Financial Frictions, Expectations and Business Cycle: Evidence from an Estimated DSGE Model *

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Abstract

In this empirical paper we incorporate financial accelerator mechanism into a version of the Smets and Wouters (2007) DSGE framework and estimate the model assuming, on the one hand, complete rationality of expectations and, alternatively, adaptive learning mechansim of expectation formation. In the estimations, we use financial data such as stock prices and finance premium as well as survey data of inflation expectations. We study how the deviation from the complete rationality assumption can modify the implications of financial frictions for the real economy and affect the empirical performance of the DSGE model. We show that the implications of the financial accelerator for the business cycle may vary depending on the approach to modeling the expectations. In particular, the results suggest that the learning scheme based on small forecasting functions is able to amplify the effects of financial frictions relative to the model with rational expectations. We also demonstrate that the model with learning outperforms the model with RE in forecasting inflation and real variables and is more successful in replicating the most recent economic downturn driven by severe financial shocks. Finally, we find that survey expectations are more consistent with the time varying mechansim of expectation formation implied by learning. Survey data contain useful information not present in the macro data alone and can improve forecasting performance of the DSGE model.

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1 Introduction

Despite the significant implications of public expectations for policy outcomes and actual macroeconomic dynamics, the issue of modeling the expectation formation mechanism in DSGE models has not received a sufficient attention. The current generation of empirical DSGE models is largely based on the assumption of Rational Expectations (RE), which implies that agents possess complete knowledge about the economy (the model, its parameters as well as stochastic structure) and therefore rely on "true" (model-consistent) forecasts in their decision-making process. At the same time, modern economies face various uncertainties and feature periods with changing economic structure and policy environment. In particular, the dynamic growth of financial markets and the implementation of more sophisticated financial instruments, which requires instant analysis and adjustment to new information, have complicated the task of efficient and up-to-date pricing and credit decisions. During the most recent financial crisis, central banks had to implement unconventional policy actions, which made it more difficult to predict and evaluate the effects of such a policy on the real economy. In other words, in practice, agents possess only limited information about the current state of the economy and have to make their choices on the basis of forecasts produced in an environment with incomplete information. Therefore, allowing the public to gradually learn the underlying economic structure is more realistic and enables more reasonable conclusions to be drawn about the factors affecting changes in public's predictions as well as the impact of expectations on actual economic activity.

In this paper, we contribute to studying the macro-financial linkages by focusing on adaptively formed expectations (adaptive learning) as a mechanism that can potentially amplify and propagate shocks to the real economy. The latest financial turmoil has demonstrated that the impact of imbalances in the financial sector on the real economy and wealth can be far more influential than many economists have anticipated. The failure of the DSGE models, which were in use prior to the crisis, to capture and explain the link between financial and real markets has pointed to the problem of the models misspecification in this important dimension. Even the more recent studies that analyze DSGE models with financial frictions have documented the problems in replicating the observed dynamics, explaining the "surprising" origin of the crisis and its propagation channels[1]. Therefore, there is a need for further research in the field of macro-financial modeling. In

[1] Brzoza-Brzezina and Kolasa (2013) present an empirical evaluation of the two most popular approaches to implementing financial frictions into DSGE models: the Bernanke et al. (1999) setup, and the Kiyotaki and Moore (1997) model. They find that models with the Bernanke et al. framework fit data better. However, they point out that even these models do not make a clear improvement over the New Keynesian benchmark in terms of marginal likelihood and matching the VAR impulse responses.

particular, it might be helpful to consider additional features of the transmission mechanism that could potentially augment the interactions between the financial and real sectors thus making models more successful in describing the business cycle dynamics. In this work, we study the interaction between the "expectations" and "macro-financial" factors exploring the fact that public's predictions play an important role in driving financial markets. We assess how departure from the complete rationality assumption in modelling the expectations about forward-looking (in particular, financial) variables may impact agents' decisions over time, in addition to changes already induced by the presence of financial frictions, and whether the time variation of agents' predictions (beliefs) can improve the fit of the estimated model and its dynamic properties. In addition, we evaluate how augmenting the estimation of macroeconomic model with observed financial data such as asset prices and credit spread affects the estimation and forecasting results. We would like to examine whether the shocks that the model uses to fit the data consisting solely of macroeconomic variables can satisfy the enchanced set of the relationship implied by the DSGE model with financial frictions. The estimated importance of the structural macroeconomic shocks can be significantly diminished once financial data are taken into consideration. Financial data, which is available at higher frequency, may capture some of the information contained in measured expectations of macroeconomic variables. In addition, we would like to evaluate if inclusion of survey expectations and learning improves the DSGE model forecasting performance.

We incorporate adaptively formed expectations (Evans and Honkapohja, 2001, Milani, 2007 and Orphanides and Williams, 2007) into a version of the Smets and Wouters (2007) DSGE model with a financial accelerator (FA). It is assumed that agents know the structure but they are uncertain about the parameters of the model. To learn the parameters, they formulate linear econometric models based on their economic perceptions and re-estimate these models as soon as new information arrives. Following Slobodyan and Wouters (2012b), we assume that forecasting models are re-estimated every period on the basis of the Kalman filter learning algorithm. We estimate the model using Bayesian methods and assess the joint role of financial frictions and the departure from the complete rationality assumption for the U.S. business cycle. On the basis of the estimation and simulation results, we assess how adaptive learning (AL) algorithm can modify the transmission mechanism of the model with RE and investigate ability of AL to generate additional macroeconomic fluctuations in line with real data.In this paper, we follow Slobodyan and Wouters (2012b) and assume that agents' forecasts can be also based on very small forecasting models, in particular on a model where expected value of a forward-looking variable depends on a constant and two lags of this variable.

The main result of our work suggests that learning based on small forecasting models can amplify the effect of financial frictions and strengthen the propagation of shocks to the real economy. In particular, we demonstrate that in a model with FA, learning may create additional business cycle fluctuations in line with the real data by influencing the degree of economic persistence in asset prices and investments. Specifically, time-varying agents' perceptions about the financial and economic developments may translate into greater perceived asset price persistence and more pronounced (and persistent) responses of investment and output. Simulation results also demonstrate that the sensitivity of the economy to financial shocks increased after the middle of 1980s, thus strengthening the link between asset prices and the real economy.

Regarding the estimated parameters, we find that some of the estimated structural rigidities and shock persistence decrease. More specifically, autoregressive components of exogenous processes for price and wage mark-up shocks fall dramatically, similar to Slobodyan and Wouters (2012a,b). The decline in the persistence of the finance premium shock is not so dramatic, but still notable. In addition, shock to the net worth becomes an iid process under learning. These results imply that learning represents an alternative source of endogenous inertia and is helpful in explaining inflation, wage, finance premium and stocks dynamics. The estimated financial frictions parameters tend to decrease somewhat under learning, also indicating that deviation from rational expectation introduces endogenous persistence in the external finance premium and strengthens the impact of the FA mechanism compared to the model with RE. Finally, we find that survey expectations are more consistent with the time varying mechanism of expectation formation implied by learning. Survey data contain useful information not present in the macro data alone and improve forecasting performance of the DSGE model.

The results of our work are generally in line with the AL and financial frictions literature. Merola (2013) estimates a Smets and Wouters model, augmented with a FA and the observed data on credit spread. She shows that the model is suitable to capture much of the historical developments in US financial markets that led to the financial crisis. In particular, the model can account for the output contraction in 2008, as well as widening in corporate spreads and supports the argument that financial conditions have amplified the US business cycle and the intensity of the recession. In the AL literature, a number of studies have already demonstrated that learning can improve the fit of macroeconomic models. In particular, Milani (2007, 2008) and Sargent, Williams and Zha (2006) have shown that introducing AL can generate the levels of persistence observed in U.S. data. Slobodyan and Wouters (2012a) assess whether learning may overcome the problem of DSGE models misspecification, pointed up in Del Negro et al. (2007). They demonstrate that the best-performing learning model may closely approximate the optimal DSGE-VAR (in terms of marginal likelihood and impulse response functions), which proves that the model-consistent expectations imposed by the RE hypothesis are too restrictive. The authors also illustrate that specification of initial beliefs plays an important role for this result. Slobodyan and Wouters (2012b) incorporate less-than-rational beliefs into the Smets and Wouters (2007) model and find that the impact of AL on macro dynamics is more pronounced when the agents' information set is more restrictive than the one implied by RE. In small forecasting models, learning can explain episodes of inflation dynamics in the U.S. and also lowers the persistence of some of the exogenous shocks. Rychalovska and Slobodyan (2010) estimate a set of small to medium-sized DSGE models for the Euro Area. They also find that assuming adaptive expectations results in better model fit than if RE is used, especially when the agents use very little information to form their beliefs. Therefore, the conclusion that AL based on small forecasting models outperforms MSV learning and RE models seems to be a robust one, at least for the U.S. and European data.

The rest of the paper is organized as follows: in Section 2, we present the model. In particular, we focus on modeling the financial sector and on explaining the expectation formation mechanism; Section 3 contains the estimation methodology and results; Section 4 describes the effects of financial frictions on the transmission mechanism in the model with AL, and Section 5 concludes.

2 The model with a financial accelerator under learning

In the recent literature, the FA represents a common approach to incorporate financial frictions into DSGE models. This framework implies that endogenous developments in credit markets work to amplify and propagate shocks to the real economy. Depending on the origin/type of such an acceleration mechanism, two main strands in the literature can be distinguished. The first one implies capturing firms' balance sheet effects on investment by relying on a one-period stochastic optimal debt contract with costly state verification (Bernanke and Gertler, 1989; Carlstrom and Fuerst, 1997; Cespedes et al., 2004). The key aspect is that such a framework allows modeling of an endogenous, positive interest rate spread. In particular, Bernanke and Gertler (1989) and Bernanke et al. (1999) introduce the agency problem with asymmetric information in order to model a positive interest rate spread, i.e. an "external finance premium" defined as the difference between the cost of external sources of funding and the opportunity cost of funds internal to the firm. Due to the agency problem in lending, the external finance premium depends inversely on the borrowers' net wealth and thus will be countercyclical, enhancing swings in real variables and amplifying the effects of monetary and financial shocks. The second approach emphasizes another aspect of many possible frictions – the role of endogenous collateral constraint that links the credit capacity of borrowers to the value of their asset holdings (Kiyotaki and Moore, 1997; Iacoviello, 2005; Iacoviello and Neri, 2008).

In this paper, we incorporate financial frictions combined with an imperfectly rational expectation formation mechanism into a medium-scale DSGE model based on Smets and Wouters (2007). In the specification of the Kalman filter learning, we closely follow Slobodyan and Wouters (2012b). The financial frictions are integrated in the form of the FA as originally applied in Bernanke and Gertler (1989) and extensively explored in the recent literature. More specifically, Bernanke et al. (1999) incorporate an "external finance premium" concept into a dynamic New Keynesian model with nominal rigidities to study how credit market frictions may influence the transmission of monetary policy. They show that, under a reasonable parameterization of the model, the FA significantly amplifies the effects of shocks to the economy. The most relevant other examples include Chtistiano et al. (2010), De Graeve (2008) and Christensen and Dib (2008). In particular, in terms of its empirical relevance, Christiano et al. (2010) have found that both for the Euro Area and for the U.S. the FA plays a relevant role in amplifying shocks that move prices and output in the same direction (e.g. monetary policy shocks) as well as in explaining the business cycle. De Graeve (2008) incorporates the FA into the Smets and Wouters (2003) model and estimates the external finance premium for the U.S. economy. He finds that a model-consistent estimate of this unobservable financial variable has substantial realistic content (the estimate strongly co-moves with the proxies for the premium). Another important result of his study is that incorporating financial frictions improves the empirical performance of an otherwise standard DSGE model. Christensen and Dib (2008) reach a similar conclusion.

The model contains a number of nominal and real rigidities such as monopolistic competition on goods and labor markets, Calvo price and wage stickiness, habit formation in consumption and capital adjustment costs. Following the seminal contributions of Smets and Wouters (2003, 2007) and Christiano et al. (2005), these structural rigidities have become widely used in order to match the observed properties of the main macroeconomic series.

The economy consists of households, final and intermediate goods producers, a monetary authority and a financial sector. Intermediate-sector firms are monopolistically competitive. They produce differentiated goods, decide on labor and capital input and set prices according to the Calvo (1983) model. Households supply homogenous labor to an intermediate labor union, which differentiates the labor services. Since there is a certain monopoly power over labor, unions can set wage rates. I assume that unions face Calvo-type frictions in setting wages. In addition, nominal rigidities in wage- and price-setting are enhanced by the assumption that prices that are not re-optimised are partially index-linked to past inflation rates.

The financial sector is represented by capital good producers, financial intermedies and entrepreneurs. Capital producers accumulate new capital and sell it to entrepreneurs. Entrepreneurs rent capital stock to intermediate firms and borrow from the bank in order to finance capital acquisitions. The bank obtains resources for lending by issuing deposits to households. As in Bernanke et al. (1999), financial contracts issued to households pay a real return which is not contingent upon the realization of the shock. Financial market imperfections are set up in the form of asymmetric information between entrepreneurs and the banks. Due to this friction, the optimal financial contract, which maximizes the entrepreneur's payoff subject to the required rate of return of lenders, implies the existence of an endogenous external finance premium that depends on the entrepreneur's leverage ratio. As in Kiyotaki and Moore (1997), the financial frictions of Bernanke et al. (1999) are based on the idea that asset price variability affects the entrepreneurial financial position and therefore drives credit market imperfections. Introducing AL and incomplete information into such a framework brings in additional dynamics in asset prices and the external finance premium. As a result, imperfectly rational beliefs can become an important driving force behind the fluctuations on financial markets and modify the impact of financial frictions on the real economy.

The model is detrended with a deterministic trend γ that represents a labor-augmenting growth rate in the economy. The non-linear system is then linearized around the stationary steady state of the detrended variables. Lower-case variables denote detrended variables expressed in real terms. In this section, we focus mainly on describing the financial sector as well as the learning algorithm. For the formal presentation of the rest of the micro-foundations, we would like to refer to the original Smets and Wouters (2007) model. The log-linearized model equations are also summarized in the Appendix.

2.1 Financial sector

2.1.1 Capital-goods producers

Following Bernanke et al. (1999) and Christiano et al. (2003), capital-goods producers, owned by households, produce new capital goods which are sold to entrepreneurs at price Q_t. Capital-goods producers are competitive and take the price as given. They combine investment goods, purchased from the final good producers, with the existing capital stock, rented from the entrepreneurs, to produce new capital goods, K_{t+1}. As in Bernanke et al. (1999), it is assumed that the rental rate for the existing capital is zero, since the operation takes place within one period. Capital-goods producers are subject to quadratic adjustment costs specified as function $S(\frac{I_t}{I_{t-1}})$, with $S^{''}(.) > 0$. In addition, the capital production technology is affected by an investment-specific shock ε_t^i. The optimization problem of capital-goods producers, in real terms, consists of choosing the level of investment I_t to maximize the real expected profits:

$$\max_{I_t} E_t \left\{ \sum_{s=0}^{\infty} \beta^s \frac{\lambda_{t+s}}{\lambda_t} \left[Q_{t+s} I_{t+s} \varepsilon_{t+s}^i - I_{t+s} - Q_{t+s} I_{t+s} \varepsilon_{t+s}^i S(\frac{I_{t+s}}{I_{t+s-1}}) \right] \right\}, \tag{1}$$

where λ_t denotes the marginal utility of the real income of the household. The solution to the problem is:

$$\varepsilon_t^i Q_t \left(1 - S(\frac{I_t}{I_{t-1}})\right) = 1 + \varepsilon_t^i Q_t S^{'}(\frac{I_t}{I_{t-1}}) \frac{I_t}{I_{t-1}} - E_t \left\{ \beta \frac{\lambda_{t+1}}{\lambda_t} \varepsilon_{t+1}^i Q_{t+1} S^{'}(\frac{I_{t+1}}{I_t}) \left(\frac{I_{t+1}}{I_t}\right)^2 \right\}. \tag{2}$$

Equation (2) relates the price of capital to investment and adjustment costs. In the absence of adjustment costs, Q_t is constant and equal to one. After detrending and log-linearization of (2), the dynamics of investment are given by:

$$\widehat{i}_t = \frac{1}{(1+\overline{\beta}\gamma)} (\widehat{i}_{t-1} + (\overline{\beta}\gamma)\widehat{i}_{t+1} + \frac{1}{\gamma^2 S''} \widehat{Q}_t \ ) + \widehat{q}_t, \tag{3}$$

where S'' is the steady-state elasticity of the capital adjustment cost function and $\overline{\beta} = (\beta/\gamma^{\sigma_c})$, where β is the discount factor applied by households. As in Christiano et al. (2005), a higher elasticity of the cost of adjusting capital reduces the sensitivity of investment $(\widehat{i}_t)$ to the real value of the existing capital stock $(\widehat{Q}_t)$. Finally, $\widehat{q}_t$ represents a disturbance to the investment–specific technology process and is assumed to follow a first–order autoregressive process with an iid–normal error term: $\widehat{q}_t \ = \rho_q \widehat{q}_{t-1} + \epsilon_t^i$.

The evolution of the capital stock is represented by the following expression:

$$\widehat{k}_t = \ (1 - \frac{i_*}{\overline{k}_*}) \ \widehat{k}_{t-1} \ + \ \frac{i_*}{\overline{k}_*} \widehat{i}_t + \frac{i_*}{\overline{k}_*} (1+\overline{\beta}\gamma)\gamma^2 S'' \widehat{q}_t. \tag{4}$$

2.1.2 Entrepreneurs and banks

In the original Smets and Wouters (2007) model, financial markets do not incorporate endogenous forms of inefficiency. In particular, households can borrow in any quantity at the rate that might exceed the risk-free rate R_t set by the central bank due to the exogenous premium ε_t^b. Modeling endogenous credit imperfections requires distinguishing between borrowers and lenders and the existence of a conflict between the two parts. Therefore, new types of agents have to be introduced. Following Bernanke et al. (1999), entrepreneurs, who are risk neutral and survive until the next period with probability $\varkappa$, use their own funds (the net worth, N_{t+1}) and loans from the bank (B_{t+1}) to finance capital that is rented to the production sector. Competitive banks finance the loans by accepting deposits from the households at the risk-free rate. The financial intermediation between the banks and entrepreneurs is subject to friction based on the agency problem, which leads to the existence of the interest rate premium. In particular, after the purchase of the capital stock, each entrepreneur receives an idiosyncratic productivity shock ω that affects the capital holdings (ωK_{t+1}) and the return on capital holdings (ωR^K_{t+1}). Banks have to pay a "state verification" (monitoring) cost to infer the realized return. As a result, entrepreneurs have to pay an external finance premium over the riskless rate in order to borrow funds.

In the formal representation of the entrepreneurs' problem, we follow Christiano et al. (2010) and somewhat deviate from the original BGG specification, assuming that entrepreneurs are not directly involved in the production of intermediate goods. Thus, we keep the model closer to the original Smets and Wouters set-up, where monopolistically competitive firms choose capital, labor and set prices. One of the implications of such a departure is that the net worth of entrepreneurs comes from profits (including capital gains) accumulated from previous capital investment. In the original BGG specification, the net worth also includes an income from labor, which entrepreneurs supply inelastically to the general labor market. However, Bernanke et al (1999) note that the labor income source of the net worth, under reasonable parametrization, does not have a significant impact on the FA effects and the model dynamics.

At the end of period t, entrepreneurs purchase capital K_{t+1} from capital-goods producers at price Q_t. Thus, the amount of borrowed funds is given by $B_{t+1} = Q_t K_{t+1} - N_{t+1}$. After observing the $t+1$ shock, the entrepreneur decides on the degree of capital utilization (U_{t+1}) and rents a part of the capital services to intermediate-goods firms at rate $\widehat{r}^k_{t+1}$. A non-depreciated capital stock is then sold at price Q_{t+1}. The amount of effective capital that entrepreneurs can rent to firms is $K^S_{t+1} = U_{t+1} K_{t+1.}$ The income from renting capital services is $r^k_{t+1} U_{t+1} K_{t+1}$,while the (real) cost of changing capacity utilization is $a(U_{t+1}) K_{t+1}$, where a is a convex function with $a^{'}, a^{''} > 0$. The entrepreneur, who receives an idiosyncratic productivity shock ω, chooses U_{t+1} to solve:

$$\max_{U_{t+1}} \left[r^k_{t+1} U_{t+1} - a(U_{t+1}) \right] \omega K_{t+1}. \tag{5}$$

The (aggregate) solution implies that

$$r^k_{t+1} = a'(U_{t+1}). \tag{6}$$

Then optimal log-linear representation of capital utilization condition takes the form:

$$\widehat{u}_t = ((1-\psi)/\psi)\widehat{r}_t^k, \tag{7}$$

where ψ is the elasticity of the capital utilization cost function. The log linearized relation for capital services is given by

$$\widehat{k^S}_{t+1} \;=\; \widehat{u}_{t+1} \;+\; \widehat{k}_{t+1}. \tag{8}$$

The average (aggregated over all the entrepreneurs) rate of return on capital purchased at time t is given by

$$E_t R_{t+1}^k = E_t \left[\frac{r_{t+1}^k U_{t+1} - a(U_{t+1}) + Q_{t+1}(1-\tau)}{Q_t} \right], \tag{9}$$

where τ is the depreciation rate. Expression (9) equates the marginal return of capital, given by the right-hand-side terms, to the real expected interest rate on external funds. The log-linearized relation that describes the dynamics of the average expected real return to capital is given by

$$E_t \widehat{R}_{t+1}^K = \frac{1-\tau}{\overline{R}^K} E_t \widehat{Q}_{t+1} + \frac{\overline{r}^k}{\overline{R}^K} E_t \widehat{r}_{t+1}^k - \widehat{Q}_t, \tag{10}$$

where $\overline{R}^K$ denotes the steady-state return to capital and $\overline{r}^k$ is the steady-state rental rate.

The equilibrium condition on financial markets is derived from the optimal-debt contract problem, which maximizes the welfare of the entrepreneur, combined with the zero-profit condition of the bank. The details of the financial contract specification and derivations can be found in Appendix A of Bernanke et al. (1999). The optimality condition, which determines the link between the external financing costs, capital purchases and entrepreneurial financial position, is given by

$$E_t R_{t+1}^k = E_t \left[s(\frac{N_{t+1}}{Q_t K_{t+1}}) \varepsilon_t^b R_t \right]. \tag{11}$$

Equation (11) indicates that the cost of external financing is composed of the premium for borrowed external funds represented by a function $s(\frac{N_{t+1}}{Q_t K_{t+1}})$[2], the risk-free interest rate and an exogenous disturbance that describes fluctuations in the risk premium not captured

[2]Bernanke et al. (1999) define the external finance premium as a ratio of default costs to quantity borrowed:

$$\frac{\mu \int^{\varpi_t} \omega R_t^K Q_{t-1} K_t dF(\omega)}{Q_{t-1} K_t - N_t}$$

by the financial frictions of Bernanke et al. (1999). Therefore, the FA mechanism is driven by the risk premium which consists of both endogenous and exogenous components. The exogenous shock ε_t^b measures to what extent Bernanke-type financial frictions explain the dynamics of the risk premium[3]. The log-linearized condition (11) gives the representation of the external finance premium in the following form:

$$\widehat{prem}_t = E_t \widehat{R}_{t+1}^K - (\widehat{R}_t^n - E_t[\widehat{\pi}_{t+1}] + \widehat{\varepsilon_t^b}) = -el \left\{ E_t \left[\widehat{N}_{t+1} - \widehat{Q}_t - \widehat{k}_{t+1} \right] \right\}, \tag{12}$$

where el represents the elasticity of the external finance premium to the change in the financial conditions. The equation above indicates that, in equilibrium, an entrepreneur purchases capital up to the point where the expected real return on capital is equal to the marginal cost of external finance. The higher the fraction of the project value financed by the entrepreneur's internal funds (the higher the net worth N relative to the gross value of capital QK), the lower the capital market friction and the lower the corresponding risk premium. It is easy to see that the model with a FA can be reduced to the Smets and Wouters (2007) model. In particular, the absence of financial frictions implies the case when entrepreneurs have sufficient net worth to finance the demand for capital stock. In such a situation, the risk of default associated with borrowing external funds vanishes, the risk-free rate and the real return to capital coincide. Thus, after setting the amount of borrowing and/or the parameter elasticity of the external finance premium to zero and combining equations (10) and (12), we obtain a standard relation which determines the asset price dynamics in the Smets and Wouters (2007) model.

The law of motion for the aggregate financial wealth of entrepreneurs is given by

$$N_{t+1} = \varkappa V_t + W_t^e, \tag{13}$$

where $\varkappa$ is the entrepreneurial survival rate and W_t^e is the transfer to all the entrepreneurs who are in business in period t. This transfer provides new entrant entrepreneurs with the funds needed to buy some capital. The aggregate net worth of surviving entrepreneurs V_t is equal to the difference between the revenue from capital holding in time t and the cost of borrowing carried over from the previous period (the rate of interest paid by entrepreneurs on loan contracts B_t signed in time $t-1$), averaged across all the entrepreneurs:

$$V_t = \left[R_t^K Q_{t-1} K_t - E_{t-1} R_t^K \left(Q_{t-1} K_t - N_t \right) \right]. \tag{14}$$

The log-linearization of combined equations (13) and (14) leads to the expression of en-

[3]Gilchrist et al. (2009) interprete this financial disturbance as a shock to the supply of credit that captures the efficiency of the financial intermediation process or a shock to the financial sector that boosts the external finance premium beyond the level warranted by the current economic conditions.

trepreneurial net worth in the form of the following accumulation equation:

$$\widehat{N}_{t+1} = \varkappa \overline{R}^K \left[\frac{\overline{K}}{\overline{N}} \left(\widehat{R}_t^K - E_{t-1} \widehat{R}_t^K \right) + E_{t-1} \widehat{R}_t^K + \widehat{N}_t \right] + \varepsilon_t^{nw}, \tag{15}$$

where $\overline{K}/\overline{N}$ is the steady-state ratio of capital to net worth, i.e. the inverse of the leverage ratio and ε_t^{nw} is the shock to the net worth, which follows a stationary AR(1) process. Equation (15) demonstrates that, in general terms, the endogenous variations in net worth in the next period come from unexpected changes in the real return on capital. These unexpected changes might come from the shocks that reduce the rental rate of capital or the market value of capital. As a result of such shocks, the entrepreneurial net worth will drop, leading to a fall in investment. As will be shown below, learning process may impact this traditional FA channel. In particular, both the rental rate and the price of capital are forward-looking variables, affected by expectations. Departure from the complete rationality assumption by allowing agents to formulate predictions that reflect their own perceptions about the economy can modify the evolution of financial variables. This, in turn, may have a substantial effect on the entrepreneurs' financial positions and a real economy[4]. Combining equations (12) and (15), the net worth can be expressed as a function of the risk-free interest rate and the exogenous and endogenous finance premia:

$$\widehat{N}_{t+1} = \varkappa \overline{R}^K \left[\frac{\overline{K}}{\overline{N}} \widehat{R}_t^K - \left(\frac{\overline{K}}{\overline{N}} - 1 \right) \left(\widehat{R}_{t-1} + \widehat{\varepsilon_{t-1}^b} \right) - el \left(\frac{\overline{K}}{\overline{N}} - 1 \right) \left(\widehat{k}_t + \widehat{Q}_{t-1} - \widehat{N}_t \right) + \widehat{N}_t \right] + \widehat{\varepsilon}_t^{nw}. \tag{16}$$

The values of the parameters $\varkappa$, $\overline{K}/\overline{N}$ and el determine the impact of financial frictions on the real economy. The higher the entrepreneurial survival rate and the capital to the net worth steady-state ratio, the more persistent the evolution of net worth will be. Combined with the higher elasticity of the external finance premium, this would imply a stronger response of the wedge between the expected return on capital and the risk-free rate. Therefore, shocks affecting entrepreneurial net worth would have greater real effects.

2.2 Monetary policy and equilibrium

The model is completed by adding the following empirical monetary policy reaction function:

$$\begin{aligned} \widehat{R}_t^n \;=\; & \rho_R \widehat{R}_{t-1}^n \;+\; (1-\rho_R)(r_\pi \widehat{\pi}_t \;+\; r_y \widehat{ygap}_t) \\ & + r_{\Delta y}(\widehat{ygap}_t - \widehat{ygap}_{t-1}) \;+ r_t. \end{aligned} \tag{17}$$

[4]As I will demonstrate, in this model, the variability of asset prices due to learning is one of the main sources of the additional volatility of the net worth and the finance premium, especially if firms are highly leveraged.

The monetary authority follows a generalized Taylor rule responding to inflation and the output gap terms (current and lagged). The latter is defined as the difference between actual and potential output. The output gap is approximated by $\widehat{ygap}_t = \widehat{y}_t - \widehat{A}_t$. The parameter ρ_R captures the degree of interest rate smoothing. I assume that the monetary policy shock (r_t) follows a first-order autoregressive process with an iid–Normal error term: $\widehat{r}_t \; = \rho_r \widehat{r}_{t-1} + \epsilon_t^r$.

The log-linear representation of the resource constraint is given by

$$\widehat{y}_t \;=\; \frac{(\overline{R}^K - 1 + \tau)k_*}{y_*}\widehat{u}_t + \widehat{\mu}_t^{bank} + \; \frac{c_*}{y_*}\widehat{c}_t \; + \; \frac{i_*}{y_*}\widehat{i}_t \; + \widehat{g}_t \;, \tag{18}$$

where $\widehat{\mu}_t^{bank} = (k_*/y_*)(\overline{R}^K - \overline{R})(1 - \overline{N}/\overline{K})(\widehat{R}_t^K + \widehat{Q}_{t-1} + \widehat{k}_t)$ measures the bank monitoring cost; $\widehat{g}_t$ is exogenous government spending, which is assumed to follow a first–order autoregressive process with an iid–normal error term and is also affected by the productivity shock as in Smets and Wouters (2007): $\widehat{g}_t \; = \rho_g \widehat{g}_{t-1} \; + \rho_{ga}\epsilon_t^a + \epsilon_t^g$. The relation between domestic productivity and government spending is motivated by the fact that, in estimation, the exogenous spending component also includes net exports, which may be affected by domestic productivity developments.

2.3 Introducing adaptive learning

We depart from the assumption of RE and assume that agents possess incomplete knowledge about the economic environment (model structure and parameters). Since agents do not know the Rational Expectation Equilibrium (REE), they are unable to produce model-based predictions of the path of forward-looking variables and have to form their own beliefs about future developments on the basis of the information they observe. As in Marcet and Sargent (1989) and Evans and Honapohja (2001), agents gradually learn the "true" parameters of the model by adapting their expectations with the use of a certain learning algorithm. In this section, we present a general description of a Kalman filter learning setup[5]. For more details on the description and implementation of this learning process, see Slobodyan and Wouters (2012b).

The model described in Section 2 and summarized in the model Appendix can be

[5] The alternative, widely used, learning method is the constant gain Recursive Least Squares (RLS). Sargent and Williams (2005) demonstrate that both learning methods mentioned above are asymptotically equivalent on average. However, their transitory behavior may differ significantly. In particular, the Kalman filter tends to result in a faster adjustment of agents' beliefs due to the fact that the learning gain is not assumed to be constant.

represented in the following structural form:

$$A_0 \begin{bmatrix} y_{t-1} \\ w_{t-1} \end{bmatrix} + A_1 \begin{bmatrix} y_t \\ w_t \end{bmatrix} + A_2 E_t y_{t+1} + B_0 \epsilon_t = const., \tag{19}$$

where the vector y_t includes endogenous variables of the model and w_t is an exogenous AR(1) process :

$$w_t = \Gamma w_{t-1} + \Pi \epsilon_t. \tag{20}$$

In general form, the RE solution of the model (19) is given by the following expression:

$$\begin{bmatrix} y_t \\ w_t \end{bmatrix} = \mu + T \begin{bmatrix} y_{t-1} \\ w_{t-1} \end{bmatrix} + R\epsilon_t, \tag{21}$$

where T and R are time-invariant non-linear functions of the model structural parameters Θ. Under RE, the intercept μ is normally a vector of zeros [6]. The stochastic structure, represented by the vector w_t and innovations ϵ_t, is determined by 8 exogenous shocks and 2 measurement errors. The model variables summarized by vector y can be grouped into the state variables y^s that appear with a lag, forward variables y^f that appear with a lead, and the so-called static variables[7]. Agents have to forecast 7 forward-looking variables, such as: consumption, investment, hours worked, price and wage inflations, return on capital and asset prices. Deviation from the RE assumption implies that agents do not have sufficient information about the model and its parameters to formulate the REE-based expectations $E_t y_{t+1}$. Thus, they cannot derive the law of motion (21). To obtain the solution, they have to formulate the so-called Perceived Law of Motion (PLM), which relates the value of the forward variable j to the model state variables X_j using a reduced-form linear function:

$$y^f_{j,t} = \beta_{j,t-1} X_{j,t-1} + u_{j,t}. \tag{22}$$

The forecasting model is then represented in the SURE format and composed of the variable-specific equations of the form (22). In this paper, in period t, agents predict only next-period variables which are present in Euler equations of firms and consumers. Thus, I implement the *Euler equation learning*, promoted by Evans and Honkapohja (2001) [8]. The error term $u_{j,t}$ in (22) consists of linear combination of the true model errors ϵ_t and has a variance-covariance matrix Σ. Matrix X_j includes a set of variables that are used to form predictions about forward-looking variable j. Thus, X_j may consist of all the state variables of the model y^s or a subset of y^s. Note, that the functional form of (22) generally

[6] It can be a nonzero vector only for observable variables that are not detrended.

[7] y^f and y^s could intersect.

[8] An alternative type of learning - long-horizon learning, advocated by B. Preston (2005)- implies that agents forecst economic variables infinitely many periods ahead.

corresponds to the predictions obtained under RE. What learning brings into the set-up is the time variation in β_j due to periodical updates of beliefs, and the possibility for the coefficients β_j to diverge from the values implied by the RE solution (21). In addition, the vector of beliefs β_j may also include the constant which would mean that agents make inferences not only about the persistence of economic processes but also about growth rates. Finally, the information set X_j can be significantly more restrictive than the vector of all the state variables y^s, which would be used under RE.

The Kalman filter learning algorithm consists of the following steps:

1. Formulation of the initial beliefs. Following Slobodyan and Wouters (2012a and 2012b), we apply the standard assumption in the learning literature and assume that initial beliefs $\beta_{1|0}$ are consistent with the REE[9]. Thus, the initial values for the vector of beliefs β and variance-covariance matrix Σ, needed to initialize the Kalman filter, are based on the matrix of second moments Ω of the model variables, which is derived for the REE given by (21) and evaluated for the parameter vector Θ. The moments imply a relationship between the forward-looking variables y^f and the variables X used in forecasting equations. Then, using the corresponding rows and columns of Ω, $\beta_{1|0}$ is obtained as the projection of X on y[10]:

$$\beta_{1|0} = E\left[X'X\right]^{-1} \cdot E\left[X'y\right],$$

where the expectations $E[]$ are taken from the RE solution. In the estimation procedure, initial beliefs always correspond to the REE for the currently evaluated (in the posterior optimization or MCMC steps) parameter vector[11]. Therefore, this approach of choosing the initial beliefs is the closest to the pure rational expectations. Given $\beta_{1|0}$, the variance-covariance matrix is calculated as:

$$\Sigma = E\left[\left(y_t^f - X_{t-1}\beta_{1|0}\right)\left(y_t^f - X_{t-1}\beta_{1|0}\right)^T\right].$$

2. Update. The Kalman filter is used to obtain the updated estimates of the vector

[9] Slobodyan and Wouters (2012a) discuss alternative ways of evaluating the initial beliefs, which may significantly deviate from the REE. Specifically, initial beliefs can be constructed from the pre-sample regressions or derived on the basis of optimized posterior probablility as in Sargent et al. (2006). Slobodyan and Wouters find that dynamics under the learning process can be very sensitive to initial beliefs. In the situation when agents' beliefs significantly deviate from the beliefs implied by the REE, model dynamics under learning can be driven rather by transition from the initial beliefs to the equilibrium beliefs than by the standard updating due to new information.

[10] We use the fact that estimator β_{OLS} is unbaised.

[11] In this respect, initial beliefs are called to be "model-consistent".

of beliefs β and the error covariance matrix P:

$$\begin{aligned} \beta_{t/t} &= \beta_{t/t-1} + K_t \widetilde{z}_t, \\ P_{t/t} &= \left(I - K_t X_{t-1}\right) P_{t/t-1}, \end{aligned} \tag{23}$$

where the innovation or measurement residual $\widetilde{z}_t = y_t^f - \beta'_{t/t-1} X_{t-1}$, the innovation (or residual) covariance $S_t = \Sigma + X'_{t-1} P_{t/t-1} X_{t-1}$; and the optimal Kalman gain $K_t = P_{t/t-1} X_{t-1} S_t^{-1}$. Updating of the beliefs at any t depends on the forecast error, data (best estimates of the variables at time $t-1$) and on the initial beliefs.

3. Prediction. According to agents' beliefs, the coefficients β follow a vector autoregressive process:

$$\left(\beta_t - \overline{\beta}\right) = F \cdot \left(\beta_{t-1} - \overline{\beta}\right) + v_t, \tag{24}$$

where F is a diagonal matrix with $\rho \leq 1$ on the main diagonal, $\overline{\beta}$ is the the initial beliefs and v_t are i.i.d. errors with variance-covariance matrix V . Parameter ρ, which is assumed to be the same for all the seven forecast variables, measures the intensity of updates and therefore is referred to the learning "gain" parameter, which is estimated. Using (24), the forecast of the evolution of β can be obtained as follows: $\left(\beta_{t+1/t} - \overline{\beta}\right) = F \cdot \left(\beta_{t/t} - \overline{\beta}\right)$. The predicted estimate covariance is given as: $P_{t+1/t} = F P_{t/t} F' + V$. The variance-covariance matrix V of shocks v_t and an initial guess for $P_{1|0}$ are taken to be proportional to the GLS estimator of the variance of β[12]: $V = \sigma \left(X'\Sigma^{-1}X\right)^{-1}$and $P_{1|0} = \gamma \left(X'\Sigma^{-1}X\right)^{-1}$, where scaling parameters σ and γ can be calibrated or estimated.

The best estimates of the beliefs $\beta_{t/t-1}$ generated in the prediction step of the Kalman filter are then used to calculate expectations $E_t y_{t+1}$ of forward–looking variables according to forecasting equations (22). Substituting these expectations into the structural representation of the model (19), we obtain the Actual Law of Motion (ALM) of the system in a purely backward–looking form:

$$\begin{bmatrix} y_t \\ w_t \end{bmatrix} = \mu_t + T_t \begin{bmatrix} y_{t-1} \\ w_{t-1} \end{bmatrix} + R_t \epsilon_t. \tag{25}$$

Introducing AL does not affect the initial steady state of the system, i.e. at time $t = 0$ we start from the RE equilibrium solution given by equation (21).

In general terms, the transmission under learning can be summarized as follows. Following the innovations in exogenous disturbances and subsequent changes in the variables included in the information set, agents start to adjust their belief coefficients iterating over equation (23). In expressing their perceptions about the impact of the shock on the economy, they have two instruments in their disposal: to update the autoregressive

[12] β_{GLS} is an efficient estimator for the SURE model.

coefficients, which determine the persistence of the forecast variables, and the mean of the variable. Since the effect of the learning process operates through expectations, equations that define the intertemporal decisions of agents and thus directly involve the expectation terms become particularly important for transmitting the impact of the shocks to changes in the belief coefficients and afterwards onto the estimated structural parameters and the model dynamics. In particular, the forward-looking equations that largely determine the dynamics under learning are the Euler equation for consumption, inflation Philips curve, investment equation and one of the key FA equations (10), which describes the evolution of the average expected real return on capital. For example, exogenous shock to the risk premium will mostly impact expectations (and beliefs) about the real return on capital and asset prices via equations (10) and (12). In particular, following the positive risk premium shock, asset prices will show a decline. Such an observed downturn on financial markets motivates agents to revise perceptions about asset price persistence upwards. At the same time, the mean can be adjusted downwards if agents believe that the shock will have a rather pronounced negative effect on the growth rate of asset prices. These beliefs will be reflected in the expected trend in asset prices through the forecasting equation (22) and will feed into the dynamics of the endogenous variables through the ALM (25). Therefore, the simulation of the system's dynamics under learning entails calculation of a time–varying transmission mechanism determined by μ_t, T_t and R_t, which are the functions of the model structural parameters given by matrices A_0, A_1, A_2 and B_0 as well as beliefs $\beta_{t/t-1}$. The intensity of time-variation determines to what extent the equilibrium path under learning differs from the RE equilibrium. In our case, when agents' beliefs are chosen to be consistent with REE, we expect that most of the updates will be driven by adjustments of beliefs due to new information.

Note that the specification of forecasting equations implies that agents do not directly observe and distinguish between exogenous processes. Learning agents make inferences only on the basis of the observed variables affected by the shock (and the available information set can be very limited). As a result, the effects of certain shocks on the equilibrium dynamics under learning can be over-estimated or, on the contrary, realized more slowly and incompletely compared to the transmission under RE. The transmission speed depends to a significant extent on the nature of the shock. In particular, in forecasting inflation, productivity or price mark-up shocks, which have a direct impact on the marginal costs (and thus on the inflation Phillips curve relation), are realized quicker compared for example to the monetary policy shock, which is transmitted into prices via its impact on investment and demand[13].

[13] Slobodyan and Wouters (2012b) demonstrate that learning agents tend to overestimate the effect of price mark up shock. As a result, the inflation response is more persistent, which in turn determines a less aggressive monetary policy reaction and a lower decline in the aggregate demand. In this respect, the

3 Estimation strategy and results

3.1 Data and measurement equations

The model is estimated using the following macroeconomic quarterly U.S. time series: real GDP, real consumption, real investment, real wage, hours worked, GDP deflator, federal funds rate, stock prices index and the credit spread[14]. Nominal variables are deflated by the GDP deflator. Aggregate variables are expressed in per capita terms. All variables except hours, inflation, and interest rates are taken in first differences. We estimate the model for the sample period 1954:1 - 2014:2. The long data sample is chosen in order to assess the importance of time variation in the model parameters introduced by AL. The link between the model variables and the data is given by a set of the following equations:

$$\begin{bmatrix} dlGdp_t \\ dlCons_t \\ dlInv_t \\ dlWage_t \\ lHours_t \\ dlP_t \\ FedFundsR_t \\ dSP500_t \\ Spread_t \end{bmatrix} = \begin{bmatrix} \overline{\gamma_y} \\ \overline{\gamma_c} \\ \overline{\gamma_i} \\ \overline{\gamma_w} \\ \bar{l} \\ \overline{\pi} \\ \overline{r} \end{bmatrix} + \begin{bmatrix} \widehat{y}_t - \widehat{y}_{t-1} \\ \widehat{c}_t - \widehat{c}_{t-1} \\ \widehat{i}_t - \widehat{i}_{t-1} \\ \widehat{w}_t - \widehat{w}_{t-1} \\ \widehat{l}_t \\ \widehat{\pi}_t \\ \widehat{R}^n_t \\ \widehat{N}_t - \widehat{N}_{t-1} + me_{N,t} \\ \widehat{prem}_t + me_{S,t} \end{bmatrix}, \tag{26}$$

where l and dl stand for log and log difference, respectively. Unlike Smets and Wouters (2007), I estimate separately the trends for output, consumption, investment and wages growth rates, instead of imposing a common trend on these variables. $\overline{\pi} = 100(\Pi_* - 1)$ is the quarterly steady-state inflation rate and $\overline{r} = 100(\overline{\gamma}^{\sigma_c}\Pi_*/\beta - 1)$ is the steady–state nominal interest rate. Given the estimates of the average trend growth rate and the steady–state inflation rate, the latter will be determined by the estimated discount rate. Finally, $\bar{l}$ refers to steady-state hours worked. Measurement errors denoted by $me_{N,t}$ and $me_{S,t}$ are added to the observation equations of financial variables.

ability of learning to generate more gradual responses of certain variables to shocks makes the dynamics under learning more realistic and consistent with the responses produced by VAR and DSGE-VAR models.

[14] The stock price index is S&P500. The credit spread is defined as the difference between the corporate BAA yield and the AAA yield.

3.2 Bayesian estimation under adaptive learning

The log-linearized versions of the models are estimated using Bayesian methods. These methods combine a likelihood function of the data with a prior density to derive the posterior distribution of the structural parameters. The prior density contains information about the model parameters from other sources (microeconometric and calibration evidence). The estimation procedure includes: first, the estimation of the mode of the posterior distribution by maximizing the log posterior function, and second, the Metropolis–Hastings algorithm, which is used to compute the posterior distribution and to evaluate the marginal likelihood of the model[15]. Typically, from 300,000 to 500,000 MCMC draws were performed, using three chains. For estimation purposes, the model is represented in the state-space form, which combines equation (25), the so-called state equation, with the measurement equations (26). Given the state space representation of the model, the Kalman filer is used to evaluate the likelihood. For evaluating the model under imperfectly rational beliefs, we use the toolbox developed by Slobodyan and Wouters (2012b), which implements procedures for AL within the Dynare 4.24 Matlab toolbox. As described in Section 2, the estimation procedure under learning differs from the inferences under the RE due to time variation in beliefs (23), which implies the dynamic solution of μ_t, T_t and R_t[16]. The values of μ_t, T_t and R_t are used to form expectations of the next period model variables in the main Kalman filter step and are used to calculate the model likelihood. In other words, the standard Kalman filter incorporates an additional filtering stage used to update agents' beliefs (see section 2.3).

In this paper, we estimate the AL model specification with relatively small information set (given by matrix X_j in (22)) used by agents to form their beliefs about the forward-looking variables. In particular, we assume that the forecasting equation (22) for every forward-looking variable includes two lags and a constant[17]. Thus, agents form and update their beliefs about the persistence and the mean of the variables. In addition, the forecasting equation for investment includes also lagged stock prices and the asset prices equation incorporates the first lag of investment. Therefore, agents use the financial data in predicting real activity and vise versa.

The priors for the estimation are chosen following Smets and Wouters (2003, 2007). These papers present a detailed description of the estimation methodology as well as the justification for the choice of priors. The priors for additional parameters related to the financial frictions are based on calibration exercises and previous literature (Bernanke et al., 1999, Merola, 2013 and De Graeve, 2008). In particular, $\overline{R}^K$ is calibrated at 1.0129.

[15]For more details on Bayesian estimation of DGSE models, see An and Schorfheide (2007).

[16]These matrices do not change over time during the estimation under RE.

[17]Parsimonious AR(2) form of forecasting equations has been chosen in order to ensure simple structure and interpretation.

Parameters $\varkappa$, $\overline{k}/\overline{N}$, and el are assumed to have Normal priors with sufficient standard deviations.

3.3 Estimation results. Model fit

Table 1 reports the logarithms of marginal data densities for the various estimated specifications. We compare the results for models with RE and AL. In addition, each version was estimated on the dataset that incudes financial data (stock prices and credit spread) and the dataset that includes only macroeconomic variables.

Table 1: Model Comparison in Terms of Marginal Likelihood

Model specification	*with fin.data*	*w/o fin.data*
RE	-2101.21	-1400.87
Kalman filter AL	-2088.61	-1380.75

The estimation results suggest that the model with Kalman filter learning fits the data better than the model specification based on RE, estimated with or without financial data. Therefore, RE hypothesis appears to be restrictive. This result is in line with the previous studies which anaysed the performance of the models with learning in the model specifications which do not incorporate financial frictions (Slobodyan and Wouters, 2012a and 2012b). In order to shed more light on the estimation results, we analyse the relative likelihood (evaluated at the posterior mode) of alternative model specifications as a function of time. We would like to find out how introducing AL changes the empirical performance of the model over time relative to the model with RE. In other words, it is important to understand whether the transmission mechanism of the learning model can be more suitable for describing the macro-dynamics during certain economic episodes. Figure 1 shows the cumulative likelihood for AL model estimated with and without financial data relative to the corresponding model with RE[18]. The upward trend of the cumulative difference line indicates that on average the likelihood of the learning model on this time interval is better relative to the likelihood implied by the model based on RE. Figure 1 indicates that, on average, the RE model, estimated with or without financial data, fits the data better before the middle of 1970s. Therefore, it appears that during this period other shocks than those which impact and propagate through the expectations dominated in driving the macroeconomic dynamics. In particular, we could think about fiscal shocks, which do not directly affect the intertemporal choices of agents. Such a conjecture is in line with the empirical evidence, which suggests that from late 50-s and until

[18] I compute the difference in likelihood for AL and RE models, and plot the cumulative sum of this difference.

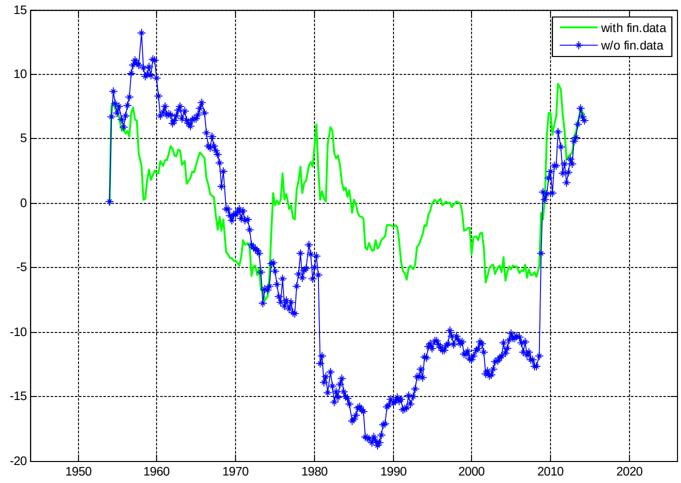


Figure 1: Cumulative sum of the likelihood difference: RE vs AL

the appointment of Volcker the fiscal authority was dominating in the policy conduct. The late 1970-s were marked by a change in the policy regime. In particular, monetary policy became more systematic and started to play an important role in stabilizing economic activity and inflation. This change in policy regime is reflected in Figure 1. The graph also demonstrates that the aptitude of the learning model to describe the data-generating process has improved in the second part of the sample. This could imply that shocks affecting the expectations (price and wage mark-up shocks, as well as financial shocks) became more important in driving the macroeconomic dynamics. For both data samples, with and without financial data, learning significantly outperforms the model with RE during the most recent financial crisis.

3.4 Parameter estimates

In this subsection, we discuss the impact of adaptively formed expectations on the estimated parameters. We present the posterior optimization results for the RE and the learning models. The results of the model comparison are presented in Table 2.

Table 2 demonstrates that there are important changes in the estimated structural parameters and the stochastic processes depending on the assumed expectation formation mechanism (RE or AL). Naturally, the most pronounced impact of learning will be on the parameters that enter the expectation equations such as the consumption Euler equation, price and wage Phillips curves, as well as the equations that determine the average return

Table 2: Comparison of RE and AL models in terms of the estimated parameters

Parameters		Prior distribution			Posterior, RE model		Posterior, AL model	
		Type	Mean	St.d	Mode	St.d	Mode	St.d
Shocks								
exo.risk prem.	σ_b	I.Gam	0.1	2	0.341	0.045	0.791	0.19
net worth	σ_{nw}	I.Gam	0.1	2	0.515	0.151	1.195	0.242
investment	σ_q	I.Gam	0.1	2	0.583	0.05	0.415	0.051
price markup	σ_p	I.Gam	0.1	2	0.145	0.012	0.192	0.01
wage markup	σ_w	I.Gam	0.1	2	0.355	0.019	0.326	0.017
AR.coeff-s								
exo.risk prem.	ρ_b	Beta	0.5	0.2	0.879	0.025	0.68	0.145
net worth	ρ_{nw}	Beta	0.5	0.2	0.607	0.125	-	-
investment	ρ_q	Beta	0.5	0.2	0.221	0.093	0.45	0.06
price markup	ρ_p	Beta	0.5	0.2	0.938	0.024	0.091	0.071
wage markup	ρ_w	Beta	0.5	0.2	0.948	0.028	0.031	0.023
pr.markup,ma	μ_p	Beta	0.5	0.2	0.853	0.045	-	-
w.markup,ma	μ_w	Beta	0.5	0.2	0.934	0.032	-	-
Str.params								
adjust. cost	φ	Norm	4	1.5	4.391	0.676	3.752	1.005
habit	η	Beta	0.7	0.1	0.515	0.062	0.558	0.048
Calvo wages	ξ_w	Beta	0.5	0.1	0.812	0.031	0.764	0.044
Calvo prices	ξ_p	Beta	0.5	0.1	0.756	0.034	0.833	0.033
index. wages	ι_w	Beta	0.5	0.15	0.635	0.124	0.351	0.113
index. prices	ι_p	Beta	0.5	0.15	0.214	0.076	0.215	0.098
int.rate smooth	ρ_r	Beta	0.75	0.1	0.848	0.017	0.907	0.017
pol rule, inflat.	r_π	Norm	1.5	0.25	1.807	0.144	1.368	0.186
cap./net worth	k/N	U	1	3.5	1.845	0.127	1.726	0.126
survival rate	$\varkappa$	U	0.9	1	0.962	0.01	0.952	0.011
elast.risk prem	el	U	0	0.5	0.085	0.012	0.083	0.012
gain (AL)	g	Beta	0.9	0.025	-	-	0.982	0.007
Log(ML)					-2101.21		-2088.61	

to capital and the external finance premium[19].

In particular, we find that some of the estimated structural rigidities and shock persistence decrease. More specifically, autoregressive components of exogenous processes for price and wage mark-up shocks fall dramatically. The decline in the persistence of the shock to the risk premium is not so pronounced, but still notable. In addition, the variance of the investment shock and the parameter of investment adjustment costs decline somewhat under learning. These results imply that learning is helpful in explaining inflation, wage and investment dynamics. Modeling adaptive expectations of these variables introduces "endogenous" persistence, which has an empirically appealing economic inter-

[19] Apart from the financial sector, the model is largely based on Smets and Wouters (2007) and Slobodyan and Wouters (2012b). Therefore, we can observe a very similar pattern in deviations of the parameters estimated under learning from the parameters obtained under RE.

pretation. In addition, the exogenous net worth shock, which together with the finance premium shock can be viewed as a proxy for the degree of the financial-market inefficiencies, becomes the i.i.d process under learning. However, the variability of these shocks increases. Thus, it appears that the time-varying transmission mechanism can explain part of the persistence in the external finance premium and the net worth, but cannot generate sufficient volatility in these financial variables through evolution of agents' beliefs.

The degree of wage rigidity and wage indexation show a decline. The parameters of price rigidity increases slightly and the habit formation parameter stays essentially the same. The parameter of the interest rate smoothing increases and the parameter measuring the response to changes in inflation in the policy rule shows a decline. Thus under learning, the interest rate tends to adjust slower in response to shocks and changes in inflation. We may conclude that learning represents an important source of endogenous inertia, but it can only partially substitute for the "mechanical" source of rigidities and the persistence of some of the disturbances.

Three parameters – capital-to-net-worth ratio, entrepreneurial survival rate and the elasticity of the external finance premium are jointly responsible for the FA effects in the model. The higher value of these parameters strengthens the impact of financial frictions on the real economy. Comparing the results for RE and AL models presented in Table 2, we can see that the estimated capital-to-net-worth ratio, survival probability and the parameter elasticity tend to decrease somewhat under learning indicating that endogenous persistence in the external finance premium introduced by learning is not induced by the increase of financial frictions parameters. Our estimates of the elasticity (0.085 in the posterior mode for the RE model and 0.083 for AL) are in line compared to the regression and calibration results from the previous literature. In particular, Bernanke et al. (1999) calibrates $el = 0.05$ based on a realistic value of monitoring costs and bankruptcy rates. Christensen and Dib (2008) estimate this parameter at 0.042. However, they calibrate the remaining financial parameters at a lower level. De Graeve (2008) reports a value of elasticity of 0.1047. Therefore, the estimated overall impact of financial frictions has a comparable magnitude across different studies.

3.5 Forecast evaluation and comparison

Forecasting performance is an important criterion in the assessment of a model's credibility and usefulness for policy analysis. In this section, we compare the forecast accuracy of the estimated DSGE models with RE and with learning. We calculate predictions for 9 macroeconomic and financial time series: output, consumption, investment, real wages, inflation, hours, interest rate, finance premium and stock prices. All the variables except the inflation, hours, interest rate and finance premium are in growth rates. The forecasts are computed for two sample period: from 1970Q1 to 2014Q2 and on a shorter sample from 2004Q2 to 2014Q2, focusing on the most recent economic crisis. As a criterion of the forecast accuracy we use a traditional measure - RMSE which is computed for one to eight step ahead predictions. The results of the forecast comparison are presented in Table 3. In addition, we calculate the RMSE for the models estimated without financial data. The results are shown in the Table 3a. The results demonstrate that the model with AL shows a superior predictive performance for the real variables such as output, consumption and investment, compared to the model with RE. Table 3a indicates that using financial data as observable variables in the estimation under RE hypothesis brings an improvement in short-term (1 to 3 quarters) forecasting of investment and labour dynamics. The RMSE of the model with learning appears to be less sensitive to the inclusion of financial data into the estimation procedure.

Table 3. Point forecast accuracy. Model with financial data

Forecast sample:1970-2014						
	1Q		4Q		8Q	
	RE	AL	RE	AL	RE	AL
Output gr.	0.339	0.307	0.442	0.386	0.401	0.38
Consumption gr.	0.33	0.305	0.383	0.383	0.34	0.359
Investment gr.	0.765	0.747	1.293	1.133	1.273	1.144
Labour	0.281	0.221	1.223	1.004	2.112	1.747
Inflation	0.096	0.101	0.123	0.125	0.156	0.14
Wage	0.49	0.507	0.509	0.509	0.511	0.505
Interest rate	0.053	0.049	0.183	0.182	0.321	0.321
Stock prices gr.	3.317	3.419	3.454	3.396	3.471	3.448
Finance premium	0.151	0.15	0.313	0.316	0.34	0.365

Forecast sample:2004-2014						
	1Q		4Q		8Q	
	RE	AL	RE	AL	RE	AL
Output gr.	0.715	0.648	0.933	0.815	0.845	0.802
Consumption gr.	0.697	0.644	0.808	0.809	0.718	0.757
Investment gr.	1.614	1.578	2.727	2.391	2.685	2.414
Labour	0.591	0.465	2.579	2.118	4.455	3.686
Inflation	0.203	0.214	0.26	0.264	0.329	0.296
Wage gr.	1.034	1.071	1.073	1.073	1.078	1.067
Interest rate	0.111	0.103	0.386	0.385	0.678	0.677
Stock prices gr.	6.998	7.213	7.286	7.165	7.322	7.275
Finance premium	0.318	0.316	0.66	0.667	0.717	0.77

Table 3a. Point forecast accuracy. Models without financial data

Forecast sample:1970-2014						
	1Q		4Q		8Q	
	RE	AL	RE	AL	RE	AL
Output gr.	0.335	0.294	0.398	0.39	0.361	0.397
Consumption gr.	0.335	0.303	0.333	0.384	0.324	0.383
Investment gr.	0.92	0.751	1.274	1.144	1.139	1.136
Labour	0.323	0.221	1.292	1.005	2.075	1.831
Inflation	0.099	0.103	0.136	0.128	0.171	0.141
Wage	0.497	0.509	0.508	0.508	0.506	0.504
Interest rate	0.055	0.048	0.214	0.162	0.374	0.271

Forecast sample:2004-2014						
	1Q		4Q		8Q	
	RE	AL	RE	AL	RE	AL
Output gr.	0.708	0.62	0.84	0.823	0.761	0.838
Consumption gr.	0.707	0.64	0.702	0.812	0.682	0.808
Investment gr.	1.94	1.584	2.687	2.413	2.403	2.397
Labour	0.681	0.467	2.724	2.121	4.377	3.862
Inflation	0.21	0.216	0.286	0.27	0.361	0.297
Wage gr.	1.049	1.075	1.071	1.072	1.068	1.063
Interest rate	0.116	0.101	0.452	0.342	0.788	0.572

Figures 2 and 3 compare the one-step-ahead forecasting performance of the RE and AL model. In particular, we contrast the forecasts and the actual dynamics of inflation, output growth, investment growth and stock prices growth.

As described in the Section 2.3, forecasting models imply that agents have very limited information set and cannot discriminate between the shocks that hit the economy in a specific moment of time. Admittedly, such an assumption cannot be considered as very realistic, but allowing agents to learn the structural shocks would complicate the estimation procedure dramatically. A simple form of forecasting equations may facilitate or deteriorate the forecasting performance of AL comparing to the model with RE. In particular, rational agents have the possibility to anticipate future policy reactions and incorporate this knowledge into the predictions. In such cases, learning may react quicker and be more successful in predicting the abrupt changes in the certain variables. On the other hand, learning agents, who observe only limited set of variables, might be unable to identify the shock in time and therefore could react with a certain delay. As a result, the overall economic impact might be underestimated.

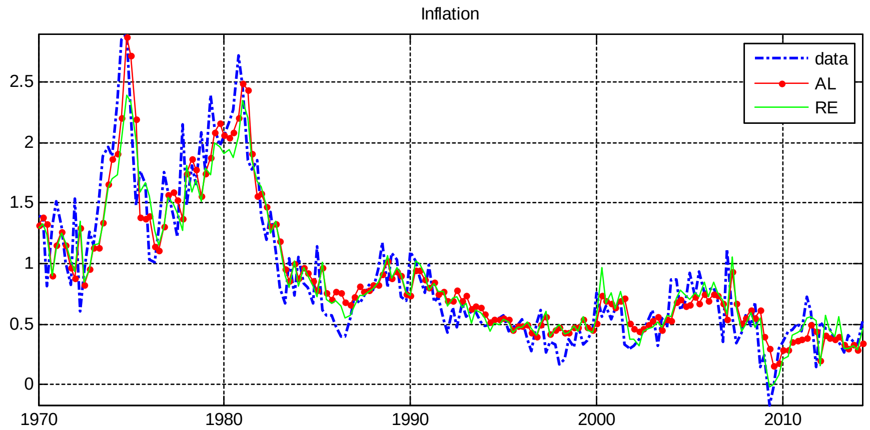
Let us consider several economic episodes during which AL showed better and worse forecasting performance relative to the model with RE.

Figures 2 and 3 illustrate that the AL model was more successful in predicting inflation in the middle of 1970s and the beginning of the 1980s (see also Figure 1, which shows an improvement in relative likelihood during these periods). Slobodyan and Wouters (2012b) explain this result by more realistic modeling of private sector inflation expectations. More specifically, they show that inflation persistence implied by the learning model displays significant time variation (increase in 1970s and 1980s) due to the impact of price and wage markup shocks, which were active during this time period.

The AL model did better in predicting the decline in output and investment growth during the recession in the beginning of the 1980s and also during the most recent economic downturn, characterized by a particularly pronounced drop in the real sector. The departure from the RE hypothesis in modeling the asset price and investment expectations plays a prominent role in explaining this result. In particular, the latest financial distress was driven by an adverse shock originated in financial sector. The disturbance was directly transmitted into financial variables - net worth and asset prices, leading to the sizable and fast updates of the asset price expectations, which feed back into FA mechanism and reinforced the impact of the shock. The amplification effect occurs because learning agents perceive decline in asset prices as being more persistent, which resulted in the more pronounced decline in the net worth and real variables compared to the model with RE.

On the other hand, AL was less successful in predicting the economic downturn during the recession of middle 1970s. Such a result can be explained by the fact that the information set available to the learning agents appeared to be too restrictive for the proper identification of this recession. In particular, the 1973-74 recession brought a particularly strong decline in investment. In forecasting such a dynamics, the performance of AL is, first of all, determined by ability of agents to identify an investment-specific shock from the recent trend in investment. Following the shock, agents did react to an investment decline by adjusting their beliefs (see also Figure 4). In particular, the autoregressive coefficients in the forecasting equation for investment showed an increase, but learning agents estimated the persistence of the investment process at the lower level comparing to the agents with RE. In addition, perceptions about growth rates (the constant) were adjusted downwards, but only to a relatively small extent (compared to the recession in the 1980-s). Therefore, it appears that AL agents perceived this period as being less severe for the real economy compared to the fully rational agents.

The main results of this section can be summarized as follows. Introducing AL into the DSGE model with financial frictions can contribute to capturing the properties of real data over certain time intervals, especially in the second half of the data sample. Among the main factors determining performance of AL are the nature of shock affecting the economy and the speed of transition of the innovations to agents' expectations.



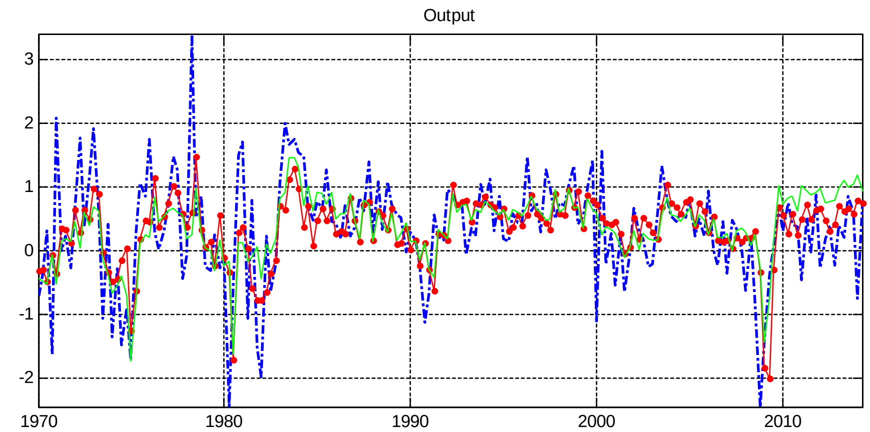


Figure 2: 1-step-ahead forecasts: inflation and output growth

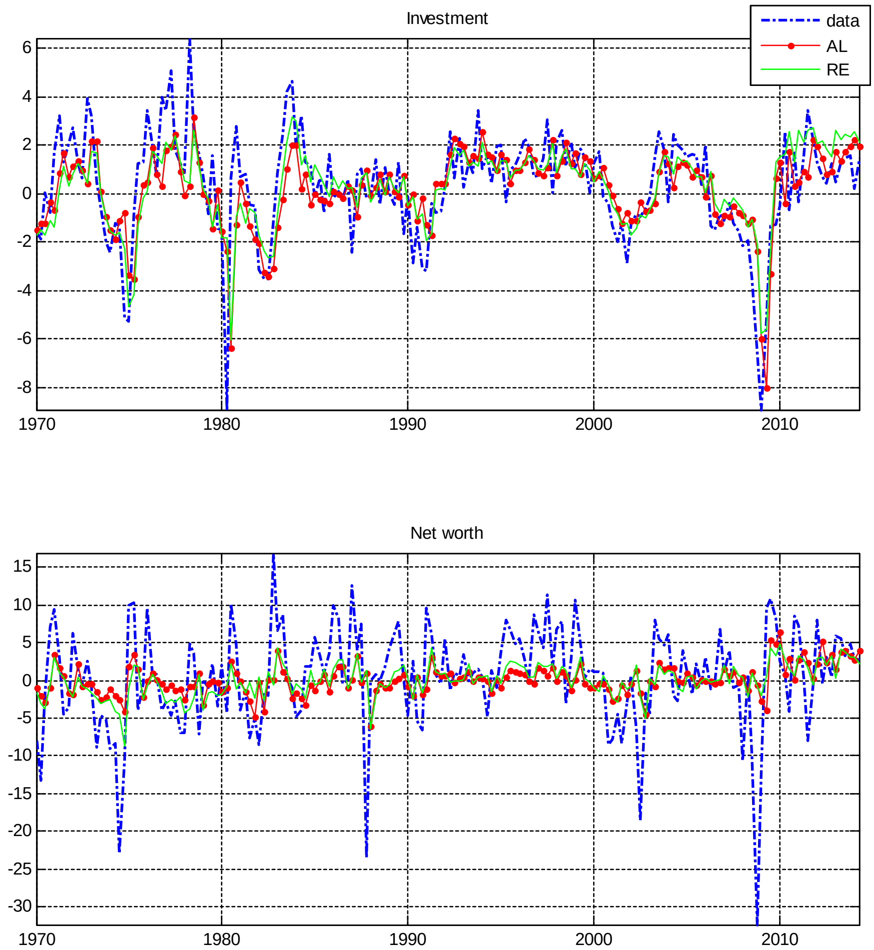


Figure 3: 1-step-ahead forecasts: investment and net worth growth

4 Financial frictions under learning. Time variation and transmission mechanism

AL can affect model fit in several ways. First, the time variation of beliefs allows the model to become time-varying (25). This could improve the empirical performance of the model if the process that generates the time series of the observed variables is itself time-varying. On the other hand, if the belief-updating process is too volatile, the uncertainty factor could lead to a deterioration of the fit. Another channel through which AL operates is through a change in the information set available to the agents. Even when beliefs are consistent with a REE and are not time-varying, the changes in the information set used by the agents to form the expectations will lead to a divergence of the transmission mechanism from that under RE. In this section, we will further examine how the expectation formation mechanism introduced by learning modifies the properties of the model and, as a result, affects the transmission of various shocks.

4.1 Evolution of agents' beliefs

In this subsection, we discuss the dynamics of expectations implied by AL model. We analyze the time variation of beliefs, which summarize the perceptions of agents about the economy and together with the model's structural parameters jointly determine the dynamic properties of the model. We also discuss factors affecting the estimates of the beliefs coefficients.

Figure 4 plots the evolution of agents' beliefs given by the coefficients of the forecasting functions of the learning model. We present the evolution of the autoregressive component, which measures the perceived persistence and is given as a sum of AR(1) and AR(2) coefficients, and a constant. In other words, we plot agents' Perceived Law of Motion (PLM) given by (22).

Figure 4 illustrates that agents perceive real consumption, labor, wages and investment as highly persistent processes with relatively stable autoregressive parameters. At the same time, similar to Slobodyan and Wouters (2012b), we find that Kalman filter learning introduces significant time variation in agents' beliefs about inflation. In particular, perceived inflation persistence displayed peaks around the mid 1970s and again around 1980, then gradually declined to a level around 0.6 since the mid 1980s. Slobodyan and Wouters (2012b) also illustrate that perceived inflation persistence is mainly driven by innovations to the price mark-up, wage mark-up and productivity shocks, which had the most pronounced impact on beliefs in the first half of the sample when inflation was relatively high.

Beliefs about asset prices and investment (both the autoregressive parameters and the

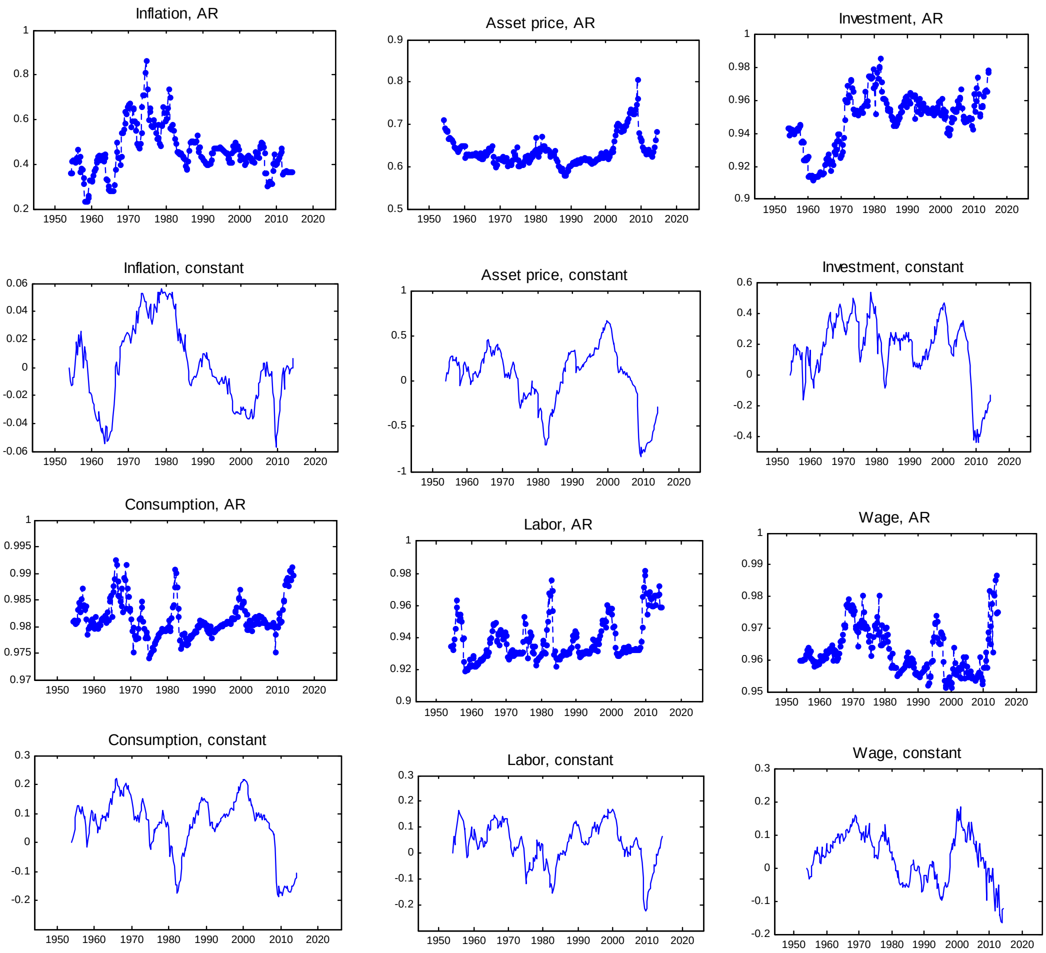


Figure 4: Evolution of beliefs under learning: sum of ar(1) and ar(2) parameters

constants) also show substantial time variation. The perceived asset price persistence evolved in the 0.5-0.85 range, with spikes in the 1970s, 1980s, 2000-s and particularly 2008. Such dynamics appears to be consistent with developments on financial markets observed during this period. In particular, the NBER recessions were recorded around 1973-1974, 1980-1982, 2000-2002 and 2008. Prior to recessions, asset prices showed an upward trend (in particular, in the middle of the 1990s) and, generally tended to be higher than the predicted values. Positive forecasting errors motivated optimistic expectations of households and lead to the upward revision of their beliefs about asset price persistence. Such an update, in turn, contributed to the further growth of asset prices and investment. As a result, in a model with AL, sudden negative shocks to asset prices would be realized in the state of high asset price and investment persistence. Therefore, such shocks may have greater real effects. During the crisis, when asset prices rapidly declined, the perceived persistence decreased as well[20].

In addition to the autoregressive components, agents also revise their beliefs about the expected means of forward-looking variables (given by a constant in the forecasting equations). The presence of a constant brings additional changes to the transmission mechanism compared to the one implied by the RE model, where all the real variables are assumed to have a common trend growth rate and inflation is centred around a fixed inflation objective. Slobodyan and Wouters (2012b) interpret the variations of the constant as deviations of agents' expectations from these steady-state values. Figure 4 illustrates that constants vary the most for asset prices and investment and reflect rather a cyclical pattern of change in these variables. Significant shifts in the expected means can add to the macroeconomic volatility and contribute to over-optimistic or -pessimistic developments in agents' expectations. Financial frictions do not have important implications for the perceived persistence of other variables – real consumption, wage and labor – making our results in this respect very similar to Slobodyan and Wouters' findings (2012b).

4.2 Financial accelerator under learning and the transmission mechanism

Implied persistence is an important determinant of the real effects of shocks hitting the economy. In particular, shocks to the inflation rate, which is perceived as a highly persistent process, will lead to stronger and longer-lasting responses of inflation. For an

[20] Slobodyan and Wouters (2012b) explain in greater detail the beliefs' updating process. In particular, for the constant term, higher (lower) than expected realizations of the forward variable result in upward (downward) revision in the constant of the forecasting equation. The updating of persistence has a state-dependent nature. For example, in periods when the inflation rate is high, revisions in persistence are positively correlated with the inflation innovations. At the same time, when a positive inflation innovation is realized in a low inflation state, the perceived inflation persistence will decline.

inflation-targeting central bank, such dynamics would imply a more aggressive monetary policy reaction, which would affect real output to a greater extent. In the FA framework, agents' perceptions about financial variables such as asset prices may have additional macroeconomic implications. If agents perceive asset prices to be more persistent, financial shocks will have a greater impact on households' financial position (net worth) and hence the external finance premium. Therefore, financial disturbances will involve higher cumulative effects on investment and output. The results presented in the previous subsection indicate that a learning model may have significant implications for the shock transmission due to the dynamic changes in implied persistence of asset prices, inflation as well as real variables relative to the model with RE.

The previous literature has already provided some insights into the transmission mechanism in models with financial frictions. Christensen and Dib (2008) study the transmission of shocks in the estimated model with RE and a FA. Unlike Bernanke et al. (1999), their model incorporates a nominal debt contract, allowing for debt deflation effects. Christensen and Dib (2008) find that the FA mechanism considerably amplifies and propagates the impact of demand-side shocks – monetary policy, money demand and preference shock – on investment and the price of capital. The implications of financial frictions for inflation and output are found to be relatively minor. De Graeve (2008) reports similar effects of the FA. In particular, the investment response to a preference and monetary policy shocks is stronger relative to the model without financial frictions. In both studies, the FA mechanism dampens the rise of investment following positive technology and investment supply shocks. This contrasts sharply with the results in Bernanke et al. (1999) and Walentin (2005), in which favourable productivity shocks reduce the premium and therefore boost investment relative to a model without financial frictions. In addition, in the De Graeve (2008) model, the dynamics of investment following investment supply shocks differs somewhat from the results documented in Bernanke et al. (1999) and other existing studies (Walentin, 2005; Christensen and Dib, 2008). He explains the difference in responses by the form of adjustment costs[21].

In this section, we compare the implications of financial frictions for the transmission mechanism in the RE and AL model. Figures 5-7 show the impulse responses under the risk premium, net worth and monetary policy shocks, respectively. In fact, the figures present the time variation of impulse responses and thus reflect the time-varying transmission mechanism under learning. The very first impulse response (denoted by the thick line) corresponds to the reaction under RE. The dynamics of inflation are similar to those documented in Slobodyan and Wouters (2012b) because the FA did not significantly affect

[21]Bernanke et al. (1999) works with capital adjustment costs, whereas De Graeve (2008) assumes investment adjustment costs. This implies a more gradual and hump-shaped response of investment.

inflation persistence in the 1970s. In particular, inflation responded much more strongly to shocks around the 1970s, when perceived inflation was very persistent.

Figure 5 shows the responses to a risk premium shock. The response of both investment and output is hump-shaped, with the peak of the response occuring 5 to eight quarters after the impact of financial shock. The reaction of output, investment and financial variables in the AL model is stronger relative to the model with RE. The peaks in the perceived asset price persistence observed in 1980s and especially in 2008 explain a dramatic fall in asset prices, which reduces the net worth. As a result, the gap between the cost of external financing and the risk-free rate shows an increase, which is more persistent compared to the model with RE. Therefore, the responses of investment and output are also amplified indicating a stronger impact of the FA on the real economy. This example clearly illustrates the mutually reinforcing interaction between the FA and AL.

Figure 6 presents the impulse responses to an adverse shock to the net worth. In contrast to the finance premium shock, disturbances to net worth have less pronounced but longer-lasting effect. Under learning, reaction of financial variables is more persistent, which is brought by endogenous inertia introduced by learning (in AL model, shock to the net worth is estimated as iid process). Similar to the finance premium shock, disturbance to the net worth implies more substantial fluctuations in real variables when asset prices are perceived to be more persistent. The longer lasting decline in investment implies a significant reduction in the supply of capital, which causes marginal costs to increase. Thus, our estimates imply that adverse shocks to the net worth are inflationary, and, therefore, may present the monetary authority with a trade-off between inflation and output. In the model with learning, the trade-off can become more severe when agents perceive inflation to be relatively more persistent.

Figure 7 presents the effects of the monetary policy shock. Following the monetary tightening, inflation, asset prices, investment and output decline. The immediate reaction of variables under learning is generally lower but more persistent. At the peak, the responses can be considerably amplified relative to the model where financial frictions interact with RE. Under learning, the interest rate adjusts more slowly due to greater inertia in interest rates and a lower response of the interest rate to changes in inflation (see Table 2). Due to the gradual adjustment of inflation expectations, actual inflation also responds more slowly. Thus, adjustment of the policy rate is not as aggressive as under RE.

The analysis of the transmission mechanism performed in this section enables several important conclusions to be drawn. First of all, learning agents perceive the developments

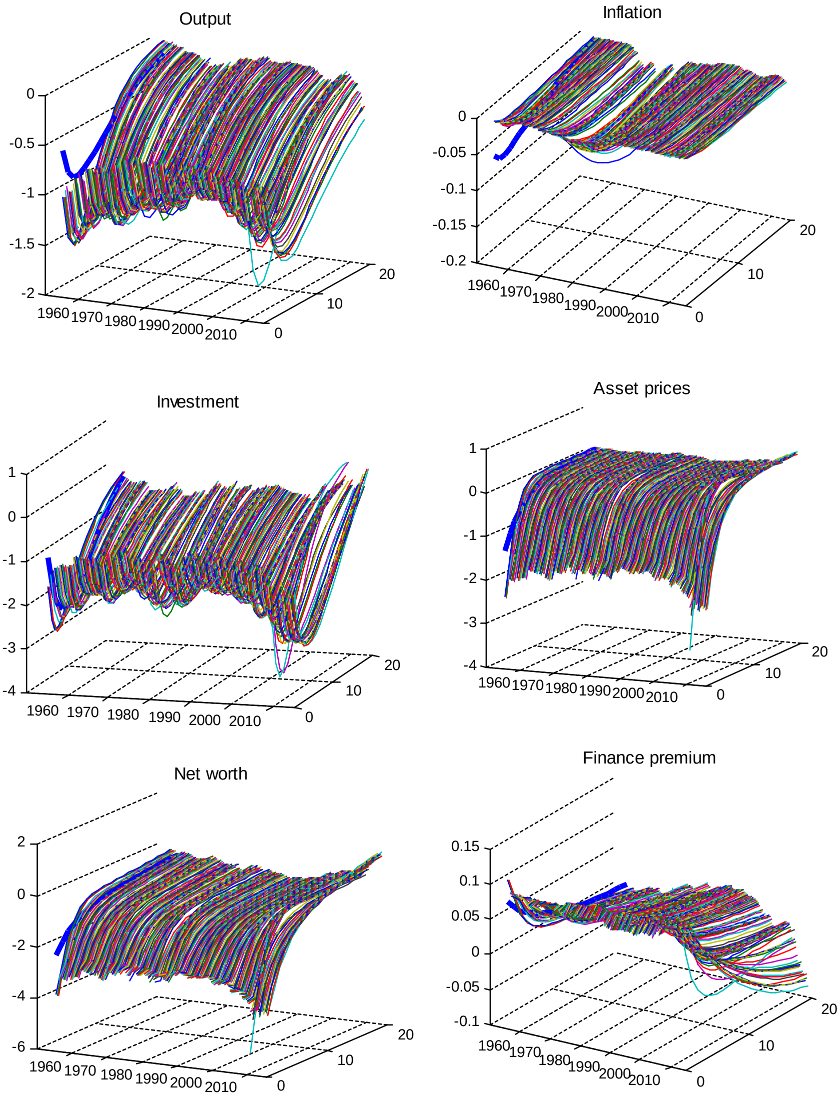


Figure 5: Impulse responses to the finance premium shock

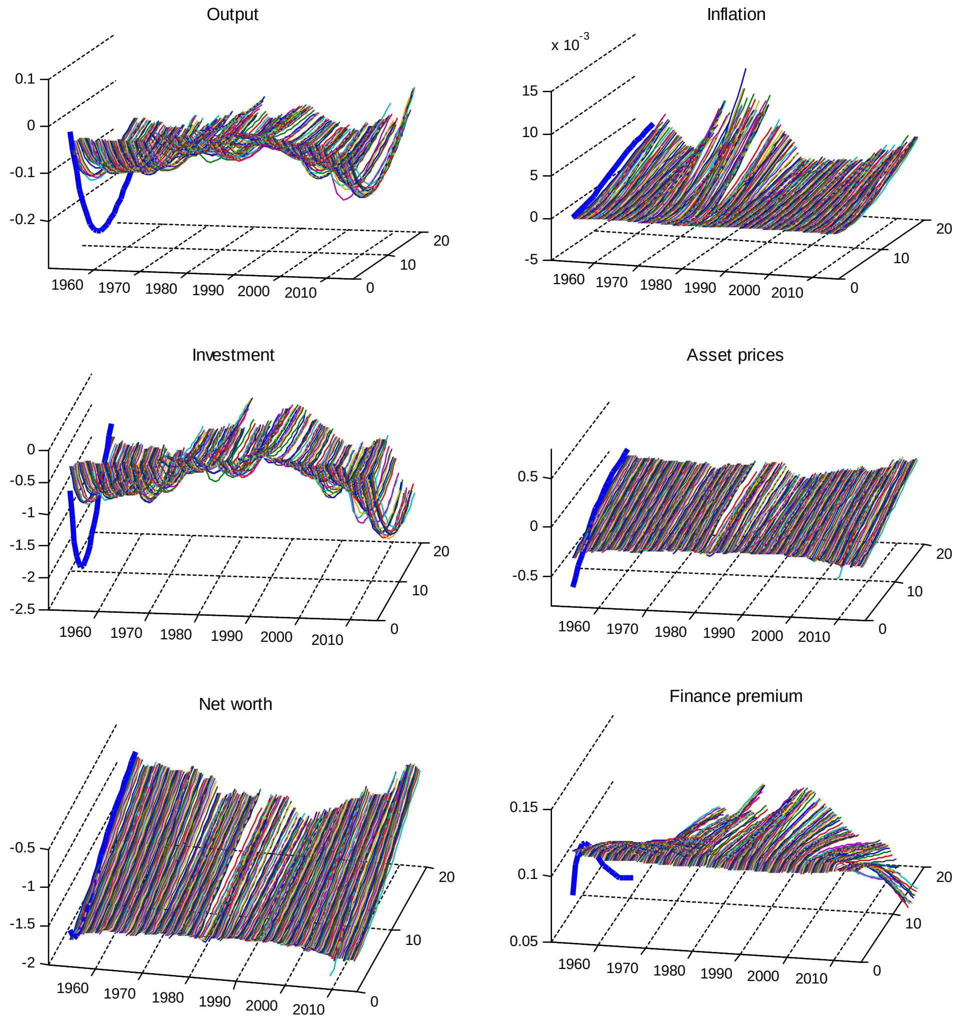


Figure 6: Impulse responses to the shock to the net worth

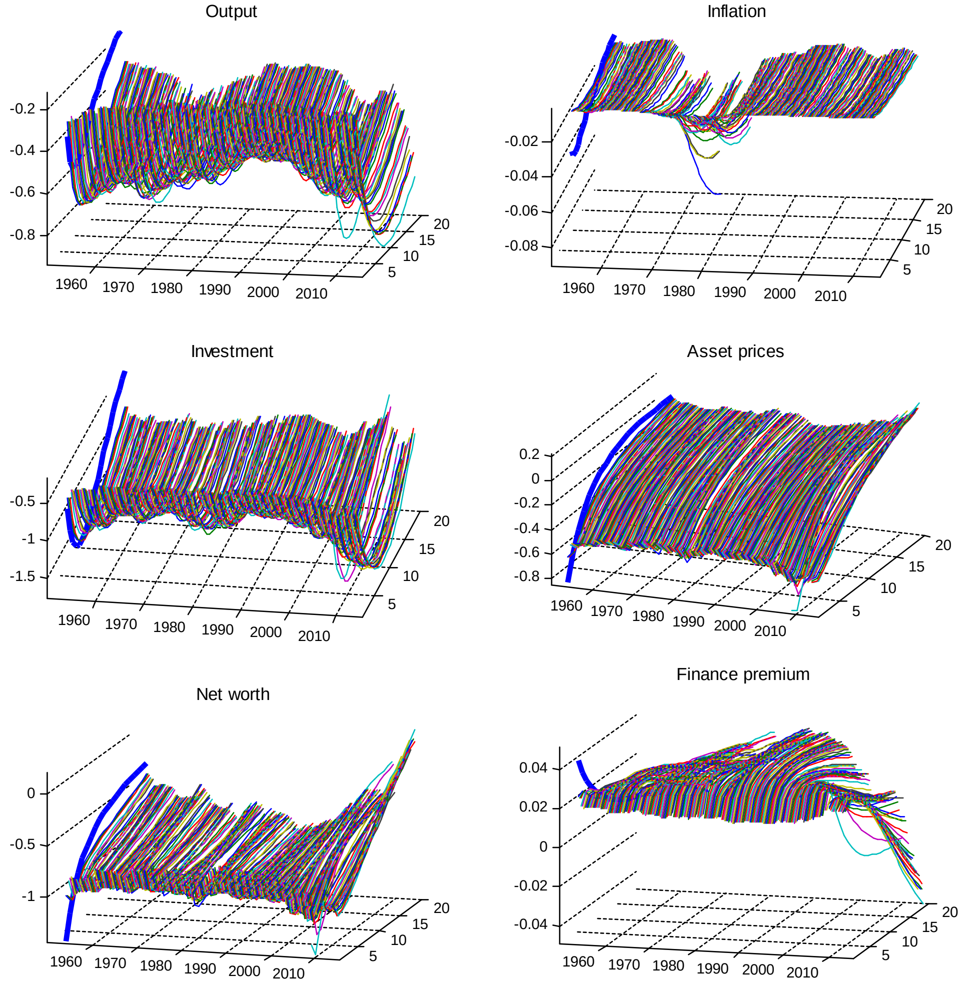


Figure 7: Impulse responses to the monetary policy shock

on financial markets as being important for explaining the business cycle. Exogenous investment shocks become less important in explaining the dynamics of investments under learning. Instead, financial shocks play more significant role. Endogenous developments in asset prices driven by adaptively formed expectations of the private agents become essential for the evolution of investment. The profile of output responses is also closely correlated with the reactions of asset prices. More persistent response on financial markets represents endogenous inertia which is not induced by structural parameters (financial friction parameters tend to decline under learning) or more persistent disturbances.

4.3 Contribution of structural shocks to business cycle fluctuations

In this section, we study the contribution of structural shocks to the forecast error variance of the main endogenous variables. We examine the relative importance of various macroeconomic and financial shocks in the model specifications estimated with and without financial data. The results, presented in Tables 4 and 4a, demonstrate that disturbances emanating from the financial sector (ε_b and ε_{nw}) play the prominent role in driving the fluctuations in real variables, thus explaining a significant portion of real contraction during the recessions. In particular, fluctuations in output are primarily shaped by productivity and finance premium shocks. The latter one accounts for about 42% of the total variation. Financial shocks become even more important for the dynamics of investment. Under learning, the role of financial shocks increases. In total, two financial shocks account for more than 50% of variation in output and investment growth. Finance premium shock becomes relatively more important under learning, whereas the role of the net worth shock in explaining the dynamics of real and financial variables diminishes.

Comparing the Tables 4 and 4a we can see that without financial variables, a significant portion of the real fluctuations in the RE model are explained by the government spending and investment specific technology shocks. At the same time, the role of the financial shocks is underestimated. Table 5 also demonstrates that introducing AL reduces the importance of the government spending and investment technology shocks and increases the role of financial shocks, even when financial variables are not explicitly included in the set of observables. Therefore, the dynamics of the model with RE appears to be more sensitive to the set of observable variables used in the estimation. The model with AL introduces endogenous variation in financial variables consistent with real data, which helps capturing the sources of business cycle fluctuations.

The price and wage markup shocks are the most significant determinants of the consumer price inflation. Productivity and demand shocks account for only about 6% of inflation volatility. Such a small relative contribution can be partially explained by the presence of structural rigidities, which makes the slope of the Phillips curve rather small. This implies that developments in marginal costs will have only limited impact on inflation unless these developments are very large and extremely persistent.

Table 4. Forecast Error Variance Decomposition.

RE model with financial data

Variables/shocks	ε_a	ε_b	ε_g	ε_q	ε_m	ε_p	ε_w	ε_{nw}
Output gr.	21.72	41.96	12.40	4.14	13.69	3.57	1.29	1.23
Investment gr.	1.32	35.58	1.18	23.44	11.91	1.72	0.79	24.05
Inflation	6.85	15.07	0.03	0.01	2.90	47.93	26.91	0.31
Interest rate	5.47	65.38	1.99	0.81	7.66	6.34	9.17	3.18
Asset prices	0.67	51.19	1.69	6.70	18.23	0.63	2.17	18.73
Net worth	0.29	29.43	0.65	2.65	10.44	0.66	1.86	54.01
Finance premium	2.16	10.65	0.94	5.29	4.28	3.69	3.64	69.34

AL model with financial data[22]

Variables/shocks	ε_a	ε_b	ε_g	ε_q	ε_m	ε_p	ε_w	ε_{nw}
Output gr.	15.00	↑ 54.56	9.56	1.23	17.14	0.67	0.23	1.61
Investment gr.	0.30	↑ 59.67	1.11	↓ 5.46	16.55	0.66	0.18	↓16.07
Inflation	3.86	9.42	1.13	0.04	21.62	48.79	↓12.64	2.51
Interest rate	2.17	71.32	5.31	0.49	11.66	0.37	1.43	7.25
Asset prices	0.05	↑ 74.04	0.78	1.28	14.16	0.49	0.30	↓8.90
Net worth	0.84	↑ 42.31	0.32	0.44	7.57	0.24	0.39	↓47.90
Finance premium	5.79	↑ 18.18	1.13	0.89	1.64	0.01	0.51	71.85

[22] Arrows indicate the direction of the change (increase or decrease) of the contribution of the particular shock relative to the model with RE.

Table 4a. Forecast Error Variance Decomposition.

RE model without financial data

Variables/shocks	ε_a	ε_b	ε_g	ε_q	ε_m	ε_p	ε_w
Output gr.	17.79	24.46	21.26	25.22	6.36	2.96	1.95
Investment gr.	1.57	19.29	1.33	73.55	3.56	0.51	0.20
Inflation	15.1	2.66	1.1	2.29	2.62	47.91	28.33
Interest rate	19.90	16.21	4.82	20.61	14.49	12.63	11.34
Asset prices	0.46	56.16	0.94	37.26	4.89	0.14	0.14
Net worth	0.26	78.45	0.59	15.95	4.44	0.13	0.18
Finance premium	4.85	72.03	0.95	16.72	1.51	3.15	0.79

AL model without financial data

Variables/shocks	ε_a	ε_b	ε_g	ε_q	ε_m	ε_p	ε_w
Output gr.	12.27	↑61.36	↓9.35	↓1.43	14.78	0.64	0.16
Investment gr.	0.19	↑74.23	1.42	↓7.95	↑15.43	0.63	0.15
Inflation	2.63	4.13	0.60	0.03	8.55	75.83	8.23
Interest rate	↓2.66	↑77.75	3.84	↓0.64	13.54	↓0.48	1.10
Asset prices	0.02	↑84.43	0.96	↓1.99	12.05	0.39	0.16
Net worth	1.42	↑83.94	0.68	↓1.21	11.58	0.34	0.73
Finance premium	27.0	55.84	4.69	↓5.99	3.65	0.58	2.21

4.4 Using survey data of inflation expectations: Implications for model fit and forecasting performance

The state of inflation expectations greatly influences actual inflation and thus the central bank's ability to achieve price stabilization. Therefore, policy makers closely monitor public's perceptions about the evolution of the price change and examine factors affecting the mechanism of expectation formation. Survey measures are often used to examine the inflation expectations of private agents. In the previous sections, we have demonstrated that, in macro models, the mechanism of expectation formation has important implications for the transmission mechanism and the model dynamics. In particular, we showed that AL can enchance the propagation mechanism of the DSGE model and generate persistence that is otherwise caused by either nominal frictions or by the dynamics of exogenous stochastic processes.

In this section, we add survey data as an additional observable variable and investigate whether a DSGE model with a financial accelerator can simulataniously fit macro, financial and survey data of inflation expectations. Using inflation expectations in the estimation imposes additional challenge for the model to fit the data since the model-based agents' beliefs should be consistent with the survey measure. Important questions which can be addressed in this context: what is the more suitable conceptual framework for thinking about inflation expectations? To what extent the chosen learning algorithm is consistent with the observed survey data? Does survey data contain additional information that is not present in the macro data alone?

As a measure of the inflation expectations, we use a quarterly expectations for the rates of inflation from the Survey of Professional Forecasters (SPF). The estimation sample starts in 1969q1[23]. We compare the estimates under alternative assumptions about the expectation formation (RE and AL) and investigate the forecasting performance of the models estimated with and without the survey data. In estimations using survey data, the set of the model equations is supplemented with the additional measurement equation for expectations:

$$dlP_{t,t+1}^{SPF} = \overline{\pi} + E_t\widehat{\pi}_{t+1} + \epsilon_t,$$

where $dlP_{t,t+1}^{SPF}$ is the log difference of SPF inflation expectations and ϵ_t is the AR(1) measurement error. Therefore, survey data is viewed as a noisy measure of the model-consistent inflation expectations.

When survey data is used as an observable in the estimations, it introduces additional cross-equation restrictions and affects the identification of the parameters in the model equations which contain the inflation expectation terms: price and wage Phillips curves, consumption Euler equation, financial accelerator equation (12), which determines the dynamics of the risk premium. Therefore, the use of survey data can impact the dynamics of inflation, wage as well as the real variabes (via financial accelerator mechanism). Table 5 contains posterior estimates of the parameters and the log marginal likelihood for the models with RE and AL, estimated with and without survey data. The results of the Table 5 demonstrate, that learning outperforms RE in estimation with and without survey data. Using survey data among the observables in the estimations with RE reduces the importance of such nominal frictions as wage and price indexation. At the same time, the calvo price, habit formation, adjustment costs as well as financial accelerator parameters slightly increase. In addition, the autoregressive coefficients of the price mark-up and the exogenous risk premium shocks increase. Therefore, we may conclude that, overall, there is a tendency for the structural rigidities and persistence of stochastic processes to raise when survey measure is included in the dataset. Such a result may indicate that survey expectations introduce more volatility into the model-based beliefs implied by the RE hypothesis. This generates additional dynamics into inflation, consumption

[23] The SPF data is available starting 1968q3.

and investment processes, which is possible to reconcile with real data only through the increase of the structural rigidities. For the models with AL, we can see that the pattern of the parameter change is similar, except for the elasticity of the external finance premium, which somewhat declines. Moreover, the autoregressive coefficients of the price mark-up and the exogenous risk premium shocks decline. The estimated AL gain parameter is slightly higher when survey data is included among the observables. This implies more dynamic beliefs' updating process. Therefore, it appears that the AL algorithm is more consistent with the actual expectations formation mechanism. Survey data contains useful information for determining the learning agents' forecasting models for inflation and investment. The outcomes of the point forecast accuracy exercise, presented in Tables 6 and 6a, confirm this result. In particular, we can see that the AL model which uses the survey data better predicts the dynamics of investment (as well as financial variables). SPF inflation expectations are also better predicted by the model with learning.

Table 5. Model comparison with and without survey data.

Parameters		Dataset with survey data				Dataset without survey data			
		Posterior, RE		Posterior, AL		Posterior, RE		Posterior, AL	
		Mode	St.d	Mode	St.d	Mode	St.d	Mode	St.d
Shocks									
exo.risk prem.	σ_b	0.303	0.037	0.748	0.087	0.373	0.056	0.784	0.088
net worth	σ_{nw}	0.363	0.154	0.046	0.018	0.394	0.159	0.047	0.02
investment	σ_q	0.551	0.046	0.429	0.031	0.552	0.046	0.411	0.034
price markup	σ_p	0.161	0.012	0.182	0.012	0.128	0.012	0.15	0.012
wage markup	σ_w	0.39	0.024	0.341	0.017	0.386	0.024	0.343	0.019
AR.coeff-s									
exo.risk prem.	ρ_b	0.916	0.022	0.716	0.05	0.867	0.036	0.724	0.045
net worth	ρ_{nw}	0.647	0.17	-	-	0.658	0.159	-	-
investment	ρ_q	0.116	0.079	0.468	0.058	0.12	0.081	0.446	0.055
price markup	ρ_p	0.967	0.011	0.22	0.101	0.958	0.02	0.384	0.114
wage markup	ρ_w	0.054	0.037	0.029	0.02	0.058	0.039	0.037	0.024
pr.markup,ma	μ_p	0.928	0.02	-	-	0.874	0.044	-	-
Str.params									
adjust. cost	φ	4.202	0.724	3.675	0.509	4.185	0.688	3.358	0.454
habit	η	0.532	0.079	0.602	0.027	0.514	0.085	0.564	0.033
Calvo wages	ξ_w	0.859	0.017	0.781	0.029	0.861	0.019	0.765	0.03
Calvo prices	ξ_p	0.797	0.026	0.798	0.029	0.744	0.034	0.782	0.03
index. wages	ι_w	0.614	0.121	0.515	0.132	0.737	0.104	0.609	0.141
index. prices	ι_p	0.116	0.046	0.222	0.096	0.284	0.094	0.165	0.089
int.rate smooth	ρ_r	0.832	0.02	0.865	0.02	0.827	0.021	0.883	0.02
pol rule, inflat.	r_π	1.662	0.139	1.472	0.165	1.70	0.139	1.464	0.18
cap./net worth	k/N	1.918	0.133	1.968	0.124	1.887	0.13	1.926	0.126
survival rate	$\varkappa$	0.963	0.013	0.938	0.009	0.958	0.013	0.924	0.011
elast.risk prem	el	0.09	0.012	0.069	0.007	0.09	0.013	0.075	0.008
gain (AL)	g			0.989	0.003			0.98	0.003
Log(ML)			-1545.9		-1523.1		-1607.8		-1580.6

Table 6. Point forecast accuracy. Estimation sample: 1969q1-2014q2.

Dataset with survey data. Forecast sample: 2004-2014						
	1Q		4Q		8Q	
	RE	AL	RE	AL	RE	AL
Output	0.602	0.595	0.767	0.709	0.737	0.699
Consumption	0.643	0.665	0.754	0.733	0.709	0.69
Investment	1.427	1.291	2.311	2.259	2.309	2.207
Labour	0.519	0.461	2.177	2.118	3.764	3.601
Inflation	0.195	0.201	0.256	0.261	0.307	0.301
Inflation expect.	0.11	0.09	0.152	0.136	0.201	0.177
Wage	1.048	1.074	1.064	1.076	1.06	1.068
Interest rate	0.114	0.113	0.359	0.409	0.604	0.73
Stock prices	6.929	6.924	7.316	7.206	7.351	7.386
Finance premium	0.317	0.303	0.651	0.601	0.699	0.662

Table 6a. Point forecast accuracy. Estimation sample: 1969q1-2014q2.

Dataset without survey data. Forecast sample: 2004-2014						
	1Q		4Q		8Q	
	RE	AL	RE	AL	RE	AL
Output	0.631	0.593	0.793	0.767	0.737	0.771
Consumption	0.667	0.658	0.766	0.786	0.696	0.759
Investment	1.479	1.347	2.369	2.464	2.321	2.363
Labour	0.576	0.453	2.401	2.191	4.017	3.931
Inflation	0.211	0.249	0.261	0.32	0.309	0.307
Wage	1.033	1.094	1.063	1.103	1.06	1.094
Interest rate	0.12	0.11	0.405	0.335	0.68	0.597
Stock prices	6.947	6.957	7.289	7.245	7.341	7.412
Finance premium	0.317	0.307	0.649	0.621	0.698	0.699

5 Conclusions

In this paper, we compare the implications of a FA mechanism for the real economy in models with alternative assumptions about expectation formation. We perform a Bayesian estimation of a medium-scale DSGE model with financial frictions assuming, on the one hand, complete rationality of expectations and, alternatively, adaptive learning algorithm based on small forecasting functions. We evaluate and compare the model fit, estimated parameters and the transmission mechanism as well as forecasting performance. The estimation results suggest that adaptively formed expectations add to improved model fit.

We show that the implications of a FA for the business cycle may vary depending on the expectation assumptions (RE or learning). The results suggest that a learning scheme based on small forecasting functions is able to amplify the effects of financial frictions relative to a model with RE. We show that the model dynamics under learning are driven to a significant extent by the time variation of agents' beliefs about the evolution of financial variables. Specifically, we demonstrate that perceived asset price persistence in a learning model with simple forecasting equations varies through the cycle and thus differs significantly from the levels implied by the RE. During periods when agents perceive asset prices as being relatively more persistent, shocks that affect this variable lead to more pronounced macro-economic outcomes. The asset price persistence appears to be particularly important for explaining investment dynamics. In particular, increased asset price persistence implies a more pronounced (and persistent) response of investment under the risk premium shocks. Therefore, we argue that learning may play a significant role in driving and amplifying macro-economic fluctuations; it introduces an important time variation and strengthens the real effects of the FA compared to the assumption of RE.

We also demonstrate that financial data helps in identifying structural financial shocks. In estimations perforemd without financial variables, the contribution of financial shocks is underestimated. Furtherore, we show that the model with learning outperforms the model with RE in forecasting inflation and real variables and is more successful in replicating the most recent economic downturn driven by severe financial shocks. Finally, we find that survey expectations are more consistent with the time varying mechansim of expectation formation implied by learning. Survey data contain useful information not present in the macro data alone and improve forecasting performance of the DSGE model.

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6 Model appendix

6.1 Households and Labor Markets

Household j chooses consumption, hours worked and savings so as to maximize a utility function, non-separable[24] in two arguments – a CES basket of consumption-good varieties and labor services:

$$E_t \sum_{s=0}^{\infty} \beta^s \left[\frac{1}{1-\sigma_c} (C_{t+s}(j) - \eta C_{t+s-1})^{1-\sigma_c} \right] \exp\left(\frac{\sigma_c - 1}{1+\sigma_l} L_{t+s}(j)^{1+\sigma_l} \right), \tag{27}$$

where σ_c and σ_l are preference parameters and η is an external habit-formation parameter, which introduces the dependence of the household consumption on the lagged aggregate

[24] A sensitivity check demonstrated that the use of the separable (in consumption and labor) form of the utility function, employed in Smets and Wouters (2003), does not significantly affect the estimation results or the conclusions of the paper.

consumption. Households can save by depositing funds in the bank and by buying government bonds. These assets (denoted, in total, as AT) are perfect substitutes and earn the same riskless nominal interest rate R^n. Households also obtain dividends from owing intermediate and capital goods producers as well as from labor unions. Therefore, the budget constraint of the representative household takes the form:

$$C_{t+s}(j) + \frac{AT_{t+s}(j)}{\varepsilon_t^b R_{t+s}^n P_{t+s}} - T_{t+s} = \frac{W_{t+s}(j)L_{t+s}(j)}{P_{t+s}} + \frac{AT_{t+s-1}(j)}{P_{t+s}} + \frac{Div_{t+s}}{P_{t+s}}, \tag{28}$$

where c_t is the exogenous premium on the bonds' return, W_{t+s}^h is the nominal wage, T_{t+s} are lump-sum taxes or subsidies and Div_{t+s} are dividend payments.

The first-order conditions with respect to consumption and assets result in the Euler equation, which after model detrending and log-linearization takes the following form:

$$\begin{aligned}\widehat{c}_t = & \frac{1}{(1+(\eta/\gamma))} E_t\left[\widehat{c}_{t+1}\right] + \frac{(\eta/\gamma)}{(1+(\eta/\gamma))}\widehat{c}_{t-1} \\ & - \frac{(1-\eta/\gamma)}{\sigma_c(1+(\eta/\gamma))}(\widehat{b}_t + \widehat{R}_t^n - E_t[\widehat{\pi}_{t+1}]) - \frac{(\sigma_c - 1)(w_*^h L/c_*)}{\sigma_c(1+(\eta/\gamma))}(E_t\left[\widehat{L}_{t+1}\right] - \widehat{L}_t).\end{aligned} \tag{29}$$

The backward-looking term arises in the consumption equation due to the assumptions of external habit formation captured by the parameter η. Therefore, current consumption ($\widehat{c}_t$) depends on a weighted average of past and expected future consumption. The consumption process is also affected by the expected growth in hours worked ($E_t\left[\widehat{L}_{t+1}\right] - \widehat{L}_t$) (due to the non-separable in consumption and labor form of the utility function), the ex–ante real interest rate ($\widehat{R}_t^n - E_t[\widehat{\pi}_{t+1}]$) and a disturbance term $\widehat{b}_t$. γ is the deterministic trend, which arises as a result of model detrending[25]. $\widehat{b}_t$ is assumed to follow a first–order autoregressive process with an iid–normal error term: $\widehat{b}_t = \rho_b \widehat{b}_{t-1} + \epsilon_t^b$. Variables with stars denote the steady-state values.

As in Smets and Wouters (2007), labor markets consist of labor unions, who allocate and differentiate labor supplied by households, and labor packers, who buy labor from the unions, package it into a Kimball (1995) composite aggregator L_t that is resold to intermediate goods producers. Unions have market power over labor services and set wages that are subject to nominal rigidities along the lines of Calvo (1983). Every period only a $(1-\xi_w)$ fraction of intermediate labor unions can readjust wages. The chosen wage rate set by the union maximizes the stream of future (discounted) wage income for all the time periods when the union is stuck with that wage in the future. The first-order conditions to problems (27) and (28) with respect to hours worked combined with the solution to the profit-maximization problem of the intermediate labor union and the law

[25]Detrended real variables are obtained by dividing the nominal variables by a deterministic trend: $c_t = C_t/\gamma^t$, $w_t = W_t/(\gamma^t P_t)$ etc.

of motion of the aggregate wage result in the following wage equation:

$$\begin{aligned}\widehat{w}_t \ &= \frac{1}{(1+\overline{\beta}\gamma)}(\widehat{w}_{t-1}+\overline{\beta}\gamma E_t\left[\widehat{w}_{t+1}\right]-(1+\overline{\beta}\gamma\iota_w)\widehat{\pi}_t+\iota_w\widehat{\pi}_{t-1}+\overline{\beta}\gamma E_t\left[\widehat{\pi}_{t+1}\right] \\ &+\frac{(1-\xi_w\overline{\beta}\gamma)(1-\xi_w)}{\xi_w((\phi_w-1)\varepsilon_w+1)}[\frac{1}{1-\eta/\gamma}\widehat{c}_t-\frac{\eta/\gamma}{1-\eta/\gamma}\widehat{c}_{t-1}\ +\sigma_l\widehat{L}_t-\widehat{w}_t\ ]+\widehat{\lambda_{w,t}}\ ,\end{aligned} \tag{30}$$

where $\overline{\beta} = \beta/\gamma^{\sigma_c}$ and β is a discount factor applied to households. Due to nominal wage stickiness and the partial indexation of wages to inflation, real wages adjust only gradually to the desired wage mark–up. ξ_w is a wage stickiness parameter. Parameter ι_w measures the degree of indexation. If wages are perfectly flexible ($\xi_w = 0$), the real wage is a constant mark–up over the marginal rate of substitution between consumption and leisure. When wage indexation is zero (ι_w), real wages do not depend on lagged inflation. In addition to wage stickiness, the speed of adjustment to the desired mark-up depends on the demand elasticity for labor, which is a function of the steady-state labor market mark-up ($\phi_w - 1$) and the curvature of the Kimball labor-market aggregator ε_w. The wage–mark up disturbance ($\widehat{\lambda}_{w,t}$) is assumed to follow an ARMA (1,1) process with an iid–normal error term: $\widehat{\lambda}_{w,t} = \rho_w\widehat{\lambda}_{w,t-1} - \mu_w\epsilon_{w,t-1} + \epsilon_t^w$.

6.2 Production sector: Firms

The production sector consists of final- and intermediate -goods producers. Final -goods producers buy intermediate goods $Y_t(i)$, aggregate them into a composite final good Y_t and resell to consumers in a perfectly competitive market. The solution to the profit-maximization problem of these firms is standard and determines the demand function for intermediate inputs $Y_t(i)$. Intermediate -goods producers, who operate under monopolistic competition, rent capital from entrepreneurs at the rate R_t^k, hire labor from labor packers and use a typical Cobb-Douglas production function augmented with fixed costs:

$$Y_t(i) = \varepsilon_t^a K_t^S(i)^\alpha \left[\gamma^t L_t(i)\right]^{1-\alpha} - \gamma^t\Phi, \tag{31}$$

where $K_t^S(i)$ is capital services used in production, $L_t(i)$ is aggregate labor input, α is the share of capital in production and Φ is a fixed cost. γ^t respresents the labor-augmenting deterministic growth rate in the economy and ε_t^a is total factor productivity. The log-linearized aggregate supply equation (31) takes the form:

$$\widehat{y}_t \ = \Phi(\ \alpha(\widehat{k^S}_t)\ +\ (1-\alpha)\widehat{L}_t + \widehat{A}_t), \tag{32}$$

where the total factor productivity ($\widehat{A}_t$) is assumed to follow a first-order autoregressive process: $\widehat{A}_t \ = \rho_a\widehat{A}_{t-1} + \epsilon_t^a$. The solution to the cost-minimization problem yields the conditions that determine the labor demand function in the following log-linear form:

$$\widehat{L}_t = \widehat{k^S}_t \ - \ \widehat{w}_t \ + \ \widehat{r}_t^k. \tag{33}$$

Equation (33) implies that the rental rate of capital is negatively related to the capital-labor ratio and positively to the real wage (both with unitary elasticity). The marginal cost is the same for all firms and represented by the following relation:

$$\widehat{mc}_t \ = \ (1-\alpha)\ \widehat{w}_t + \alpha\ \widehat{r}_t^k - \widehat{A}_t. \tag{34}$$

Similar to wages, in each period, only a fraction of firms $(1-\xi_p)$ can re-optimize prices. In the environment of price rigidities, the optimal price will maximize the expected discounted stream of future profits for the firm for all states of nature when the firm cannot reset the price optimally. Thus, the current inflation rate will depend on current *and* future expected marginal costs. Non-reoptimized prices are partially index-linked to past inflation, which gives rise to the backward-looking term in the inflation equation. Profit maximization by price–setting intermediate firms gives rise to the following New-Keynesian Phillips curve:

$$\widehat{\pi}_t = \frac{1}{(1+\overline{\beta}\gamma\iota_p)}(\iota_p\widehat{\pi}_{t-1} + \overline{\beta}\gamma E_t\left[\widehat{\pi}_{t+1}\right] + \frac{1}{((\phi_p-1)\varepsilon_p+1)}\frac{(1-\xi_p\overline{\beta}\gamma)(1-\xi_p)}{\xi_p}(\widehat{mc}_t)) + \widehat{\lambda_{p,t}}, \tag{35}$$

where ι_p denotes the indexation coefficient. The inflation equation demonstrates that the speed of adjustment to the desired mark-up depends on the degree of price stickiness ξ_p, the curvature of the Kimball goods market aggregator ε_p and the steady state mark-up $(\phi_p - 1)$. The price mark–up disturbance $(\widehat{\lambda}_{p,t})$ is assumed to follow an ARMA(1,1) process: $\widehat{\lambda}_{p,t} = \rho_p\widehat{\lambda}_{p,t-1} - \mu_p\epsilon_{p,t-1} + \epsilon_t^p$, where ϵ_t^p is an iid–Normal price mark–up shock.