,t} dz - \sum_{i=1}^{n} \sigma_{p,i,t} dq_{i,t} \tag{3}$$

where dz is a Wiener process scaled by $\sigma_{w,t}$, and $dq_{i,t}$ a Poisson process scaled by $\sigma_{p,i,t}$. The Wiener process models small fluctuations around the trend, while the Poisson process models rare catastrophic events. The use of the Poisson process in the modelling of agents' optimal consumption and savings decisions has been introduced by Wälde (1999) and later used in the study of natural disasters by Müller-Fürstenberger & Schumacher (2015) and Bretschger & Vinogradova (2017), and in a slightly different set-up by Ikefuji & Horii (2012). As Müller-Fürstenberger & Schumacher (2015), we will assume the

Poisson process to be endogenous possibly both through its intensity and its probability, and to depend on risk-mitigation spending. The probability of a shock is assumed to be of the form $\mathbb{E}dq_{i,t} = \lambda_i f_i dt$ with λ_i a constant and f_i a function of abatement activities τ_j, $j = 1, ..., m$ to be defined. We also denote $\tilde{K}_i$ the stock of capital after a shock of the i^{th} process occurred, with $\forall i$, $0 < \tilde{K}_{i,t} < K_t$, so that the size of a shock for the process i at time t is $\sigma_{p,i,t} = K_t - \tilde{K}_{i,t}$. Bretschger & Vinogradova (2017) also consider the case of an endogenous variance for the Wiener process. Although possible in this model, for the sake of simplicity we keep the Wiener process independent of risk-mitigation spending as this feature does not bear critical implications.

The objective of the central planner is to maximize its utility (1) subject to the stochastic law of capital accumulation (3). The solution method is detailed in the appendix. It makes use of useful contributions in the resolution of stochastic problems in continuous time (e.g. Merton, 1971; Wälde, 1999; Sennewald & Wälde, 2006) and how it applies to Epstein-Zin-Weil preferences in an endogenous growth model (see Epaulard & Pommeret, 2003). It is shown in the appendix that if we define:

$$X(K, C, \tau) = V_k[(1 - \sum_{j=1}^{m} \tau_{j,t})Y - C] + \frac{1}{2}V_{kk}\sigma_w^2 + \sum_{i=1}^{n} \lambda_i f_i \left(V(\tilde{K}_i) - V(K) \right) \tag{4}$$

with $V_k = \partial V(K)/\partial K$ and $V_{kk} = \partial^2 V(K)/\partial K^2$, then the Hamilton-Jacobi-Bellman equation of this problem can be expressed as:

$$\rho \frac{\epsilon(1-\gamma)}{\epsilon - 1} V(K_t) = \max \left[\frac{\epsilon}{\epsilon - 1} \frac{C_t^{\frac{\epsilon-1}{\epsilon}}}{[(1-\gamma)V(K_t)]^{\frac{\epsilon-1}{\epsilon(1-\gamma)}-1}} + X(K, C, \tau) \right] \tag{5}$$

and the associated first order conditions with respect to C and τ_j are:

$$\frac{C_t^{-\frac{1}{\epsilon}}}{[(1-\gamma)V(K_t)]^{\frac{\epsilon-1}{\epsilon(1-\gamma)}-1}} + X_C = 0$$

$$X_{\tau_j} = 0 \qquad \forall j$$

with X_C and X_{τ_j} the derivatives of X with respect to C and τ_j, hence:

$$C_t^{-\frac{1}{\epsilon}} = V_k \left[(1-\gamma)V(K_t)\right]^{\frac{\epsilon-1}{\epsilon(1-\gamma)}-1} \tag{6}$$

and:

$$\text{for } j = 1, ..., m \quad : \quad YV_k = \sum_{i=1}^{n} \lambda_i \left[f_i \frac{\partial V(\tilde{K}_i)}{\partial \tilde{K}_i} \frac{\partial \tilde{K}_i}{\partial \tau_j} + \frac{\partial f_i}{\partial \tau_j} \Big(V(\tilde{K}_i) - V(K) \Big) \right] \tag{7}$$

The previous equations highlight the trade-off between the different uses of resources. Equation (6) gives the optimal arbitrage between the benefits and the opportunity cost of consumption. Equation (7) simply states that at the optimum the marginal cost of risk-mitigation spending (on the left hand side) should be equal to the marginal benefits from reducing disaster frequency and intensity (on the right hand side). This framework remains flexible and enables to study a large variety of risks in different economic settings. In the next section, I start with the benchmark case of exogenous disasters (i.e. no risk-mitigation activity) to present in the simplest way the mechanisms driving the link between disasters and growth and how they depend on preferences. Then, I turn to the case of disasters of endogenous probability. A more comprehensive set-up with both disasters of endogenous probability and intensity is presented in the appendix.

3 Benchmark: exogenous disasters

3.1 Specification

In this section, we consider the simple case of a unique process ($n = 1$), and take the probability of a disaster as fixed ($m = 0$, i.e. no risk-mitigation instrument available), and their intensity as a constant fraction of the capital stock. Specifically, we take $f = 1 + \delta$, i.e. $\mathbb{E}(dq_t) = \lambda(1 + \delta)dt$, and $\tilde{K} = \omega K$ with $\omega \in [0; 1]$ a constant. The variance of the Wiener process is assumed to linearly depend on the level of the capital stock, with $\sigma_w = \sigma K$, so that fluctuations remain proportional to the size of the economy. Finally, we assume production follows from an AK technology. This last assumption is made for two reasons. First, it is technically convenient as it will prove to provide sufficient linearity to the problem to obtain closed-form solutions. Second, the AK specification is relevant in our setting as it captures the "no-rebound" effect observed empirically for natural disasters. As shown by Hsiang & Jina (2014) using the example of hurricanes, natural disasters cause permanent output losses that are not compensated by higher growth rates in the aftermath, nor in two following decades. These evidences therefore suggest that the AK specification is best fitted to model the effect of disasters on long-run growth.

3.2 Optimal resources allocation

The shape of the problem leads to the following guess for the value function (Weil, 1990; Epaulard & Pommeret, 2003):

$$V(K) = \psi^{\frac{1-\gamma}{1-\epsilon}} \frac{K^{1-\gamma}}{1-\gamma} \tag{8}$$

with ψ a constant to be determined. Substituting the guess (8) into the first order condition with respect to C (6) derived in the previous section gives:

$$C^* = \psi K \tag{9}$$

and going back to the HJB equation (5) we can solve for ψ, the optimal share of capital consumed:

$$\psi = \rho\epsilon + (1-\epsilon)\left(A - \frac{\gamma\sigma^2}{2} - \lambda(1+\delta)\frac{(1-\omega^{1-\gamma})}{1-\gamma}\right) \tag{10}$$

and from the law of capital accumulation defined by equation (3) we can determine the optimal saving rate $s^* = S^*/Y$:

$$s^* = \frac{1}{A}\left[\epsilon(A-\rho) + (1-\epsilon)\left(\frac{\gamma\sigma^2}{2} + \lambda(1+\delta)\frac{(1-\omega^{1-\gamma})}{1-\gamma}\right)\right] \tag{11}$$

Consumption and savings are therefore constant fractions of capital and output on the optimal path. Interestingly, the consumption share is decreasing with risk — i.e. higher σ or λ, lower ω — and risk aversion — higher γ — if and only if $\epsilon < 1$. Symmetrically, when $\epsilon < 1$ the saving rate is increasing with risk and risk aversion. This situation can be interpreted as *precautionary savings*, while the opposite one ($\epsilon > 1$) can be interpreted as *precautionary consumption*. The arbitrage between precautionary savings and consumption depends on the relative importance of an income and a substitution effect caused by an increase in risk. When the IES (ϵ) takes a low-value, aversion to inter-temporal fluctuations ($1/\epsilon$) is high, in which case a higher risk of a catastrophe (and therefore a higher risk of being poorer) in the future incentivizes some transfers from current to future consumption. This income effect can be more than compensated by a substitution effect when agents are little averse to fluctuations. In this second situation, when capital is more at risk, the incentives to consume rather than accumulate are higher and an increase in risk leads to more consumption in the present at the expense of savings. The role of the inter-temporal elasticity of substitution in determining the link between risk and consumption/savings decisions has been early emphasized by Leland (1968) and Sandmo (1970), and more recently in the case of natural disasters by Müller-Fürstenberger & Schumacher (2015), Bakkensen & Barrage (2016), Bretschger & Vinogradova (2017) and Akao & Sakamoto (2018). However, because they use a time-additive power utility function, these papers cannot disentangle the effect of risk aversion from aversion to fluctuations. The use of non-expected utility enables to clarify these previous results and better identify the role of each parameter. As illustrated by the following comparative statics, we see that the *sign* of the effect of risk on consumption and savings only depends on the value of ϵ relative to 1:

$$\frac{\partial \psi}{\partial \lambda} = -A\frac{\partial s^*}{\partial \lambda} = -(1-\epsilon)(1+\delta)\frac{(1-\omega^{1-\gamma})}{1-\gamma}\begin{cases} < 0, & \text{if } \epsilon < 1. \\ \geq 0, & \text{otherwise.} \end{cases}$$

$$\frac{\partial \psi}{\partial \omega} = -A\frac{\partial s^*}{\partial \omega} = (1-\epsilon)\lambda(1+\delta)\omega^{-\gamma}\begin{cases} > 0, & \text{if } \epsilon < 1. \\ \leq 0, & \text{otherwise.} \end{cases}$$

while the *magnitude* of this effect positively depends on the risk aversion coefficient γ since (see proof # 1 in the appendix):

$$\forall \gamma \neq 1, \quad \frac{\partial \frac{1-\omega^{1-\gamma}}{1-\gamma}}{\partial \gamma} = \frac{ln(\omega)\omega^{1-\gamma}(1-\gamma) + (1-\omega^{1-\gamma})}{(1-\gamma)^2} > 0$$

$$\frac{\partial \omega^{-\gamma}}{\partial \gamma} = -ln(\omega)\omega^{-\gamma} > 0$$

Thus, if a low IES implies that precautionary savings dominate over precautionary consumption, a high value of the RRA simply magnifies this effect but does not play on its sign. The restriction imposed by the time-additive expected utility that $\gamma = 1/\epsilon$ therefore leads to a mis-interpretation of the effect of preferences on the relationship between risk and consumption/savings decisions. In the even more special case where both of these parameters converge to 1, (i.e. when utility is logarithmic as in e.g. Golosov et al., 2014) the results further simplify and disasters do not have any effect on agents savings decisions. Thus, even in this simple benchmark, the generalization to Epstein-Zin-Weil preferences already appears useful as it offers a richer characterization of the effects of risk on individuals' decisions.

3.3 Optimal growth and the effects of disasters

The previous results suggest that the effect of disaster *risks* on growth is ambiguous. In some situations, higher risk can foster capital accumulation, and thus economic growth. However, even in this case it remains unclear what is the long-run aggregate impact of disaster *risks* and *strikes* on growth. To examine this issue, we first compute the stochastic growth rate of the economy from the law of capital accumulation as stated by equation (3):

$$\begin{aligned}\frac{dC^*}{C} &= (A-\psi)dt + \sigma dz - (1-\omega)dq_t \\ &= \left[\epsilon(A-\rho) + (1-\epsilon)\frac{\gamma\sigma^2}{2} + \frac{1-\epsilon}{1-\gamma}\lambda(1+\delta)(1-\omega^{1-\gamma})\right]dt + \sigma dz - (1-\omega)dq_t\end{aligned} \tag{12}$$

The first term in dt is the trend growth rate, and σdz represents the fluctuations around this trend. When the economy is hit by a shock, consumption decreases by $(1-\omega)$. Note that in a deterministic model without shocks, we obtain the standard Keynes-Ramsey formula where A is the marginal return on capital : $(dC/C)^{*det} = \epsilon\,[A-\rho]\,dt$. Finally, because $\mathbb{E}(dz) = 0$ and $\mathbb{E}(dq_t) = \lambda(1+\delta)dt$, the expected growth rate of this economy, which is also the average long-run growth rate g^* is :

$$g^* = \mathbb{E}\left(\frac{dC^*}{C}\right) = \left[\epsilon(A-\rho) + (1-\epsilon)\frac{\gamma\sigma^2}{2} + \frac{1-\epsilon}{1-\gamma}\lambda(1+\delta)(1-\omega^{1-\gamma}) - \lambda(1+\delta)(1-\omega)\right]dt \tag{13}$$

The previous formula enables to disentangle the effect on growth of disaster *risks,* i.e. the mechanisms through which the anticipation of disasters may affect economic decisions, from the effect of disaster *strikes* captured by the last term of the right-hand side of equation (13). The sensitivity of the expected growth rate to disasters can be analyzed by looking at the following comparative statics:

$$\frac{\partial g^*}{\partial \lambda} = (1+\delta)\left[(1-\epsilon)\frac{1-\omega^{1-\gamma}}{1-\gamma} - (1-\omega)\right]dt \tag{14}$$

$$\frac{\partial g^*}{\partial \omega} = \lambda(1+\delta)\left[1-(1-\epsilon)\omega^{-\gamma}\right]dt \tag{15}$$

From these results, the effects of disaster risks and strikes on growth appear clear. Disaster strikes have an obvious negative effect on average long-run growth: when the probability or the intensity of disasters increases — higher λ, lower ω — the expected drop in output due to shocks is larger and so expected growth declines. However, this effect must be weighted against the ambiguous impact of disaster risks on growth. This effect is driven by precautionary savings or consumption, and is therefore positive when $\epsilon < 1$ and negative otherwise. In both cases, it is magnified for higher values of risk aversion γ. When $\epsilon < 1$, since precautionary savings need to compensate for the losses caused by disaster strikes, our results show that disasters and growth can be positively linked in the long-run *if and only if* ϵ is sufficiently small and γ is sufficiently large, i.e. if the economy displays both high risk aversion and high aversion to inter-temporal fluctuations. These results give theoretical support to the empirical findings of Sawada et al. (2011) who found negative effects of disasters on short-run growth, but positive effects in the long-run as was found by Skidmore & Toya (2002) from a cross-sectional analysis. Indeed, when precautionary savings dominate, despite their negative immediate impact disasters may encourage capital accumulation and thus promote growth in the long-run. As noted by Bakkensen & Barrage (2016), whether cross-sectional or panel analysis are used to assess empirically the impact of disasters may affect the results as these methods will essentially capture different effects. While cross-sectional studies identify the potentially positive effect of disaster risks on growth, studies using panel data with fixed effect identify the negative effect of disaster strikes.

Two last comments deserve attention. First, it should be noted that disasters generate large transfers between generations. These transfers are due both to the impact of disaster risks — that either favor consumption or savings — on the deterministic pattern of growth and to the stochastic realization of disasters. Second, although higher risk may in some situations be growth enhancing, it unambiguously reduces welfare. This result holds even ignoring the impact of disasters on human lives, and considering only their effect on the stock of capital. Thus, and as pointed out by Akao & Sakamoto (2018) and Bakkensen & Barrage (2016), there are cases in which growth and welfare vary with opposite signs as a response to risk. This last result is important to stress as a positive link between disasters and growth *should not* be interpreted as disasters being welfare-improving.

4 Disasters of endogenous probability

4.1 Specification

In this section we turn to the situation in which resources can be allocated to reduce the risk of disasters through a unique instrument τ (i.e. $m = 1$). In particular, we assume risk-mitigation spending can reduce the *probability* of disasters. The specification is the same as in the previous section, except for f that we now assume to be a function of τ such that $f = 1 + \delta - \tau^\alpha$ with $0 < \alpha < 1$ the inverse of the efficiency of risk-mitigation spending[1]. This specification therefore assumes that the probability of a catastrophe depends on the share of output spent in risk-mitigation. If the entire output was spent to mitigate risk, the probability of a shock would fall to $\lambda\delta$, the probability to face a non-avoidable catastrophe. Absent any abatement activity, the probability would go up to $\lambda(1+\delta)$. If the model remains general with respect to the type of disasters considered, one can understand $\lambda(1-\tau^\alpha)$ as the probability of an environmental disaster, while $\lambda\delta$ corresponds to the probability of non-environmental disasters such as a stock market collapse, a pandemic or a war. Disasters of endogenous probability have been extensively used in the literature, including in several papers by Barro (2009, 2015) and Ikefuji & Horii (2012). As in the previous section, damages will be assumed to be a constant fraction of the capital stock. I show in the appendix that the model can alternatively be solved for disasters of endogenous intensity as done by Müller-Fürstenberger & Schumacher (2015) and Bretschger & Vinogradova (2017), as well as for multiple disasters and multiple instruments. Since these specifications yield similar intuitions, I focus here on the simplest scenario.

4.2 Optimal resource allocation

Applying the new specification, the two first order conditions (6) and (7) together with the HJB equation (5) yield:

$$C^* = \psi K \tag{16}$$

and:

$$\tau^* = \left(\frac{(1-\omega^{1-\gamma})\lambda\alpha}{A(1-\gamma)}\right)^{\frac{1}{1-\alpha}} \tag{17}$$

with:

$$\psi = \rho\epsilon + (1-\epsilon)\left((1-\tau^*)A - \frac{\gamma\sigma^2}{2} - \lambda f^* \frac{(1-\omega^{1-\gamma})}{1-\gamma}\right) \tag{18}$$

[1] Since $0 < \tau < 1$, a lower value of α means more mitigation can be performed with less resources.

and consequently the saving rate $s^* = S^*/Y$ is:

$$s^* = 1 - \tau^* - \frac{1}{A}\left[\rho\epsilon + (1-\epsilon)\left((1-\tau^*)A - \frac{\gamma\sigma^2}{2} - \lambda f^* \frac{(1-\omega^{1-\gamma})}{1-\gamma}\right)\right] \tag{19}$$

As in the previous section, the IES appears to be the critical determinant in the arbitrage between precautionary savings and consumption. Aversion to risk again plays on the magnitude of these effects, but the link now also depends on the effect of risk on risk-mitigation spending. With respect to risk-mitigation, total spending are found to be a constant share of output on the optimal path. The comparative statics below (equations 20-23) show that the share τ^* is strictly increasing with disaster risk (higher λ, lower ω) and risk aversion (γ), but aversion to fluctuations plays no role:

$$\frac{\partial \tau^*}{\partial \lambda} = \frac{\lambda^{\frac{\alpha}{1-\alpha}}}{1-\alpha}\left(\frac{(1-\omega^{1-\gamma})\alpha}{A(1-\gamma)}\right)^{\frac{1}{1-\alpha}} > 0 \tag{20}$$

$$\frac{\partial \tau^*}{\partial \omega} = \frac{-\omega^{-\gamma}}{1-\alpha}\left(\frac{\lambda\alpha}{A}\right)^{\frac{1}{1-\alpha}}\left(\frac{1-\omega^{1-\gamma}}{1-\gamma}\right)^{\frac{\alpha}{1-\alpha}} < 0 \tag{21}$$

$$\frac{\partial \tau^*}{\partial \gamma} = \frac{1}{1-\alpha}\left(\frac{\lambda\alpha}{A}\right)^{\frac{1}{1-\alpha}}\left(\frac{1-\omega^{1-\gamma}}{1-\gamma}\right)^{\frac{\alpha}{1-\alpha}}\left(\frac{ln(\omega)\omega^{1-\gamma}(1-\gamma) + (1-\omega^{1-\gamma})}{(1-\gamma)^2}\right) > 0 \tag{22}$$

$$\frac{\partial \tau^*}{\partial \alpha} = \left(\frac{(1-\omega^{1-\gamma})\lambda\alpha}{A(1-\gamma)}\right)^{\frac{1}{1-\alpha}}\frac{1}{(1-\alpha)^2}\left[ln\left(\frac{(1-\omega^{1-\gamma})\lambda}{A(1-\gamma)}\right) + \frac{1-\alpha}{\alpha}\right] \tag{23}$$

Proof # 1 in the appendix shows that the sign of 22 is always positive. The only ambiguous effect is the one of the risk-mitigation efficiency parameter α. As shown in the appendix (see proof # 2) for low values α has a positive effect on τ^*, but above a certain threshold $\bar{\alpha}$ its effect becomes negative. This non-monotonic relationship can be interpreted as a trade-off between more incentives to spend resources in mitigation when it is more efficient (substitution effect) against the possibility to mitigate more with less resources as the efficiency increases (level effect).

4.3 Optimal growth and the effects of disasters

The law of capital accumulation in equation (3) enables again to compute the stochastic growth rate:

$$\frac{dC^*}{C} = [(1-\tau^*)A - \psi]dt + \sigma dz - (1-\omega)dq_t \tag{24}$$

and thus the expected growth rate (which is also the average long-run growth rate) of this economy:

$$g^* = \mathbb{E}\left(\frac{dC^*}{C}\right) = [(1-\tau^*)A - \psi - \lambda f^*(1-\omega)]dt \tag{25}$$

This formula provides some novel intuitions relative to the one of the previous section. To better understand the new mechanisms at play, one can decompose the effect of disasters on the average long-run growth rate. Differentiating the expected growth rate with respect to λ, we have:

$$\frac{1}{dt}\frac{\partial g^*}{\partial \lambda} = \underbrace{-A\frac{\partial \tau^*}{\partial \lambda}}_{<0} \underbrace{- \frac{\partial \psi}{\partial \lambda}}_{?} \underbrace{- f^*(1-\omega)}_{<0} \underbrace{- \lambda(1-\omega)\frac{\partial f^*}{\partial \lambda}}_{>0} \tag{26}$$

and similarly with respect to ω:

$$\frac{1}{dt}\frac{\partial g^*}{\partial \omega} = \underbrace{-A\frac{\partial \tau^*}{\partial \omega}}_{>0} \underbrace{- \frac{\partial \psi}{\partial \omega}}_{?} \underbrace{+ \lambda f^*}_{>0} \underbrace{- \lambda(1-\omega)\frac{\partial f^*}{\partial \omega}}_{<0} \tag{27}$$

What do we learn from these comparative statics? All terms in equations (26) and (27) are detailed in the appendix. For both equations, the first two terms can be associated with the effect of disaster *risks*, while the last two correspond to the effect of disaster *strikes*. In the following we focus on the second equation, the derivative of expected growth with respect to ω, the share of capital remaining after a catastrophe. This derivative therefore captures the effect on expected growth of a *reduction* in disaster intensity. Similar intuitions can alternatively be derived from the comparative static with respect to λ.

First, when ω increases, disaster strikes become less harmful to the economy as a smaller share of capital $1-\omega$ is destroyed. This effect is captured by the term $\lambda f^* > 0$ in equation (27). How much this effect matters solely depends on the frequency of catastrophes. For more frequent disasters, a reduction of their intensity has larger positive effects on expected growth through this damages term. However, the reduction of disaster intensity has a second, indirect effect on expected growth through expected damages. Indeed, as ω increases, less efforts are performed to mitigate risks. As a result, the equilibrium frequency of disasters λf^* increases and so do expected damages. This second effect is captured by the last term in equation (27), $-\lambda(1-\omega)\frac{\partial f^*}{\partial \omega} < 0$. A higher value of ω has therefore an ambiguous impact on expected damages since less intense catastrophes also lead to less stringent mitigation policies and thus to more frequent disasters. In particular, an increase in ω will reduce expected damages from disaster strikes if and only if $f^* > (1-\omega)\partial f^*/\partial \omega$. Contrary to the previous section with exogenous disasters, allowing for the possibility to mitigate catastrophes therefore leads to less obvious results as more intense disasters will drive more careful policies and could *in fine*, for some parameter values, reduce expected damages.

Turning to disaster risks, we first see — as in the previous section — that disaster intensity may either favor or dampen growth through the consumption savings decision. This effect is captured by the term $-\partial\psi/\partial\omega$ that, for realistic parameter values, is positive if and only if $\epsilon > 1$. This result again says that when the IES is above unity, aversion to fluctuations is low and agents are willing to increase their savings when risk is lowered (and alternatively increase current consumption when risk increases). But in addition to the consumption-savings effect, disaster risks now also affect expected growth through the trade-off between risk-mitigation and savings, given by the term $-A\frac{\partial \tau^*}{\partial \omega} > 0$. Indeed, as τ^* is strictly decreasing in ω, for less intense catastrophes less resources are spent to reduce their probability, which leaves more for savings. Thus, while in the case of exogenous disasters risk was fostering growth if and only if $\epsilon < 1$, this condition is not sufficient anymore when mitigation is possible. Since higher risk now also leads to a transfer from savings to risk-mitigation, a net increase in savings due to risk becomes possible under slightly more restrictive conditions over ϵ. Thus, the standard result of the disaster literature that takes $\epsilon < 1$ as a sufficient condition for disasters to foster capital accumulation is not robust to the introduction of endogenous risk-mitigation policies.

Overall, the introduction of an instrument to reduce disaster probability has an ambiguous effect on growth. If some resources are shifted from capital accumulation to risk-mitigation, in the long run this negative effect might be compensated by the reduction of expected damages from disasters. In a different set-up, Ikefuji & Horii (2012) also found an ambiguous effect on growth of introducing a pollution tax to reduce disaster probability. The underlying mechanisms in this model are different than theirs, but these results bring new evidences that the impact of risk-mitigation policies on growth is ambiguous, even though they positively impact welfare.

4.4 Disasters and welfare

From the solution obtained for ψ, we can study the marginal effect of disaster parameters on welfare. As Barro (2009), I compute the marginal rate of substitution between proportionate changes in production (Y) and in disaster probability (λ):

$$-\frac{\partial V(K)}{\partial \lambda}\frac{\partial Y}{\partial V}\frac{1}{Y} = \frac{1}{\psi}\left[(1+\delta)\frac{(1-\omega^{1-\gamma})}{1-\gamma} - \lambda^{\frac{\alpha}{1-\alpha}}\frac{(\alpha^{\frac{\alpha}{1-\alpha}} - \alpha^{\frac{1}{1-\alpha}})}{1-\alpha}\left(\frac{1-\omega^{1-\gamma}}{A^{\alpha}(1-\gamma)}\right)^{\frac{1}{1-\alpha}}\right] \tag{28}$$

This expression thus corresponds to the share of production society is willing to give up for a reduction in disaster frequency. Similarly, for disaster intensity we have:

$$-\frac{\partial V(K)}{\partial \omega}\frac{\partial Y}{\partial V}\frac{1}{Y} = -\frac{\omega^{-\gamma}}{\psi}\left[\lambda(1+\delta) - \lambda^{\frac{1}{1-\alpha}}\frac{(\alpha^{\frac{\alpha}{1-\alpha}} - \alpha^{\frac{1}{1-\alpha}})}{1-\alpha}\left(\frac{1-\omega^{1-\gamma}}{A(1-\gamma)}\right)^{\frac{\alpha}{1-\alpha}}\right] \tag{29}$$

In both cases, comparative statics do not provide straightforward results as their sign depends on parameters' value. In the next section, the calibration will enable to discuss further these results.

Beyond the marginal effect of disasters, one can also be interested in the welfare benefits of the policy instrument. Following the method proposed by Lucas (1987, 2003), I denote Γ the permanent increase in consumption (in percentages) that would be necessary in the scenario without policy instrument to make the agent indifferent with the scenario where the instrument is available. Formally, Γ solves:

$$V(K)|_{\tau=\tau^*} = V((1+\Gamma)K)|_{\tau=0} \tag{30}$$

As shown in the appendix, taking the expression of the value function we can characterize Γ analytically. If we denote the consumption share of capital on the optimal path with and without policy instrument respectively $\psi_* = \psi|_{\tau=\tau^*}$ and $\psi_0 = \psi|_{\tau=0}$, then we have:

$$\begin{aligned}\Gamma &= \left(\frac{\psi_*}{\psi_0}\right)^{\frac{1}{1-\epsilon}} - 1 \\ &= \left(1 + \frac{(1-\epsilon)(\alpha^{\frac{\alpha}{1-\alpha}} - \alpha^{\frac{1}{1-\alpha}})\left(\frac{\lambda(1-\omega^{1-\gamma})}{A^{\alpha}(1-\gamma)}\right)^{\frac{1}{1-\alpha}}}{\rho\epsilon + (1-\epsilon)\left(A - \frac{\gamma\sigma^2}{2} - \lambda(1+\delta)\frac{(1-\omega^{1-\gamma})}{1-\gamma}\right)}\right)^{\frac{1}{1-\epsilon}} - 1\end{aligned} \tag{31}$$

As $\alpha \in]0;1[$, one can easily show that Γ is increasing with risk (higher λ and σ, lower ω) and risk aversion (γ) and decreasing with the degree of impatience (ρ). The effect of the IES (ϵ) however is ambiguous and is further discussed in the next section where parameters are calibrated.

5 Quantitative assessment

The previous sections have presented the model and shown its numerous implications. The objective of this section is to illustrate these results quantitatively. The calibration of the model should essentially answer three questions. First, if analytic findings have shown that both a positive and a negative effect of disasters on expected growth were *possible*, one can wonder how *plausible* are each of these two scenarios. In particular, we will try to assess to what extent individuals should have a strong distaste for risk and fluctuations to perform enough precautionary savings to cover the expected output losses from disaster strikes. Second, while disasters have an unambiguous negative impact on welfare, it is important to assess how large these effects are. Third, this calibration should evaluate to what extent using the more restrictive log-utility and time-additive power utility functions affect our understanding of the link between disasters, economic growth, optimal policies and welfare.

5.1 Set-up

5.1.1 A country/region extension

The main challenge when calibrating a disaster model is to account for both the low probability but large magnitude of these events on the people impacted, and their rather high frequency but small impact at the aggregate level (see for instance Strobl, 2011). Although powerful to explain the equity premium, the extreme environmental disasters of Barro (2015) — that realize on average once every 100 years and destroy 21% of the capital stock — do not match with observed aggregate damages at a country level. The same can be said of the estimates of future global disasters that Pindyck & Wang (2013) infer from market data. In order to reconcile these low probability and large impact events with aggregate data, I therefore slightly extend the model presented in section 4. I consider a country composed of H distinct regions. Each region has its own capital stock k^h, so that the country's aggregate capital stock is $K = \sum_{h=1}^{H} k^h$. I assume all regions share the same characteristics, i.e. all parameters are identical, but are subject to local shocks following independent Poisson processes dq_t^h that they mitigate with their own instrument τ^h.[2] All communities therefore solve the same problem, and on the optimal path they differ only by their level of capital k^h and by the timing of the shocks they face. The law of aggregate capital accumulation on the optimal path is thus:

$$\begin{aligned}\frac{dK}{K}^* &= \sum_{h=1}^{H} \frac{dk^h}{K}^* = \sum_{h=1}^{H} \frac{(y^h - \tau^h y^h - c^h)^*}{K} dt + \sum_{h=1}^{H} \frac{\sigma_w^{h,*}}{K} dz - \sum_{h=1}^{H} \frac{\sigma_p^{h,*}}{K} dq^h \\ &= [A(1-\tau^*) - \psi]dt + \sigma dz - \sum_{h=1}^{H} \frac{(1-\omega)k^h}{K} dq^h\end{aligned} \tag{32}$$

since $\sigma_w^{h,*} = \sigma k^{h,*}$ and $(y^h - \tau^h y^h - c^h)^* = [A(1-\tau^*) - \psi]k^{h,*}$. As in the previous section, the aggregate capital stock grows deterministically at the rate $A(1-\tau^*) - \psi$, and follows fluctuations of size σ. However, it is now subject to shocks of size $(1-\omega)k^h/K$ with a probability $\mathbb{E}(\sum_{h=1}^{H} dq_t^h)/dt = H\lambda f^*$ per unit of time. If there are many regions, then at the aggregate level the probability of a shock is high, but its average magnitude $(1-\omega)/H$ is low.

5.1.2 Calibration

The model is calibrated so as to represent the United States, disaggregated at the county level ($H = 3,142$). The U.S. is an interesting case study as it is one of the countries most impacted by natural disasters (Shi et al., 2015). In particular, the U.S. is by far the country most prone to material damages from cyclones: between 1990 and 2016 it has been hit by only 4% of storms worldwide, but accounts for 60% of global tropical cyclones damages (Bakkensen & Mendelsohn, 2016).

The baseline values of the parameters used in the calibration are given in Table I. Following Barro (2009) — and consistent with U.S. data — I assume that the marginal return from capital is 6.9%, and the standard deviation of normal shocks 2%. In the main specification I also assume that the coefficient of relative risk aversion is $\gamma = 3$, and the inter-temporal elasticity of substitution is $\epsilon = 1$. The value of these two parameters is discussed below, and the implications of alternative choices examined, in particular for values of ϵ above or below unity. The efficiency of the risk-mitigation technology is taken to be $\alpha = 1/4$ such that cutting by two the risk of a disaster would cost around 6% of GDP. As we know relatively little about this parameter, this is of course subject to debate but it should serve as a starting

[2] In this country/region context, these instruments can be thought of as local measures to reduce the risks of environmental disasters, such as investments to build dikes to prevent floods, stricter norms for more resilient buildings, etc. Considering small areas, we can assume that efforts to tackle the sources of climate change have a negligible impact.

point for our analysis. Finally, as Barro (2009) I set the rate of time preferences in order to match the expected growth rate. I target a rate of 1.75% that will imply a consumption-income ratio of 70%. Thus, the value of ρ should not represent the ethical discount rate discussed in the climate literature, but rather the value that best explains the data. Table VII in the appendix reports the corresponding values for various levels of risk and preferences.

In order to calibrate the risk of environmental disasters, I need to infer the probability (λ) and expected intensity ($1-\omega$) of these events at the county level. Based on observations from 1930 to 2010, Boustan et al. (2017) find that *severe* disasters have occurred on average 0.307 times every decade in each U.S. county.[3] Although some parts of the U.S. are more impacted than others, they stress that disasters are geographically widespread within the country. Based on this evidence, I assume that the *ex ante* probability of an environmental disaster for a given county each year is $\lambda = 3.07\%$. With respect to the magnitude of these disasters, Boustan et al. (2017) show that these events result in a decline by 5.2% of housing prices in the counties impacted. Although this number does not perfectly reflect productive capital destruction, it can serve as a useful proxy to calibrate disaster impacts at the county level. Other recent studies have assessed the impact of environmental disasters. Looking at the long-run impact through a reduction of the growth rate, Hsiang & Jina (2014) found that the probability of a cyclone reducing 7.4% of income was 5.8% in countries prone to these events. In China, Elliott et al. (2015) estimate that an average damaging typhoon destroys 1.9% of property values where it strikes, but they report destruction up to 64% for the most extreme events. As explained in FEMA (2010), disaster-related damages largely depend on building types and may therefore differ from a country to another. In addition, the actual losses critically depend on insurance coverage. In order to investigate a larger spectrum of situations, I therefore calibrate two additional scenarios. From the expected damages proxied from Boustan et al. (2017), I consider situations where the probability of a disaster is lower, but their intensity on the people impacted is larger. I will refer to the main calibration as a "Moderate disasters" scenario ($\lambda_M = 3.07\%$, $1-\omega_M = 5.2\%$), and alternatively consider a second "Large disasters" scenario with $\lambda_L = 1.064\%$ and $1-\omega_L = 15\%$, and a third "Extreme disasters" scenario with $\lambda_E = 0.3991\%$ and $1-\omega_E = 40\%$. All three scenarios therefore display identical expected damages, that are more or less spread over time and between agents. Considering these three scenarios enables me to

[3] These are defined as disasters leading to 25 or more deaths in total. Their dataset includes all types of environmental disasters, and are based on the FEMA roster compeleted with other sources for events that occurred prior 1964.

draw the link between the model and both the rather moderate disasters of the empirical literature, and the more catastrophic events often considered in climate models. From the point of view of empirical research, the later scenarios can also be thought of as more disaggregated cases where we focus on the smaller population of the most impacted people with scarce insurance.

Finally, I set the ratio of non-environmental over environmental disaster probability (δ) to 1 in all three scenarios. This parameter does not bear critical implications here, but this simple benchmark yields a probability of non-environmental disasters ($\delta\lambda = 3.07\%$ in the main scenario) that is consistent with the likelihood that Barro & Ursua (2008) report for such events.

Table I: Parameters used in the calibration (main specification).

Parameter	Notation	Value
Risk aversion coefficient	γ	3
Intertemporal elast. of subst.	ϵ	1
Gross return from capital	A	0.069
Damages from moderate disasters	$1-\omega_M$	5.2%
Damages from large disasters	$1-\omega_L$	15%
Damages from extreme disasters	$1-\omega_E$	40%
Ex ante probability of a moderate env. dis.	λ_M	3.07%
Ex ante probability of a large env. dis.	λ_L	1.064%
Ex ante probability of an extreme env. dis.	λ_E	0.3991%
Ratio non-enviromental / environmental disasters	δ	1
St. dev. of normal shocks per year	σ	2%
Inverse of technology efficiency	α	0.25
Number of regions	H	3,142

Taking the parameters' values in Table I, one can compute the main variables of interest. The results are reported in Table II. For the main specification ("Moderate disasters"), we obtain that about 70% of production should be consumed at each period on the optimal path, and 0.12% spent in risk-mitigation. The effect of such investment is to decrease the probability of an environmental disaster by around a fifth, from 3.07% to 2.51%. Although disasters destroy 5.2% of the capital stock in the counties they hit, on average they represent only 0.0017% of U.S. capital stock and occur in 79 counties each year. The expected yearly aggregate damage — and GDP loss — of environmental disasters is thus 0.13%.

Table II: Variables computed at parameters' baseline value.

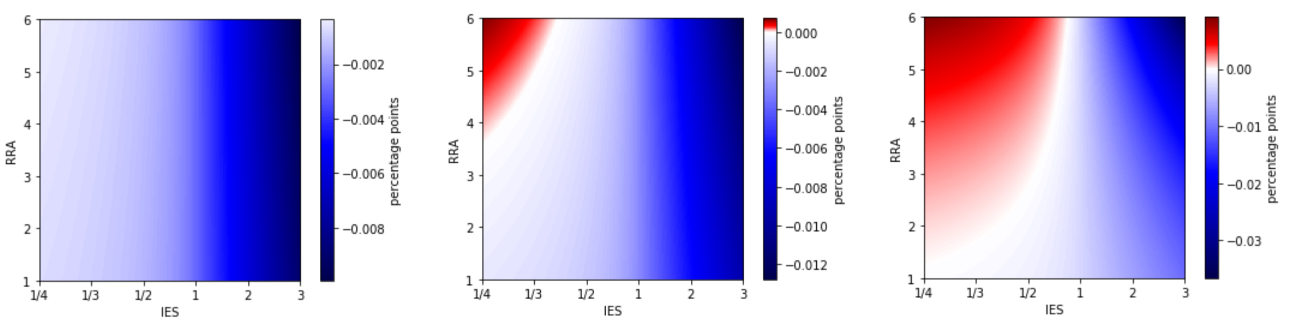
Variable	Notation	Moderate dis.	Large dis.	Extreme dis.
Share of production consumed	ψ/A	70.32%	70.32%	70.25%
Share of production in risk-mitigation	τ^*	0.12%	0.14%	0.30%
Reduction in prob. of an env. disaster	$(\tau^*)^\alpha$	18.4%	19.5%	23.4%
Expected aggregate damages from env. disaster (per year)	$(1-\omega)\lambda[1-(\tau^*)^\alpha]$	0.130%	0.128%	0.122%

5.2 How likely is it that disasters foster economic growth?

The literature has not reached a clear consensus over the *true* value of the RRA (γ) and the IES (ϵ). In an attempt to explain the equity premium puzzle, Mehra & Prescott (1985) argue that a reasonable upper bound for the relative risk aversion coefficient is 10. Barro (2009) shows that within a model displaying rare catastrophic events, a value between 3 and 4 is enough to explain the equity premium, and closer to micro evidences. With respect to the IES, the value is even more debated and there exists contrasted evidences on whether it should be taken as above or below unity. It has been shown by Bansal & Yaron (2004) that in order to explain numerous properties of asset pricing one needs to have simultaneously $\gamma > 1$ and $\epsilon > 1$, which is at odds with expected utility, and in our case suggests that precautionary *consumption* should be favored in front of higher risks on capital. Yet, most studies on micro data argue that a value of ϵ lower than unity better represents people's preferences (see Attanasio & Weber, 2010; Havránek, 2015). The choice of high values for both γ and ϵ is also problematic as it implies an implausibly high timing premium, i.e. individuals' willingness to pay for an earlier resolution of risk becomes too large (see Epstein et al., 2014).

This paper does not intend to settle this debate. The objective is rather to highlight the implications of the values of these parameters when studying rare catastrophic events within an endogenous growth framework. To do so, Figure 1 plots for the three scenarios and for different values of aversion to risk (γ) and inter-temporal elasticity of substitution (ϵ) the effect on growth of introducing disasters to the model. That is, it computes the difference between the expected growth rate of the model as calibrated in Table I, and the one of the same model with $\lambda = 0$ (or $\omega = 1$). On each figure, the red area is associated with a net positive impact of disasters on expected growth, while the blue area signals a negative impact. When disasters are "moderate", it clearly appears that the values of γ and ϵ leading to a positive effect of disasters on growth are far beyond what is commonly admitted as plausible in the literature. For "large" disasters, we see that precautionary savings over-compensate the negative impact of disaster strikes for high values of aversion towards risk and fluctuations. For instance, assuming $\gamma = 4$, one could expect disasters to foster growth for $\epsilon < 0.27$. Although such a value is small compared to standard estimates of the IES, a calibration of a power utility with RRA=1/IES=4 would thus predict a positive impact of "large" disasters on growth. Such a positive effect is obtained for even lower coefficients in the case of "extreme" disasters: assuming again $\gamma = 4$, $\epsilon < 0.64$ is sufficient to get a positive effect of disasters on expected growth. Interestingly, these results are barely sensitive to the calibration of disaster frequency, although the difference in growth rates is exacerbated in both directions for more frequent events. Expected growth being linear in λ, this parameter affects the relative importance of risk for growth, but quantitatively it has no remarkable effect on the link between preferences and expected growth. However, the results critically depend on the value assigned to disaster intensity. Intuitively, this effect of ω is due to the concavity of the value function which exacerbates the response to disaster risks relative to the impact of disaster strikes for high expected damages.[4]

Figure 1: Difference between long-run growth in a disaster vs. disaster free economy.



Moderate dis. Large dis. Extreme dis.

Note: When all parameters are calibrated following Table I except for γ and ϵ, the expected long-run growth rate is higher in the disaster than in the disaster free economy if and only if γ and ϵ lie in the red area.

Thus, while in theory *extremely intense*–low probability events could be associated with higher growth rates through precautionary savings, more frequent and less intense disasters should lead to lower growth in this framework. Based on stylized facts derived from the empirical literature, the combination of

[4]The effects of λ and ω can be most easily understood in the case of exogenous disasters by looking at the comparative statics in equations (14) and (15). In particular, if the derivative of expected growth with respect to ω will always be relatively close to zero because of the term λ factoring the expression, its sign is very sensitive to the value of risk and risk aversion, hence the highly non-linear effect of disaster intensity on precautionary savings.

such extreme risks and high distaste for fluctuations and risk appears unlikely. Although derived in a specific framework, this evidence suggests that precautionary savings may not be sufficient to explain the positive link between disasters and growth sometimes found in cross-sectional analysis (e.g. Skidmore & Toya, 2002). If this empirical relationship is robust — i.e. not driven by omitted variable bias — future theoretical research should focus on identifying other mechanisms to explain it, such as the role human capital, endogenous technical progress (as studied in Akao & Sakamoto, 2018), or a potential substitution towards more productive capital.

5.3 How much do disasters impact welfare?

From equations 28 and 29, we can calculate the marginal rate of substitution between proportionate changes in production and in disaster probability (λ) and intensity (ω). The results are presented in Table III and IV, that report the values obtained for each of the three scenarios, and different levels of risk aversion. In our baseline calibration ($\gamma = 3$) of the "moderate scenario", the coefficient of 2.11 in Table III indicates that to keep welfare constant, an increase by 10% of disaster probability (from 3.07% to 3.377%) would need to be compensated by a permanent increase by 0.65% in production (Y). Considering the "large" disasters and "extreme" disasters scenarios, such 10% increase in disaster probability would need to be compensated by an increase in production by respectively 0.76% and 1.29%. Although expected damages are identical in all three scenarios, the concavity of the utility function implies larger welfare effects for less frequent but more intense events. This difference exacerbates for large values of risk aversion. Thus, while the estimation of the welfare effect of disasters is little sensitive to the choice of the risk aversion parameter for "moderate" events, the calibration of this parameter becomes critical when larger events are considered. For instance, the increase in production necessary to compensate a 10% increase in "extreme" disaster probability is 0.76% assuming log-utility (i.e. $\gamma \to 1$), against 1.29% with a standard calibration of $\gamma = 3$, and 14.69% using an upper bound value of $\gamma = 10$. Interestingly, although the expressions from equations 28 and 29 depend on ϵ, as long as the expected growth rate is fixed by adjusting time impatience (ρ) their value is insensitive to the choice of ϵ. While critical to understand the link between disasters and growth, in this model the IES is therefore irrelevant when it comes to their impact on welfare.

The calibration of τ^* shows how these marginal effects of disasters on welfare translate into the optimal value of the policy instrument. The values for each of the three scenarios and different levels of

Table III: MARGINAL RATE OF SUBSTITUTION BETWEEN PROPORTIONATE CHANGES IN GDP (Y) AND IN DISASTER PROBABILITY (λ).

	$\gamma \to 1$	$\gamma = 3$	$\gamma = 5$	$\gamma = 10$
Moderate disasters	2.00	2.11	2.22	2.55
Large disasters	6.08	7.14	8.46	13.37
Extreme disasters	19.01	32.38	59.48	368.04

Table IV: MARGINAL RATE OF SUBSTITUTION BETWEEN PROPORTIONATE CHANGES IN GDP (Y) AND IN DISASTER INTENSITY (ω).

	$\gamma \to 1$	$\gamma = 3$	$\gamma = 5$	$\gamma = 10$
Moderate disasters	-1.21	-1.35	-1.50	-1.95
Large disasters	-0.47	-0.64	-0.89	-1.96
Extreme disasters	-0.25	-0.67	-1.82	-22.26

risk aversion are reported in Table V, while Table VI reports Lucas' measure Γ (expressed by equation 31) of the total welfare benefits of the policy. Consistent with our previous findings, both τ^* and Γ appear to be larger and more sensitive to the parameterization of risk aversion for disasters of higher magnitude. While a standard calibration $\gamma = 3$ does not make a large difference compared to a logarithmic specification $\gamma \to 1$ for "moderate" disasters, considering "large" and "extreme" events, the benefits of the instrument appear respectively 1.3 and 2.1 times bigger.

Table V: OPTIMAL SHARE OF INCOME SPENT IN POLICY INSTRUMENT (τ^*).

	$\gamma \to 1$	$\gamma = 3$	$\gamma = 5$	$\gamma = 10$
Moderate disasters	0.11%	0.12%	0.12%	0.15%
Large disasters	0.12%	0.14%	0.18%	0.34%
Extreme disasters	0.14%	0.30%	0.70%	8.53%

Table VI: WELFARE BENEFITS OF THE POLICY (Γ).

	$\gamma \to 1$	$\gamma = 3$	$\gamma = 5$	$\gamma = 10$
Moderate disasters	0.46%	0.49%	0.53%	0.64%
Large disasters	0.49%	0.62%	0.78%	1.48%
Extreme disasters	0.62%	1.29%	3.06%	50.36%

In contrast with Lucas (2003) conclusion of low welfare costs from fluctuations, our findings indicate that the benefits from mitigating environmental disasters in the U.S. can be high even ignoring their impact on human lives, although most likely (i.e taking the "moderate disaster" scenario) lower than what Barro (2009) estimates for macroeconomic disasters. These results should raise concerns over the risk of environmental disasters, even more so as Hsiang et al. (2017) predict disaster related damages in the U.S. to be increasing with climate change. Besides the need to mitigate disasters, these results also stress the importance of insurance coverage. Comparing the welfare impact of disasters across scenarios, it clearly appears that spreading the damages would lead to large welfare gains relative to a situation where fewer people are more impacted. As shown by Swiss Re Institute[5], the natural catastrophe protection gap of the U.S. amounted to 45% between 2009-2018, leaving almost half of disaster losses uninsured. As part of an adaptation strategy, the improvement of the insurance coverage could thus be very powerful.

5.4 Does using Epstein-Zin-Weil preferences matters quantitatively?

The previous results show that using the restrictive class of time-additive power utility in dynamic stochastic models of disasters may lead not only to qualitative mis-interpretations, but also to potentially large quantitative errors. As it imposes that RRA=1/IES, and because the associated parameters have empirically different values, this constraint implies two potential problems. On the one hand, if one calibrates a power utility assuming RRA=1/IES is in the range of 3–4 to correctly capture risk aversion, he will overestimate the importance of precautionary savings. As shown above, for relatively large disasters, this could lead to wrongly conclude that disasters foster long-run growth. On the other hand, when taking lower values to better match evidences regarding the IES, it leads to underestimate the effect of disasters on welfare and the optimal effort that should be performed to mitigate them. These results confirm that our analytic evidences matter quantitatively. They also bring support to previous studies that introduced Epstein-Zin-Weil utility in Integrated Assessment Models (IAMs) of the climate literature (see Crost & Traeger, 2014; Jensen & Traeger, 2014; Cai & Lontzek, 2018), and showed numerically that it implied a higher carbon price. Although the use of non-expected utility may require intensive computations in these models, the present results suggest that the choice of the utility function should be taken cautiously. As the risks embedded in these models are usually large, the effect on the model's output may be quite important.

[5] http://files.swissre.com/natcat-protection-gap-map/index.html

6 Conclusion

This paper proposed a stylized model of endogenous growth with endogenous disasters in a framework where individuals exhibit recursive preferences. The model was fully solved analytically, and the numerous mechanisms through which disasters affect growth and welfare were highlighted with an emphasis on how they each depend on preferences for risk on the one side, and inter-temporal fluctuations on the other. The ability to disentangle these two concepts appeared critical as they each play very distinct roles. In a calibration of the model based on empirical evidence about disaster impacts in the U.S., the paper has shown that the use of non-expected utility also matters quantitatively. While a proper calibration of the model leads to rejecting the hypothesis that precautionary savings may overcompensate losses from disaster strikes, a calibration of a more restrictive power utility with high risk aversion and large disasters would induce the opposite conclusion. In addition, disasters are found to have large welfare impacts, but these effects are also sensitive to the calibration of risk aversion, hence the need to use a flexible framework to correctly calibrate this parameter.

This analysis should be taken as a first step towards a better understanding of the effect of preferences on the link between disasters, growth, and welfare. To keep the model tractable and as intuitive as possible, a certain number of potentially relevant mechanisms have been left aside. In particular, the literature has shown that when facing disasters, the possibility to switch from physical to human capital could have important implications (see Ikefuji & Horii, 2012; Bakkensen & Barrage, 2016; Akao & Sakamoto, 2018). Disasters could also positively impact productivity through a "build back better" effect (Hallegate & Dumas, 2009). The model is also silent about the role of trade as an adaptation mechanism. Finally, if our calibration exercise has shown that insurance could play an important role in mitigating the welfare cost of disasters, deeper modeling of the insurance market (as investigated by Ikefuji & Horii, 2012; Müller-Fürstenberger & Schumacher, 2015) could also provide novel insights. All these fascinating elements should be seen as avenues for future research. Given the important welfare implications of disasters, I believe a lot of efforts are needed to improve our understanding of their link with the economy.

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Appendices

A General framework

We assume preferences from consumption can be represented by the following utility function:

$$(1-\gamma)U_t = \left[C_t^{\frac{\epsilon-1}{\epsilon}} dt + e^{-\rho dt}\left((1-\gamma)\mathbb{E}U(t+dt)\right)^{\frac{\epsilon-1}{\epsilon(1-\gamma)}}\right]^{\frac{\epsilon(1-\gamma)}{\epsilon-1}} \tag{33}$$

where ρ is the pure rate of time preferences, γ the coefficient of relative risk aversion, and ϵ the intertemporal elasticity of substitution. The recursive form of the utility yields the following Hamilton Jacobi Bellman (HJB) equation:

$$(1-\gamma)V(K_t) = \max\left[C_t^{\frac{\epsilon-1}{\epsilon}} dt + e^{-\rho dt}\left((1-\gamma)\mathbb{E}V(K_{t+dt})\right)^{\frac{\epsilon-1}{\epsilon(1-\gamma)}}\right]^{\frac{\epsilon(1-\gamma)}{\epsilon-1}} \tag{34}$$

The law of capital accumulation is defined as:

$$dK_t = [Y_t - \sum_{j=1}^{m} \tau_{j,t} Y_t - C_t]dt + \sigma_{w,t} dz - \sum_{i=1}^{n} \sigma_{p,i,t} dq_{i,t} \tag{35}$$

where dz is a Wiener process with scaling term σ_w, and $dq_{i,t}$ a Poisson process with endogenous parameter, i.e. $\mathbb{E}dq_{i,t} = \lambda_i f_i dt$ with λ_i a constant and f_i a function of abatement activities to be defined. Shocks are also supposed to be of endogenous size, and we denote $\tilde{K}_i$ the stock of capital after a shock from the i^{th} Poisson process occurred. From the stochastic law of capital accumulation, one can substitute for the expectation term in equation (34) using the change of variable formula and Îto's lemma, which yields:

$$\mathbb{E}V(K_{t+dt}) = V(K_t) + \mathbb{E}dV(K_t) = V(K_t) + V_k[(1-\sum_{j=1}^{m} \tau_{j,t})Y_t - C_t] + \frac{1}{2}V_{kk}(\sigma_w dz)^2 + \sum_{i=1}^{n} \mathbb{E}\left(V(\tilde{K}_{i,t}) - V(K_t)\right) dq_{i,t}$$

$$= V(K_t) + V_k[(1-\sum_{j=1}^{m} \tau_{j,t})Y_t - C_t]dt + \frac{1}{2}V_{kk}\sigma_w^2 dt + \sum_{i=1}^{n} \lambda_i f_i \left(V(\tilde{K}_{i,t}) - V(K_t)\right) dt$$

Substituting back into the HJB equation (34) gives:

$$(1-\gamma)V(K_t) = \max\left[C_t^{\frac{\epsilon-1}{\epsilon}}dt + e^{-\rho dt}\left((1-\gamma)V(K_t) + (1-\gamma)V_k[(1-\sum_{j=1}^{m}\tau_{j,t})Y - C]dt + (1-\gamma)\frac{1}{2}V_{kk}\sigma_w^2 dt \right.\right.$$
$$\left.\left. + (1-\gamma)\sum_{i=1}^{n}\lambda_i f_i\left(V(\tilde{K_{i,t}}) - V(K_t)\right)dt\right)^{\frac{\epsilon-1}{\epsilon(1-\gamma)}}\right]^{\frac{\epsilon(1-\gamma)}{\epsilon-1}} \tag{36}$$

Then, following the strategy used by Epaulard & Pommeret (2003), we denote :

$$X(K,C,\tau) = V_k[(1-\sum_{j=1}^{m}\tau_{j,t})Y - C] + \frac{1}{2}V_{kk}\sigma_w^2 + \sum_{i=1}^{n}\lambda_i f_i\left(V(\tilde{K_{i,t}}) - V(K_t)\right)$$

where τ is the vector of all τ_j, $j = 1,...,m$. Making use of two approximations when dt is small enough, $e^{-\rho dt} \simeq 1-\rho dt$ and $(1+xdt)^a \simeq 1 + axdt$, we have:

$$(1-\gamma)V(K_t) = \max\left[C_t^{\frac{\epsilon-1}{\epsilon}}dt + (1-\rho dt)\left((1-\gamma)V(K_t)\left[1 + \frac{X(K,C,\tau)dt}{V(K_t)}\right]\right)^{\frac{\epsilon-1}{\epsilon(1-\gamma)}}\right]^{\frac{\epsilon(1-\gamma)}{\epsilon-1}}$$

$$\Leftrightarrow \quad (1-\gamma)V(K_t) = \max\left[C_t^{\frac{\epsilon-1}{\epsilon}}dt + (1-\rho dt)\left((1-\gamma)V(K_t)\right)^{\frac{\epsilon-1}{\epsilon(1-\gamma)}}\left(\left[1 + \frac{\epsilon-1}{\epsilon(1-\gamma)}\frac{X(K,C,\tau)dt}{V(K_t)}\right]\right)\right]^{\frac{\epsilon(1-\gamma)}{\epsilon-1}}$$

$$\Leftrightarrow \quad (1-\gamma)V(K_t) = \max\left[C_t^{\frac{\epsilon-1}{\epsilon}}dt + (1-\rho dt)\left[(1-\gamma)V(K_t)\right]^{\frac{\epsilon-1}{\epsilon(1-\gamma)}}\right.$$
$$\left. + (1-\rho dt)\left[(1-\gamma)V(K_t)\right]^{\frac{\epsilon-1}{\epsilon(1-\gamma)}}\frac{\epsilon-1}{\epsilon(1-\gamma)}\frac{X(K,C,\tau)dt}{V(K_t)}\right]^{\frac{\epsilon(1-\gamma)}{\epsilon-1}}$$

and because $dt^2 = 0$, we can simplify the expression:

$$(1-\gamma)V(K_t) = \max\left[\left[(1-\gamma)V(K_t)\right]^{\frac{\epsilon-1}{\epsilon(1-\gamma)}}\right.$$
$$\left. + \left(C_t^{\frac{\epsilon-1}{\epsilon}} - \rho\left[(1-\gamma)V(K_t)\right]^{\frac{\epsilon-1}{\epsilon(1-\gamma)}} + \frac{\epsilon-1}{\epsilon(1-\gamma)}\left[(1-\gamma)V(K_t)\right]^{\frac{\epsilon-1}{\epsilon(1-\gamma)}}\frac{X(K,C,\tau)}{V(K_t)}\right)dt\right]^{\frac{\epsilon(1-\gamma)}{\epsilon-1}}$$

$$\Leftrightarrow \quad (1-\gamma)V(K_t) = \max(1-\gamma)V(K_t)$$
$$\times\left[1 + \frac{\left(C_t^{\frac{\epsilon-1}{\epsilon}} - \rho\left[(1-\gamma)V(K_t)\right]^{\frac{\epsilon-1}{\epsilon(1-\gamma)}} + \frac{\epsilon-1}{\epsilon(1-\gamma)}\left[(1-\gamma)V(K_t)\right]^{\frac{\epsilon-1}{\epsilon(1-\gamma)}}\frac{X(K,C,\tau)}{V(K_t)}\right)dt}{\left[(1-\gamma)V(K_t)\right]^{\frac{\epsilon-1}{\epsilon(1-\gamma)}}}\right]^{\frac{\epsilon(1-\gamma)}{\epsilon-1}}$$

$$\Leftrightarrow \quad 0 = \max \frac{\epsilon(1-\gamma)}{\epsilon - 1} \frac{\left(C_t^{\frac{\epsilon-1}{\epsilon}} - \rho\left[(1-\gamma)V(K_t)\right]^{\frac{\epsilon-1}{\epsilon(1-\gamma)}} + \frac{\epsilon-1}{\epsilon(1-\gamma)}\left[(1-\gamma)V(K_t)\right]^{\frac{\epsilon-1}{\epsilon(1-\gamma)}} \frac{X(K,C,\tau)}{V(K_t)} \right)}{\left[(1-\gamma)V(K_t)\right]^{\frac{\epsilon-1}{\epsilon(1-\gamma)}}}$$

$$\Leftrightarrow \quad \rho\frac{\epsilon(1-\gamma)}{\epsilon-1} V(K_t) = \max\left[\frac{\epsilon}{\epsilon-1} \frac{C_t^{\frac{\epsilon-1}{\epsilon}}}{\left[(1-\gamma)V(K_t)\right]^{\frac{\epsilon-1}{\epsilon(1-\gamma)}-1}} + X(K,C,\tau)\right] \tag{37}$$

From the previous equation we obtain the following first order conditions with respect to C and τ_j:

$$\frac{C_t^{-\frac{1}{\epsilon}}}{\left[(1-\gamma)V(K_t)\right]^{\frac{\epsilon-1}{\epsilon(1-\gamma)}-1}} + X_C = 0 \tag{38}$$

$$X_{\tau_j} = 0 \qquad \forall j \tag{39}$$

with X_C and X_{τ_j} the derivatives of X with respect to C and τ_j, hence:

$$C_t^{-\frac{1}{\epsilon}} = V_k\left[(1-\gamma)V(K_t)\right]^{\frac{\epsilon-1}{\epsilon(1-\gamma)}-1}$$

and:

$$YV_k = \sum_{i=1}^{n} \lambda_i \left[f_i \frac{\partial V(\tilde{K}_i)}{\partial \tilde{K}_i} \frac{\partial \tilde{K}_i}{\partial \tau_j} + \frac{\partial f_i}{\partial \tau^j}\left(V(\tilde{K}_i) - V(K)\right)\right]$$

B Exogenous disasters

In this section we assume $n = 1$ and $m = 0$, $\tilde{K} = \omega K$ with ω constant, and $f = (1+\delta)$. We also assume $\sigma_w = \sigma K$ and $Y = AK$. The shape of the problem leads to the following guess for the value function:

$$V(K) = \psi^{\frac{1-\gamma}{1-\epsilon}} \frac{K^{1-\gamma}}{1-\gamma}$$

with ψ a constant to be determined. Substituting the guess into the first order condition derived in the previous section gives:

$$C^{-\frac{1}{\epsilon}} = \psi^{\frac{1-\gamma}{1-\epsilon}} K^{-\gamma}(1-\gamma)^{\frac{\epsilon-1}{\epsilon(1-\gamma)}-1} (\psi^{\frac{1-\gamma}{1-\epsilon}})^{\frac{\epsilon-1}{\epsilon(1-\gamma)}-1} (K^{1-\gamma})^{\frac{\epsilon-1}{\epsilon(1-\gamma)}-1} (1-\gamma)^{-\frac{\epsilon-1}{\epsilon(1-\gamma)}+1} = (\psi K)^{-\frac{1}{\epsilon}}$$

$$\Leftrightarrow \quad C^* = \psi K$$

In order to check our guess for the value function is correct, we substitute it into the HJB equation and determine the value of ψ that enables to solve the problem. Recall equation (37):

$$\rho\frac{\epsilon(1-\gamma)}{\epsilon-1}V(K_t) = \max\left[\frac{\epsilon}{\epsilon-1}\frac{C_t^{\frac{\epsilon-1}{\epsilon}}}{[(1-\gamma)V(K_t)]^{\frac{\epsilon-1}{\epsilon(1-\gamma)}-1}} + X(K,C)\right]$$

with $X(K,C) = V_k[AK-C] + \frac{1}{2}V_{kk}\sigma_w^2 + \lambda(1+\delta)\left(V(\tilde{K}) - V(K)\right)$ and $V(K) = \psi^{\frac{1-\gamma}{1-\epsilon}}\frac{K^{1-\gamma}}{1-\gamma}$, so that:

$$X(K,C) = \psi^{\frac{1-\gamma}{1-\epsilon}}K^{-\gamma}[AK-\psi K] - \frac{\gamma\sigma^2}{2}\psi^{\frac{1-\gamma}{1-\epsilon}}\frac{K^{1-\gamma}}{1-\gamma} - \lambda(1+\delta)(1-\omega^{1-\gamma})\psi^{\frac{1-\gamma}{1-\epsilon}}\frac{K^{1-\gamma}}{1-\gamma}$$

$$= \psi^{\frac{1-\gamma}{1-\epsilon}}K^{1-\gamma}\left[A - \psi - \frac{\gamma\sigma^2}{2} - \lambda(1+\delta)\frac{(1-\omega^{1-\gamma})}{1-\gamma}\right]$$

and:

$$\frac{C_t^{\frac{\epsilon-1}{\epsilon}}}{[(1-\gamma)V(K_t)]^{\frac{\epsilon-1}{\epsilon(1-\gamma)}-1}} = \frac{(\psi K)^{\frac{\epsilon-1}{\epsilon}}}{\left[\psi^{\frac{1-\gamma}{1-\epsilon}}K^{1-\gamma}\right]^{\frac{\epsilon-1}{\epsilon(1-\gamma)}-1}} = \psi\psi^{\frac{1-\gamma}{1-\epsilon}}K^{1-\gamma}$$

Hence, going back to the HJB:

$$\rho\frac{\epsilon(1-\gamma)}{\epsilon-1}\psi^{\frac{1-\gamma}{1-\epsilon}}\frac{K^{1-\gamma}}{1-\gamma} = \max\left[\frac{\epsilon}{\epsilon-1}\psi\psi^{\frac{1-\gamma}{1-\epsilon}}K^{1-\gamma} + \psi^{\frac{1-\gamma}{1-\epsilon}}K^{1-\gamma}\left[A-\psi-\frac{\gamma\sigma^2}{2} - \lambda(1+\delta)\frac{(1-\omega^{1-\gamma})}{1-\gamma}\right]\right]$$

$$\Leftrightarrow \quad \rho\epsilon + (1-\epsilon)A - (1-\epsilon)\frac{\gamma\sigma^2}{2} - (1-\epsilon)\lambda(1+\delta)\frac{(1-\omega^{1-\gamma})}{1-\gamma} = \epsilon\psi + (1-\epsilon)\psi = \psi$$

So the only remaining unknown, that is the consumption share of capital on the optimal path, is:

$$\psi = \rho\epsilon + (1-\epsilon)\left(A - \frac{\gamma\sigma^2}{2} - \lambda(1+\delta)\frac{(1-\omega^{1-\gamma})}{1-\gamma}\right) \tag{40}$$

One can then use the law of capital accumulation defined by equation (35) to compute both the optimal saving rate $s^* = S^*/Y$ and the stochastic growth rate of the economy:

$$s^* = \frac{Y-C^*}{Y} = 1 - \frac{\psi}{A} = \frac{1}{A}\left[\epsilon(A-\rho) + (1-\epsilon)\left(\frac{\gamma\sigma^2}{2} + \lambda(1+\delta)\frac{(1-\omega^{1-\gamma})}{1-\gamma}\right)\right]$$

and:

$$\frac{dK^*}{K} = \frac{dC^*}{C} = (A-\psi)dt + \sigma dz - (1-\omega)dq_t$$

$$= \left[\epsilon(A-\rho) + (1-\epsilon)\frac{\gamma\sigma^2}{2} + \frac{1-\epsilon}{1-\gamma}\lambda(1+\delta)(1-\omega^{1-\gamma})\right]dt + \sigma dz - (1-\omega)dq_t$$

Finally, using the fact that $\mathbb{E}(dz) = 0$ and $\mathbb{E}(dq_t) = \lambda(1+\delta)dt$, one can easily recover the expected growth rate and the associated comparative statics with respect to risk and risk aversion. The sign of these expression can easily be determined except for the effect of risk aversion. Indeed, the overall effect of risk aversion on expected growth $g^* = \mathbb{E}\left(dC^*/C\right)$ may be positive or negative depending on the value of the IES:

$$\frac{\partial g^*}{\partial \gamma} = (1-\epsilon)\left(\frac{1}{2} + \lambda(1+\delta)\frac{ln(\omega)\omega^{1-\gamma}(1-\gamma) + (1-\omega^{1-\gamma})}{(1-\gamma)^2}\right)dt \quad \begin{cases} > 0, & \text{if } \epsilon < 1. \\ \leq 0, & \text{otherwise.} \end{cases}$$

Proof #1 : To show this, let's define $g(\gamma) = ln(\omega)\omega^{1-\gamma}(1-\gamma) + (1-\omega^{1-\gamma})$. First, notice that $g(1) = 0$. Then, if we take the derivative of this function, we have:

$$g'(\gamma) = ln(\omega)\left[-ln(\omega)\omega^{1-\gamma}(1-\gamma) - \omega^{1-\gamma}\right] + ln(\omega)\omega^{1-\gamma}$$

$$= -[ln(\omega)]^2\omega^{1-\gamma}(1-\gamma)$$

Thus, for $\omega > 0$, $g'(\gamma) < 0$ for $\gamma < 1$ and $g'(\gamma) > 0$ for $\gamma > 1$, hence $g(1)$ is a global minimum and $g(\gamma) > 0$ for $\omega > 0$ and $\gamma \neq 1$. $\square$

C Catastrophes of endogenous probability

In this section we turn to disasters of endogenous probability. We keep the assumption that ω is fixed, but we now take $m = 1$ (i.e. there exist one risk-mitigation instrument) and $f = 1 + \delta - \tau^\alpha$ with $0 < \alpha < 1$. Production still comes from an AK technology and the Wiener process is still scaled by a standard deviation $\sigma_w = \sigma K$. The general form of the problem being the same as in the previous section, we again make the following guess:

$$V(K) = \psi^{\frac{1-\gamma}{1-\epsilon}}\frac{K^{1-\gamma}}{1-\gamma}$$

Substituting the guess into the two first order conditions, and applying our new specification, we obtain:

$$C^* = \psi K$$

and:

$$AKV_k = \lambda\alpha\tau^{\alpha-1}V(K)(1-\omega^{1-\gamma})$$

$$\Leftrightarrow \quad \tau^* = \left(\frac{(1-\omega^{1-\gamma})\lambda\alpha}{A(1-\gamma)}\right)^{\frac{1}{1-\alpha}}$$

It is straightforward to show that τ^* is increasing with λ and γ (see proof #1 above) and decreasing with ω. The effect of α is less obvious, but one can show that τ^* is an increasing function of α if and only if α is below some threshold value $\bar{\alpha}$, and decreasing otherwise.

Proof #2 : Differentiating τ^* with respect to α we get:

$$\frac{\partial\tau^*}{\partial\alpha} = \left(\frac{(1-\omega^{1-\gamma})\lambda\alpha}{A(1-\gamma)}\right)^{\frac{1}{1-\alpha}} \frac{1}{(1-\alpha)^2}\left[ln\left(\frac{(1-\omega^{1-\gamma})\lambda}{A(1-\gamma)}\right) + \frac{1-\alpha}{\alpha}\right]$$

we can see that this derivative is negative if and only if $\frac{1-\alpha}{\alpha} < -ln\left(\frac{(1-\omega^{1-\gamma})\lambda}{A(1-\gamma)}\right)$, the right hand side being a positive constant since for credible parameters values the term contained in the log will be below 1. Then, as $0 < \alpha < 1$ it is obvious that for α close to 0 the derivative will be negative, while for α close to 1 it will be positive. Hence, we have a threshold $\bar{\alpha}$ such that:

$$\frac{\partial\tau^*}{\partial\alpha}\begin{cases} > 0 \text{ for } \alpha < \bar{\alpha} \\ < 0 \text{ otherwise} \end{cases}$$

□

We can then solve for ψ. The problem is the same as in the case of exogenous shocks except that now:

$$X(K,C,\tau) = \psi^{\frac{1-\gamma}{1-\epsilon}}K^{1-\gamma}\left[(1-\tau)A - \psi - \frac{\gamma\sigma^2}{2} - \lambda(1+\delta-\tau^\alpha)\frac{(1-\omega^{1-\gamma})}{1-\gamma}\right]$$

Hence, going back to the HJB:

$$\rho\frac{\epsilon(1-\gamma)}{\epsilon-1}\psi^{\frac{1-\gamma}{1-\epsilon}}\frac{K^{1-\gamma}}{1-\gamma} = \frac{\epsilon}{\epsilon-1}\psi\psi^{\frac{1-\gamma}{1-\epsilon}}K^{1-\gamma} + \psi^{\frac{1-\gamma}{1-\epsilon}}K^{1-\gamma}\left[(1-\tau^*)A - \psi - \frac{\gamma\sigma}{2} - \lambda f^*\frac{(1-\omega^{1-\gamma})}{1-\gamma}\right]$$

$$\Leftrightarrow \quad \psi = \rho\epsilon + (1-\epsilon)\left((1-\tau^*)A - \frac{\gamma\sigma^2}{2} - \lambda(1+\delta-\tau^{*\alpha})\frac{(1-\omega^{1-\gamma})}{1-\gamma}\right)$$

and finally, substituting for τ^* we get:

$$\psi = \rho\epsilon + (1-\epsilon)\left[A - \frac{\gamma\sigma^2}{2} - \lambda(1+\delta)\frac{(1-\omega^{1-\gamma})}{1-\gamma} + (\alpha^{\frac{\alpha}{1-\alpha}} - \alpha^{\frac{1}{1-\alpha}})\left(\frac{\lambda(1-\omega^{1-\gamma})}{A^\alpha(1-\gamma)}\right)^{\frac{1}{1-\alpha}}\right]$$

Lastly, we can compute the optimal saving rate and optimal growth rate of this economy starting from the stochastic law of capital accumulation defined by equation (35):

$$s^* = \frac{Y(1-\tau^*) - C^*}{Y} = 1 - \tau^* - \frac{\psi}{A} = 1 - \tau^* - \frac{1}{A}\left[\rho\epsilon + (1-\epsilon)\left((1-\tau^*)A - \frac{\gamma\sigma^2}{2} - \lambda f^* \frac{(1-\omega^{1-\gamma})}{1-\gamma}\right)\right]$$

$$\frac{dK^*}{K} = \frac{dC^*}{C} = [(1-\tau^*)A - \psi]dt + \sigma dz - (1-\omega)dq_t$$

and so the expected growth rate is:

$$g^* = \mathbb{E}\left(\frac{dC^*}{C}\right) = [(1-\tau^*)A - \psi - \lambda f^*(1-\omega)]dt \tag{41}$$

We can then compute comparative statics to analyze the incidence of disasters. Differentiating with respect to λ yields:

$$\frac{1}{dt}\frac{\partial g^*}{\partial \lambda} = -A\frac{\partial \tau^*}{\partial \lambda} - \frac{\partial \psi}{\partial \lambda} - f^*(1-\omega) - \lambda(1-\omega)\frac{\partial f^*}{\partial \lambda} \tag{42}$$

with:

$$-A\frac{\partial \tau^*}{\partial \lambda} = -A\frac{\lambda^{\frac{\alpha}{1-\alpha}}}{1-\alpha}\left(\frac{(1-\omega^{1-\gamma})\alpha}{A(1-\gamma)}\right)^{\frac{1}{1-\alpha}} < 0$$

$$-\frac{\partial \psi}{\partial \lambda} = (1-\epsilon)\left[(1+\delta)\frac{(1-\omega^{1-\gamma})}{1-\gamma} - \lambda^{\frac{\alpha}{1-\alpha}}\frac{(\alpha^{\frac{\alpha}{1-\alpha}} - \alpha^{\frac{1}{1-\alpha}})}{1-\alpha}\left(\frac{1-\omega^{1-\gamma}}{A^\alpha(1-\gamma)}\right)^{\frac{1}{1-\alpha}}\right] \quad \begin{cases} > 0, & \text{if } \epsilon < 1. \\ \leq 0, & \text{otherwise.} \end{cases}$$

$$-f^*(1-\omega) = -(1-\omega)\left[1 + \delta - \left(\frac{(1-\omega^{1-\gamma})\lambda\alpha}{A(1-\gamma)}\right)^{\frac{\alpha}{1-\alpha}}\right] < 0$$

$$-\lambda(1-\omega)\frac{\partial f^*}{\partial \lambda} = \frac{(1-\omega)\alpha}{1-\alpha}\left(\frac{(1-\omega^{1-\gamma})\lambda\alpha}{A(1-\gamma)}\right)^{\frac{\alpha}{1-\alpha}} > 0$$

and similarly with respect to ω:

$$\frac{1}{dt}\frac{\partial g^*}{\partial \omega} = -A\frac{\partial \tau^*}{\partial \omega} - \frac{\partial \psi}{\partial \omega} + \lambda f^* - \lambda(1-\omega)\frac{\partial f^*}{\partial \omega} \tag{43}$$

with:

$$-\frac{\partial \tau^*}{\partial \omega}A = A\left(\frac{\lambda\alpha}{A}\right)^{\frac{1}{1-\alpha}}\frac{\omega^{-\gamma}}{1-\alpha}\left(\frac{1-\omega^{1-\gamma}}{1-\gamma}\right)^{\frac{\alpha}{1-\alpha}} > 0$$

$$-\frac{\partial \psi}{\partial \omega} = -(1-\epsilon)\omega^{-\gamma}\left[\lambda(1+\delta) - \lambda^{\frac{1}{1-\alpha}}\frac{(\alpha^{\frac{\alpha}{1-\alpha}} - \alpha^{\frac{1}{1-\alpha}})}{1-\alpha}\left(\frac{1-\omega^{1-\gamma}}{A(1-\gamma)}\right)^{\frac{\alpha}{1-\alpha}}\right] \quad \begin{cases} < 0, & \text{if } \epsilon < 1. \\ \geq 0, & \text{otherwise.} \end{cases}$$

$$\lambda f^* = -\lambda(1+\delta) + \lambda\left(\frac{(1-\omega^{1-\gamma})\lambda\alpha}{A(1-\gamma)}\right)^{\frac{\alpha}{1-\alpha}} > 0$$

$$-\lambda(1-\omega)\frac{\partial f^*}{\partial \omega} = -\frac{1-\omega}{1-\alpha}\left(\frac{(1-\omega^{1-\gamma})\lambda\alpha}{A(1-\gamma)}\right)^{\frac{\alpha}{1-\alpha}-1}\frac{\lambda^2\alpha^2}{A}\omega^{-\gamma} < 0$$

Finally with respect to welfare, one can start from the expression of the value function: $V(K) = \psi^{\frac{1-\gamma}{1-\epsilon}}\frac{K^{1-\gamma}}{1-\gamma}$. Then differentiating with respect to λ, ω and Y, one obtains:

$$-\frac{\partial V(K)}{\partial \lambda}\frac{\partial Y}{\partial V}\frac{1}{Y} = -\frac{\partial \psi}{\partial \lambda}\frac{\psi^{\frac{1-\gamma}{1-\epsilon}-1}}{1-\epsilon}K^{1-\gamma}\frac{1}{\psi^{\frac{1-\gamma}{1-\epsilon}}}\frac{A^{1-\gamma}}{Y^{1-\gamma}} = -\frac{\partial \psi}{\partial \lambda}\frac{1}{(1-\epsilon)\psi}$$

$$= \frac{1}{\psi}\left[(1+\delta)\frac{(1-\omega^{1-\gamma})}{1-\gamma} - \lambda^{\frac{\alpha}{1-\alpha}}\frac{(\alpha^{\frac{\alpha}{1-\alpha}} - \alpha^{\frac{1}{1-\alpha}})}{1-\alpha}\left(\frac{1-\omega^{1-\gamma}}{A^{\alpha}(1-\gamma)}\right)^{\frac{1}{1-\alpha}}\right]$$

and:

$$-\frac{\partial V(K)}{\partial \omega}\frac{\partial Y}{\partial V}\frac{1}{Y} = -\frac{\partial \psi}{\partial \omega}\frac{\psi^{\frac{1-\gamma}{1-\epsilon}-1}}{1-\epsilon}K^{1-\gamma}\frac{1}{\psi^{\frac{1-\gamma}{1-\epsilon}}}\frac{A^{1-\gamma}}{Y^{1-\gamma}} = -\frac{\partial \psi}{\partial \omega}\frac{1}{(1-\epsilon)\psi}$$

$$= -\frac{\omega^{-\gamma}}{\psi}\left[\lambda(1+\delta) - \lambda^{\frac{1}{1-\alpha}}\frac{(\alpha^{\frac{\alpha}{1-\alpha}} - \alpha^{\frac{1}{1-\alpha}})}{1-\alpha}\left(\frac{1-\omega^{1-\gamma}}{A(1-\gamma)}\right)^{\frac{\alpha}{1-\alpha}}\right]$$

In order to obtain Lucas' measure (Lucas, 1987, 2003) of the welfare benefits from the policy instrument, we again start from the expression of the value function:

$$V(K)|_{\tau=\tau^*} = V((1+\Gamma)K)|_{\tau=0}$$

$$\Leftrightarrow \qquad \psi_*^{\frac{1-\gamma}{1-\epsilon}}\frac{K^{1-\gamma}}{1-\gamma} = \psi_0^{\frac{1-\gamma}{1-\epsilon}}\frac{[(1+\Gamma)K]^{1-\gamma}}{1-\gamma} \qquad \Leftrightarrow \qquad \Gamma = \left(\frac{\psi_*}{\psi_0}\right)^{\frac{1}{1-\epsilon}} - 1$$

and since:

$$\psi_* = \psi_0 + (1-\epsilon)(\alpha^{\frac{\alpha}{1-\alpha}} - \alpha^{\frac{1}{1-\alpha}}) \left(\frac{\lambda(1-\omega^{1-\gamma})}{A^\alpha(1-\gamma)} \right)^{\frac{1}{1-\alpha}}$$

we have:

$$\Gamma = \left(1 + \frac{(1-\epsilon)(\alpha^{\frac{\alpha}{1-\alpha}} - \alpha^{\frac{1}{1-\alpha}}) \left(\frac{\lambda(1-\omega^{1-\gamma})}{A^\alpha(1-\gamma)} \right)^{\frac{1}{1-\alpha}}}{\rho\epsilon + (1-\epsilon)\left(A - \frac{\gamma\sigma^2}{2} - \lambda(1+\delta)\frac{(1-\omega^{1-\gamma})}{1-\gamma} \right)} \right)^{\frac{1}{1-\epsilon}} - 1 \tag{44}$$

D With multiple catastrophes of endogenous probability and endogenous magnitude

We now turn to the case where the capital stock is subject to shocks coming from two independent Poisson processes (i.e. $n = 2$) with different frequencies and intensities. As in section 4, the probability of a shock of type 1 is assumed endogenous to risk-mitigation activities τ_1, and $\mathbb{E}dq_t^1 = \lambda_1 f_1 dt$ with $f_1 = 1 + \delta - \tau_1^{\alpha_1}$. Its intensity is again supposed to be a fixed proportion of the capital stock and $\tilde{K_1} = \omega_1 K_1$. However, we now have an additional process whose probability will be assumed exogenous and simply equal to $\mathbb{E}dq_t^2 = \lambda_2 dt$, but whose intensity will be endogenized. The specification of this second process roughly follows the one proposed by Bretschger & Vinogradova (2017). For simplicity, we abstract from the modelling of pollution as can be found in their paper, and simply assume shocks depend on some adaptation efforts τ_2 such that $\tilde{K_2} = K - (\nu - \alpha_2\tau_2)K$. We consider τ_2 as the share of production spent in adaptation policies as it enables to reduce the negative impact of disasters but does not reduce their likelihood. The share of capital that remains after a shock is denoted $\omega_2(\tau_2) = 1 - \nu + \alpha_2\tau_2$, and ν is therefore the share of capital destroyed by disasters absent any adaptation activity. For simplicity we consider the case without Brownian motion so that $\sigma_w = 0$. As in the previous section, production is derived from an AK technology. Making a similar guess as before, we have:

$$C^* = \psi K \tag{45}$$

$$
\begin{aligned}
AKV_k &= \lambda_1 \alpha_1 \tau_1^{\alpha_1 - 1} V(K)(1-\omega_1^{1-\gamma}) \\
\Leftrightarrow \quad \tau_1^* &= \left(\frac{(1-\omega_1^{1-\gamma})\lambda_1 \alpha_1}{A(1-\gamma)} \right)^{\frac{1}{1-\alpha_1}}
\end{aligned} \tag{46}
$$

and:

$$
\begin{aligned}
AKV_k &= \lambda_2 \frac{\partial V(\tilde{K}_2)}{\partial \tilde{K}_2} \frac{\partial \tilde{K}_2}{\partial \tau_2} = \lambda_2 \omega_2^{-\gamma} V_k \alpha_2 K \\
\Leftrightarrow \quad \omega_2^* &= \omega_2(\tau_2^*) = \left(\frac{\lambda_2 \alpha_2}{A} \right)^{\frac{1}{\gamma}}
\end{aligned} \tag{47}
$$

hence:

$$
\tau_2^* = \frac{\omega_2^* - (1-\nu)}{\alpha_2} \tag{48}
$$

The expression of τ_1^* remains the same as in section 4. Interestingly, the adaptation policy τ_2^* solely depends on the efficiency of the technology α_2, and on the difference between the share of capital remaining after catastrophes at equilibrium, ω_2^*, relative to the case absent adaptation policies, $1-\nu$. The share of capital preserved at equilibrium depends positively on the probability of an adverse event λ_2, on the efficiency of adaptation technology α_2, and negatively on the interest rate A. Given that $0 < (\lambda_2\alpha_2)/A < 1$, risk aversion γ also plays positively on ω_2^*. Thus, as for the first instrument τ_1^*, risk and risk aversion positively affect the optimal instrument τ_2^*, but the efficiency of the instrument α_2 has an ambiguous effect.

Given the independence of the two catastrophes and of the two instruments, the share of output that should optimally be spent to mitigate each catastrophe is not affected by the existence of the other. Contrary to Martin & Pindyck (2015) who investigate the binary decision to undertake a project to avert or not a catastrophe when facing multiple types of disasters, standard cost-benefit analysis holds in this framework. For each catastrophe, the marginal cost of mitigation efforts should equate the marginal benefits of reducing this specific catastrophe. However, because each catastrophe impacts the trajectory of output, the amounts of resources spent in each instrument $\tau_1^* Y_t$ and $\tau_2^* Y_t$ depend on the *existence* and *realization* of other catastrophes as well. The full trajectory of output Y_t can be determined applying similar methods than the ones used in the previous specifications. With:

$$X(K,C,\tau) = \psi^{\frac{1-\gamma}{1-\epsilon}} K^{1-\gamma} \left[(1-\tau_1-\tau_2)A - \psi - \lambda_1(1+\delta-\tau_1^{\alpha_1}) \frac{(1-\omega_1^{1-\gamma})}{1-\gamma} - \lambda_2 \frac{(1-(\omega_2^*)^{1-\gamma})}{1-\gamma} \right]$$

we find:

$$\psi = \rho\epsilon + (1-\epsilon)\left((1-\tau_1^*-\tau_2^*)A - \lambda(1+\delta-(\tau_1^*)^{\alpha}) \frac{(1-\omega^{1-\gamma})}{1-\gamma} - \lambda_2 \frac{(1-(\omega_2^*)^{1-\gamma})}{1-\gamma} \right)$$

Once ψ is obtained, one can easily plug this result into the stochastic law of motion of capital and compute the stochastic and expected growth rate of this economy. The results provide similar intuitions to the ones discussed in section 4.

E Calibration

Table VII: Calibration of time impatience (ρ) to match a 1.75% expected growth rate (g^*).

		Moderate dis.	**Large dis.**	**Extreme dis.**
ϵ	γ			
1/3	$\to 1$	0.014	0.014	0.016
1/3	3	0.015	0.016	0.022
1/3	5	0.016	0.019	0.033
1/3	10	0.019	0.026	0.135
$\to 1$	$\to 1$	0.049	0.049	0.049
$\to 1$	3	0.049	0.049	0.048
$\to 1$	5	0.049	0.049	0.048
$\to 1$	10	0.049	0.048	0.043
1.5	$\to 1$	0.054	0.054	0.054
1.5	3	0.054	0.054	0.053
1.5	5	0.054	0.054	0.051
1.5	10	0.053	0.052	0.028