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Mortality hazard rates and life expectancy

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Summary We consider the relation between mortality hazards and life expectancy for men and women in the Netherlands and in England. Halving the lifetime mortality hazards increases life expectancy at birth by only 9%.

1. Introduction

Epidemiologists express the effect of covariates on individual mortality in hazard ratios, actuaries and demographers in differences in life expectancies. The former are appropriate when the covariates apply to a limited age interval, the latter when we are dealing with lifetime characteristics. It is clear that the two are related: reduced hazards mean increasing life expectancies, increased mortality rates a shorter life. The question is what this relation is.

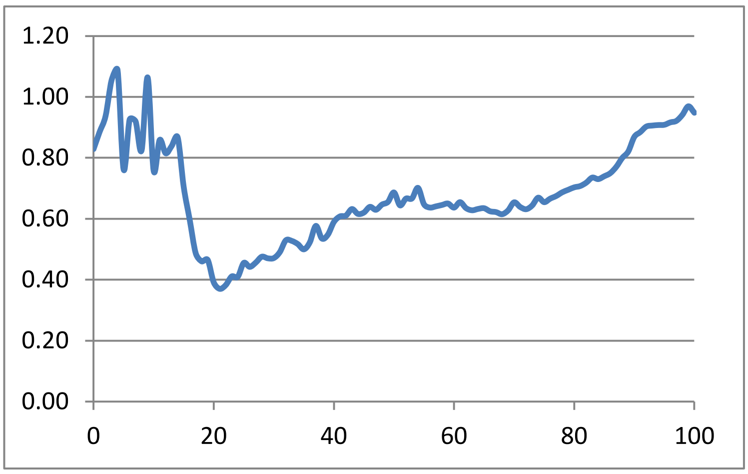
A few life distributions, like the exponential and the Weibull, permit an analytical expression for the mean – i.e. the life expectancy – in terms of the hazard, but these distributions do not apply to the entire human life span. Apart from some experiments with a Makeham-Gompertz distribution we proceed by numerical assessment, calculating the impact of variations in mortality rates from actuarial mortality tables for the Netherlands and for England [1], [2]. These mortality tables record q(t), the probability that a person who attains the age of t years dies before the next birthday. The density of life durations f(t) and the mean life expectancy are easily obtained. One may then vary all baseline q(t) in the same proportion and recalculate the life expectancy. Details of these calculations are given in the appendix.

For the latter part of the life distribution we shall also adopt a Makeham-Gompertz distribution and proceed by direct calculation of the life expectancies using numerical integration. Details are again given in the appendix.

2. Permanent changes in mortality rates

We first consider the effect of lifetime proportional variation of the mortality rate, in line with the proportional hazards model. The first example that comes to mind is the difference in gender: generally, women live considerably longer than men and they must have lower mortality rates, at least over a substantial part of their lifetime. But female and male hazards are not in constant proportion: the ratio of female to male mortality (though predominantly much smaller than 1) varies considerably with age.

Figure 1. Ratio of female to male mortality rates by age (England 2002)



Another individual characteristic that may reduce mortality rates over the entire lifetime is superior intelligence. Like the traditional indicators of Socio-Economic Status – education, occupation and income – intelligence has been shown in many independent studies to reduce individual mortality, with a standardized hazard ratio of about 0,8 – see Calvin et al [3]. In many studies intelligence is recorded at ages 10 or 12, and this is found to affect adult mortality in advanced age, up to seventy years and beyond. We may therefore assume that intelligence scores (or at least intelligence *differentials*) are a permanent individual characteristic that persists over the entire lifetime. This has been confirmed by Deary et al [4]; for a different view see Ramsden et al [5].

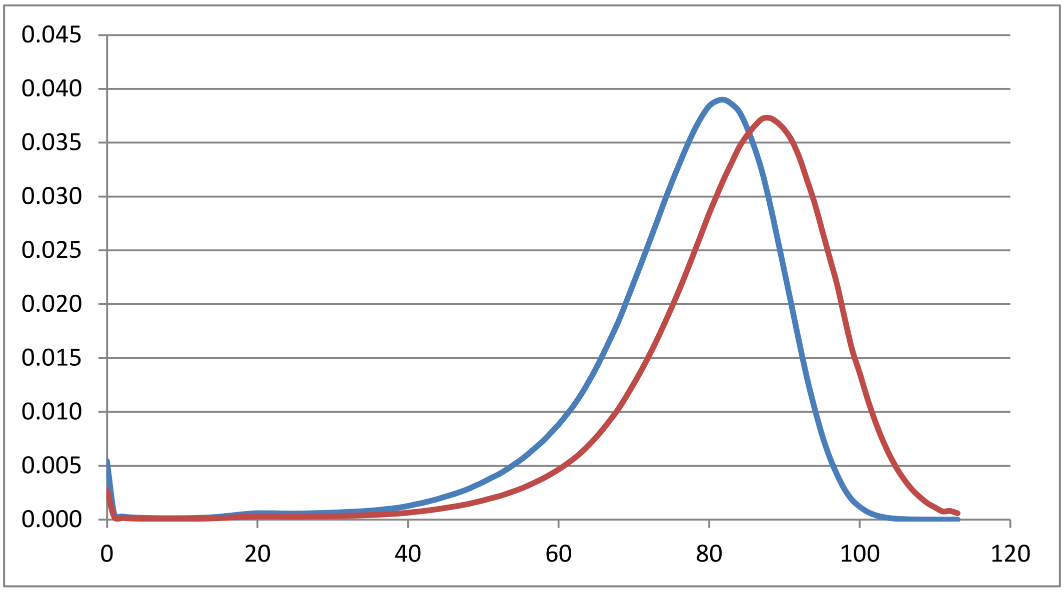
A standardized hazard ratio of 0,8 means that an intelligence differential of one standard deviation reduces mortality hazards by 20%. Most intelligence test scores are calibrated at mean 1 and standard deviation 0,15; assuming a Normal distribution, an individual intelligence of one standard deviation above the mean then means a ranking at the 84^{th} percentile of the intelligence distribution. If linear extrapolation is warranted (a moot point), a position at the upper 1% of the distribution would correspond to 2,08 standard deviations, and imply a mortality hazard of 42% below the average.

Such reduced mortality hazards must mean a longer expected life. The effect of increasing or reducing q(t) in actuarial tables over the entire lifespan by various multiplicative factors or *ratios* is shown in Table 1 and illustrated in Figure 2. This effect is smaller than one would expect: halving the lifetime mortality rates increases the life expectancy by only 9%, doubling mortality rates reduces life expectancy by a similar percentage.

Table 1. Life expectancy (in years) for various lifetime multipliers of mortality hazards (calculated from actuarial mortality tables)

Ratio	Netherlands, M	Netherlands, F	England, M	England, F
0,5	83,57	87,81	84,84	88,52
0,8	78,59	83,05	80,78	84,57
0,9	77,35	82,00	79,52	83,41
1	**76,25**	**80,96**	**78,36**	**82,36**
1,1	75,25	80,02	77,31	81,41
1,2	74,33	79,15	76,34	80,54
1,5	71,98	76,89	73,83	78,26
2,0	68,89	73,90	70,55	75,26

Figure 2. Effect on life distribution of halving mortality rates (Netherlands, men; calculated from actuarial mortality tables)



Figures 3 and 4 show the relation between these life expectancies and the logarithm of the overall mortality multiplier for the Dutch and English data, women and men. The relation is almost linear – more so in the Netherlands than in England - , with a slope of -10 (-10,28 in the Netherlands, -10,03 in England) of life expectancy (in years) in respect of the natural log(mortality rate multiplier).

Figure 3. Life expectancy as a function of log mortality rate multiplier. (Netherlands, women and men)

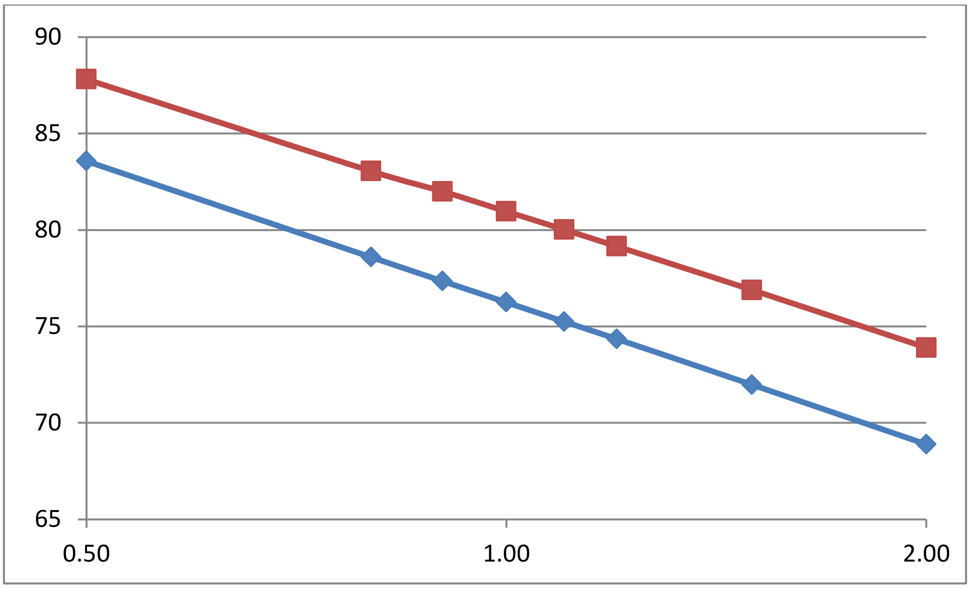
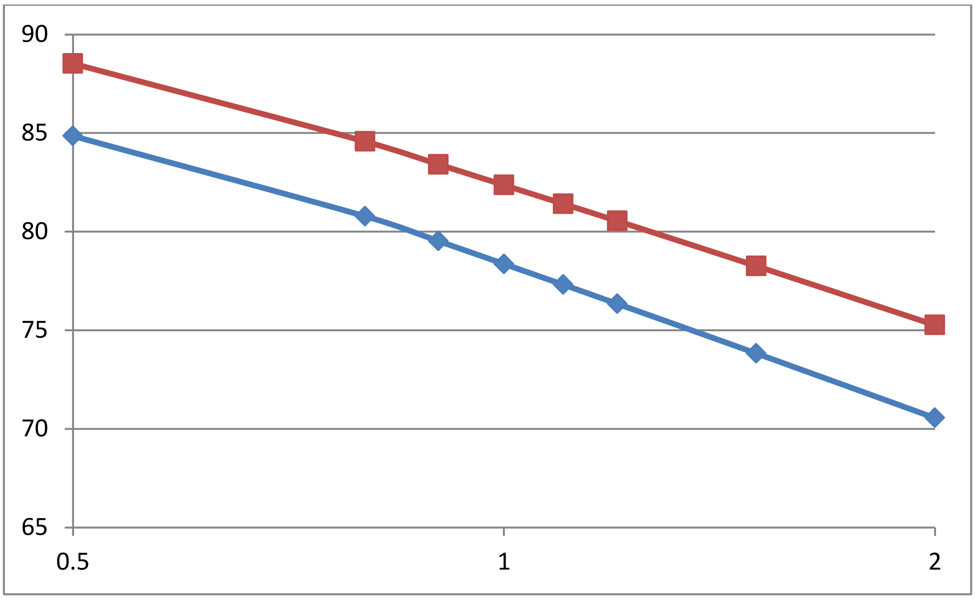


Figure 4. Life expectancy as a function of log mortality rate multiplier.
(England, women and men)



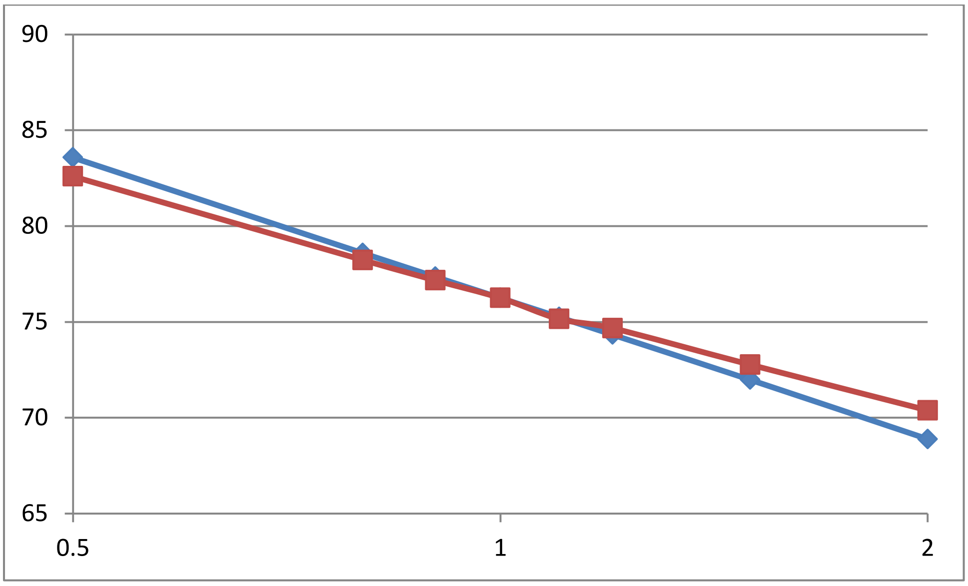
2. Temporary changes in mortality rates

We may also consider the effect of temporary variation of the mortality rate, induced by temporary changes in covariates, such as the late advent of affluence or of poverty. In a simplified scheme we assume that mortality rates are unchanged for the first 45 years of life and thereafter permanently changed by the same ratios as before. Table 3 reports the results for the Netherlands, and Figure 5 shows that the differences with the earlier outcomes for the full lifetime are quite small – the slope of life expectancy to ln(ratio) is -8,80 against -10,28 for lifetime changes. This is no surprise as the major part of all deaths occur after middle age.

Table 3. Life expectancy (in years) for various multipliers of mortality hazards beyond 45 years of age

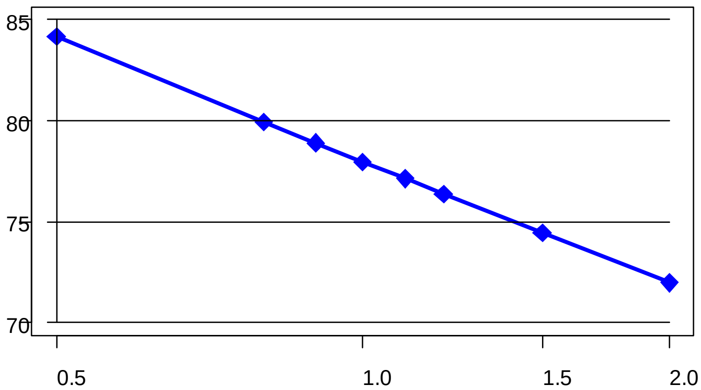
multiplicative factor	Netherlands, M	Netherlands, F
0,5	82,59	87,09
0,8	78,23	82,79
0,9	77,17	81,87
1,0	76,25	80,96
1,1	75,15	80,14
1,2	74,67	79,40
1,5	72,77	77,50
2,0	70,39	75,05

Figure 5. Same relation as Figure 3, full life and adult life only (Dutch men)



For this upper range of the life distribution we may also make use of integrating a fitted Makeham-Gompertz continuous density function. The resulting expected lifetimes for Dutch males surviving after age 45 against log of PH-factors are shown in Figure 6: we find the same linear relation of life expectancy with ln(ratio) as above, with a slope of – 8,79.

Figure 6. As Figure 5, adult life with continuous density (Dutch men)



3. References

[1] Actuarieel Genootschap: Sterftetafels GBM0005 en GBV0005.

[2] Office for National Statistics: Interim Life Tables for England, based on data for the years 2008-2010

[3] Catherine M. Calvin, Ian J. Deary, Candida Fenton, Beverly A. Roberts, Geoff Der, Nicola Leckenby and G. David Batty (2011) Intelligence in youth and all-cause-mortality: systematic review with meta-analysis. *International Journal of Epidemiology*, vol.40(3), p.626-644.

[4] Ian J. Deary, H. Lemon, J.R. Crawford and J.M. Starr (2000) The stability of individual differences in mental ability from childhood to old age: Follow-up of the 1932 Scottish Mental Survey. *Intelligence*: 28, 49-55.

[5] Sue Ramsden, Fiona M. Richardson, Goulven Josse, Michael S.C. Thomas, Caroline Ellius, Clare Shakeshaft, Mohammed L.Seghier and Caty J. Price (2011) Verbal and non-verbal intelligence changes in the teenage brain. *Nature* 10514 doi:10.138.

[6] William Makeham (1860) On the Law of Mortality and the Construction of Annuity Tables. *J. Inst. Actuaries and Assur. Mag.* **8**: 301–310.

[7] Benjamin Gompertz (1825). On the Nature of the Function Expressive of the Law of Human Mortality, and on a New Mode of Determining the Value of Life Contingencies. *Philosophical Transactions of the Royal Society of London* **115**: 513–585. doi:10.1098/rstl.1825.0026

Appendix. Method of calculation

Mortality tables

The mortality tables considered here provide values of the mortality rate q(t) at age t for t = 0 to t = 100 (England) or t = 120 (Netherlands). The calculation of the corresponding life expectancy is straightforward. With S(t) the fraction surviving at the beginning of age t and f(t) the discrete density of the truncated lifetime distribution, we have

S(0) = 1, S(t) = S(t-1) (1- q(t-1))

f(t) = S(t+1) – S(t)

life expectancy = expectation of f(t) = SUM {(t+0,5) * f(t)}

From a theoretical point of view it may be better to transform the q(t) into hazards h(t) (and back again) and look at the continuous distribution of the life-time, rather than assuming death to occur only halfway between birthdays. Then q(t) and h(t) are related as

h(t) = -ln(1 – q(t)),

q(t) = 1 – exp(-h(t)).

But over much of the age range the difference is small, and so is the effect on life expectancies.

After middle age q(t) increases with advancing age, and when it attains the value of 1 at age t^0 this means extinction beyond t^0. The life expectancies have therefore been calculated from t = 0 to t =t^0. In the baseline Dutch tables, q(t) is given up to t = 120, with t^0 113 (men) or 115 (women). There is of course little empirical evidence of mortality rates at these advanced ages and it is clear that two or three q(t) preceding t^0 have been obtained by extrapolation. Adjustments are in order when we apply multiplicative factors to all q(t): with reduced mortalities we must construct a smooth path to a later t^0, with increased mortality rates we must adjust values that would otherwise exceed 1. In either case some ad hoc extrapolation is needed at the tail end of the distribution. Apart from the extreme case where mortality rates are doubled, the life expectancy is not very sensitive to these artifices since the fraction surviving that is affected is fairly small.

The English tables end at t = 100 and have also been extended to t^0 by extrapolation.

Table A1 reports the last age t* with an *observed* value of q(t), the value of S(t*+1) (the fraction surviving at the end of t*), and the extinction age t^0, so that the reader can judge what proportion of the life distribution is involved in these arbitrary inventions. Predictably the most extreme changes and the English statistics fare worst.

Table A1. Statistics of extrapolation and smoothing

	Netherlands, men		Netherlands, women		England, men		England, women	
Factor	t*, t^0	S(t*+1)	t*, t^0	S(t*+1)	t*, t^0	S(t*+1)	t*, t^0	S(t*+1)
0,5	110, 115	0,0037	112, 118	0,0039	100, 108	0,1252	100,107	0,1475
0,8	110, 114	0,0000	112, 118	0,0001	100, 106	0,0324	100,106	0,0472
0,9	110, 114	0,0000	112, 117	0,0000	100, 106	0,0207	100,106	0,0273
1,0	110, 113	0,0000	112, 117	0,0000	100, 105	0,0082	100,105	0,0147
1,1	110, 113	0,0000	112, 116	0,0000	100, 105	0,0047	100,105	0,0110
1,2	110, 113	0,0000	112, 115	0,0000	100, 104	0,0027	100,104	0,0033
1,5	110, 112	0,0001	112, 112	0,0000	100, 103	0,0004	100,103	0,0015
2,0	103, 104	0,0037	112, 106	0,0000	100, 102	0,0000	100,102	0,0001

The Gompertz -Makeham distribution

The Gompertz–Makeham law, see [6] and [7], states that the death rate has three parameters and can be summarized by two equations, for the hazard h(t), t>0, and the survival fraction S(t), viz.

$h(t) = \alpha + \beta\,\gamma^{t}$;

$S(t) = \exp\{-\alpha.t - (\beta/\log\gamma)*(\gamma^{t} - 1)\}$.

So the hazard rate has a time-independent component α and a geometrically increasing component with age. The special case with α=0 is the Gompertz distribution, the refinement involving positive α was introduced by ; see [6] and [7]. The following formula holds in general for survival fractions, with integration over t>0:

life expectancy = $\int S(t)dt$.

There is no analytical expression for the latter integral in case of a Gompertz-Makeham distribution, but it can simply be evaluated numerically. This holds as well for the calculation of a conditional expected life-time after a certain age, given this age was reached.

The Gompertz Law fits the observed deaths in the Dutch male population well. The values of the AG-table have in fact been computed from a Gompertz-Makeham distribution with parameters

$\alpha = 0$

$\beta = 0{,}000016443$

$\gamma = 1{,}1124$

Proportional changes in the hazard are expressed by proportional changes in $\propto$ and β, with corresponding changes in S(t). The life expectancy is calculated by numerical integration of the aboce expression.

Using continuous hazard rates rather than one-year conditional mortality probabilities avoids the problem of having to account for a terminal age after which everybody dies. Using the above parameters for the Gompertz-Makeham distribution reveals that the linearity of expected terminal age against log of the PH-factor extends over an even wider interval than 0,5 to 2,0 as used above; in fact, even to values between 1/8 and 8. But such enormous variations are of little practical relevance.