On the sources of aggregate persistence in an estimated DSGE model with real-time learning

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Highlights

- There is a lack of transmission of financial market shocks to the macroeconomy under standard rational expectations (RE) models
- We suggest a transmission channel: Agents can learn from financial markets to predict macroeconomic outcomes and learning dynamics (and thus financial markets) can feed back into both the macroeconomy and financial markets
- We build a medium-scale DSGE model extended with the term structure of interest rates and adaptive learning (AL), which results in a model of multi-period forecasting inducing high persistence
- Our extension allows the term spread of interest rates to fully characterize the forward-looking variables of the model in real time

Highlights

- The estimated model shows that multi-period forecasting featured by term structure dynamics is an important source of persistent fluctuations under AL
- Our estimation results show that the importance of most endogenous sources of aggregate persistence decline dramatically when multi-period forecasting is incorporated
- Model expectations based on term structure information provides a sound characterization of the consumption growth and inflation forecasts reported in the SPF

There is growing literature analyzing the consequences of deviating from the standard assumption of RE. There are many approaches:

- Rational inattention approach (Sims, 2003; Adam, 2007; Mackowiach and Wiederholt, 2009; ...)
- Sticky information approach (Reis, 2009; ...)
- Imperfect information approach (Svensson and Woodford, 2004; Coenen, Levin and Wieland, 2005; ...)
- Limited information and real-time data (Aruoba, 2004; Pruitt, 2012; Vázquez, María-Dolores and Londoño, 2013; Casares and Vázquez, 2016;...)
- **Adaptive learning (AL) approach** (Orphanides and Williams, 2005; Branch and Evans, 2006; Milani, 2007, 2008, 2011; Eusepi and Preston, 2011; Levine, Pearlman, Perendia and Yang, 2012; Slobodyan and Wouters, 2012a, 2012b; Ormeño and Molnár, 2015;...)

The set of equations agents use to form their expectations are called the "Perceived law of motion". There are two types:

- "Minimum state variable" (MSV): expectations are a (linear) function of state-variables realizations (Milani, 2007, 2008, 2011)
- "Euler equation learning": agents use a small model to make their predictions based on observable endogenous variables (Evans and Honkapohja, 2001; Slobodyan and Wouters, 2012a, 2012b)

Eusepi and Preston (2011) show that AL based on forecasts beyond the one-period-ahead forecasts (i.e. considering *maintained beliefs over multiple horizons*) are crucial for understanding the persistence of macroeconomic dynamics: They show that the maintained beliefs hypothesis results in a huge amplification mechanism of technology shocks in a standard RBC

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We focused our attention on the AL approach followed by Slobodyan and Wouters (2012a,b) for two main reasons:

- They use Smets and Wouters (2007) model: a medium-scale DSGE model introducing many shocks and frictions
- They consider small forecasting models based on observables instead of using a MSV approach

However, we deviate from Slobodyan and Wouters (2012a,b) in two important directions:

- We consider multi-period forecasting as in Eusepi and Preston (2011) but determined by term structure dynamics
- We use small forecasting models based only on term structure information which is observed in real time

Estimated AL models *unrealistically use revised data* to characterize agents' expectations. Our paper overcomes this drawback by analyzing the potential role of the term structure of interest rates, which is observed in real time, in the learning process of economic agents

The rationale of this assumption is based on a large empirical literature -among others, Fama (1990), Mishkin (1991), McCallum (1994), Estrella and Mishkin (1997) and Ang, Piazzesi and Wei (2006)- showing evidence of the ability of the term spread to predict the future evolution of both inflation and economic activity. Indeed, in our model they are forward-looking variables (i.e. they are determined by the expected paths of macroeconomic variables)

This paper combines four building blocks: (i) medium-scale DSGE, (ii) AL, (iii) maintained beliefs incorporated through the term structure, and (iv) the SPF to discipline AL expectations; in a unified framework to contribute to the literature by

- Uncovering a transmission channel through which financial markets and learning dynamics can feed back into the macroeconomy
- Studying the contribution of term structure information in the learning processes of agents and the maintained beliefs hypothesis in order to reproduce important US business cycle features as well as forecast dynamics featured in the SPF

Agents:

- Households derive utility from their consumption relative to their habit and supply differentiated labor in monopolistic competition setting "Calvo-sticky" wages.
- Intermediate firms produce differentiated goods using labor and capital (subject to adjustment costs) in monopolistic competition and they set "Calvo-sticky" prices.
- The final good is produced using intermediate goods by firms under perfect competition
- The monetary authority follows a Taylor-type rule

We follow the work of de Graeve, Emiris and Wouters (2009) and Vázquez, María-Dolores and Londoño (2013): we extend the DSGE model by explicitly considering the interest rates associated with alternative bond maturities indexed by j (i.e. $j = 1, 2, ..., n$). From the first-order conditions characterizing the optimal decisions of the representative consumer, one can obtain the standard consumption-based asset pricing equations associated with each maturity:

$$E_t \left[\beta^j \frac{U_C(C_{t+j}, L_{t+j}) \left(exp(\xi_t^{\{j\}})(1 + R_t^{\{j\}}) \right)^j}{U_C(C_t, L_t) \prod_{k=1}^{j} (1 + \pi_{t+k})} \right] = 1, \ \ for\ j = 1,\ 2,\ ...$$

Assuming that utility function is logarithmic in consumption, after some algebra, the (linearized) consumption-based asset pricing equations can be written as

$$\left(\frac{1}{1-\frac{h}{\bar{\gamma}}}\right) c_t - \left(\frac{\frac{h}{\bar{\gamma}}}{1-\frac{h}{\bar{\gamma}}}\right) c_{t-1} =$$

$$E_t\left[\left(\frac{1}{1-\frac{h}{\bar{\gamma}}}\right) c_{t+j} - \left(\frac{\frac{h}{\bar{\gamma}}}{1-\frac{h}{\bar{\gamma}}}\right) c_{t+j-1}\right] - \left[j r_t^{\{j\}} - E_t \sum_{k=1}^{j} \pi_{t+k} + \xi_t^{\{j\}}\right], \tag{1}$$

- Agents behave as econometricians under AL: they use a linear projection scheme in which the parameters are updated to form their expectations. The forecasting model or the perceived law of motion (PLM) process is defined as follows:

$$E_t y_{t+j} = X_t \beta_{t-1},$$

where y_t is the vector containing the k forward-looking variables of the model, X_t is the matrix of the $k \times n$ regressors and β_t is the vector of the n updating parameters (it includes an intercept)

- β_t is further assumed to follow an AR(1) process around $\overline{\beta}$, where agents' beliefs are updated through a Kalman filter:

$$\beta_t - \overline{\beta} = F(\beta_{t-1} - \overline{\beta}) + v_t,$$

where F is a diagonal matrix with the learning parameter $\mid \rho \mid \leq 1$ on the main diagonal and v_t are i.i.d. errors

Slobodyan and Wouters (2012a,b) used the following PLM:

$$E_t y_{t+1} = \theta_{y,t-1} + \beta_{y,t} y_t + \beta_{y,t-1} y_{t-1}.$$

Alternatively, we consider a single term spread

$$\left\{ \begin{array}{c} E_t y_{t+1} = \theta_{y,t-1} + \beta_{y,t-1} sp_{t-1}^{\{2\}}, \quad \text{for } y = i,\ r^k,\ q,\ w \\ E_t y_{t+j} = \theta_{y,t-1}^{\{j\}} + \beta_{y,t-1}^{\{j\}} sp_{t-1}^{\{2\}}, \quad \text{for } y = c,\ \pi \text{ and } j = 0,\ 1,\ 2,\ 3 \\ E_t y_{t+j} = \theta_{y,t-1}^{\{4\}} + \beta_{y,t-1}^{\{4\}} sp_{t-1}^{\{4\}}, \quad \text{for } y = c,\ \pi \text{ and } j = 4 \end{array} \right. \tag{2}$$

We use only the short-end of the term structure information: we decided to focus on a small (parsimonious) forecasting model based only on information observed in real time

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Sample period: 1984:1-2007:4

$$X_t = \begin{bmatrix} dlGDP_t \\ dlCONS_t \\ dlINV_t \\ dlWAG_t \\ dlP_t \\ lHours_t \\ FEDFUNDS_t \\ 1-year\ yield_t \\ dlCONS_t^{e\{j\}} \\ dlP_t^{e\{j\}} \end{bmatrix} = \begin{bmatrix} \bar{\gamma} \\ \bar{\gamma} \\ \bar{\gamma} \\ \bar{\gamma} \\ \bar{\pi} \\ \bar{l} \\ \bar{r} \\ \bar{r}^{\{4\}} \\ \bar{\gamma} \\ \bar{\pi} \end{bmatrix} + \begin{bmatrix} y_t - y_{t-1} \\ c_t - c_{t-1} \\ i_t - i_{t-1} \\ w_t - w_{t-1} \\ \pi_t \\ l_t \\ r_t \\ r_t^{\{4\}} \\ E_t\left(c_{t+j} - c_{t+j-1}\right) + \epsilon_{c,t}^{\{j\}} \\ E_t\pi_{t+j} + \epsilon_{\pi,t}^{\{j\}} \end{bmatrix},$$

Table 2. Selected parameter estimates

	AL model		RE model	
	Mean	5%-95% CI	Mean	5%-95% CI
h: habit formation	0.31	(0.21,0.44)	0.92	(0.91,0.93)
φ: cost of adjusting capital	1.02	(0.69,1.37)	8.88	(8.46,9.50)
ψ: capital utilization adjusting cost	0.22	(0.14,0.29)	0.37	(0.31,0.43)
ξ_p: price Calvo probability	0.58	(0.51,0.66)	0.94	(0.93,0.95)
ξ_w: wage Calvo probability	0.60	(0.53,0.67)	0.75	(0.70,0.81)
ι_p: price indexation	0.85	(0.73,0.95)	0.11	(0.09,0.13)
ι_w: wage indexation	0.56	(0.39,0.77)	0.21	(0.15,0.27)
ρ_p: persistence of price markup shock	0.67	(0.41,0.91)	0.997	(0.994,0.999)
ρ_w: persistence of wage markup shock	0.94	(0.91,0.97)	0.83	(0.79,0.89)
log data density	216.70		186.20	

Figure 1. PLM of inflation and consumption expectations

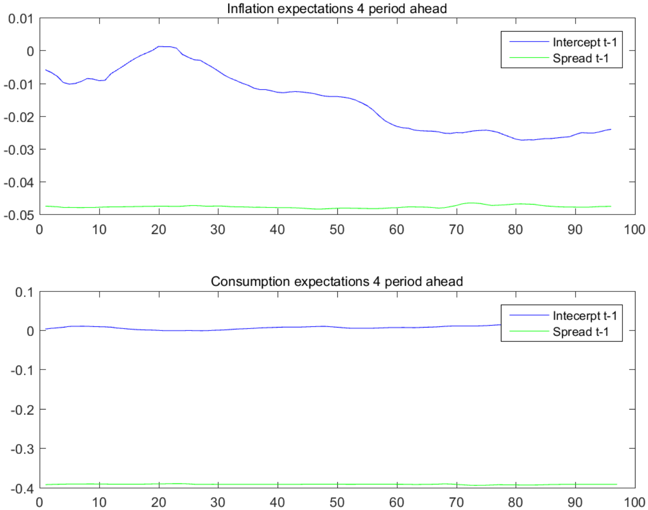


Figure 2. Impulse responses to a term-spread innovation

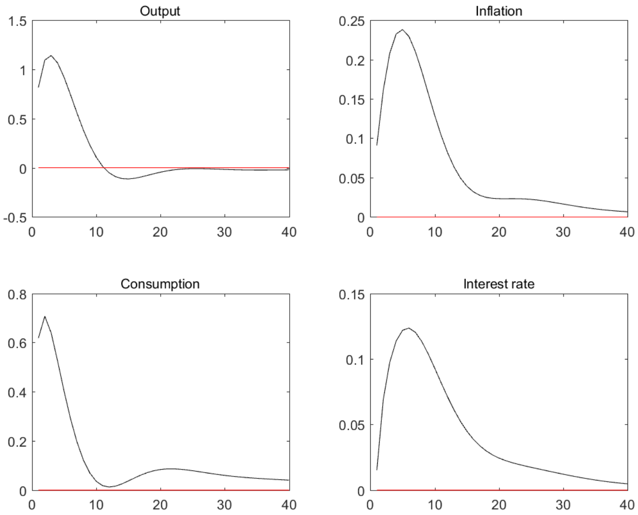


Figure 3. Impulse responses of output and inflation

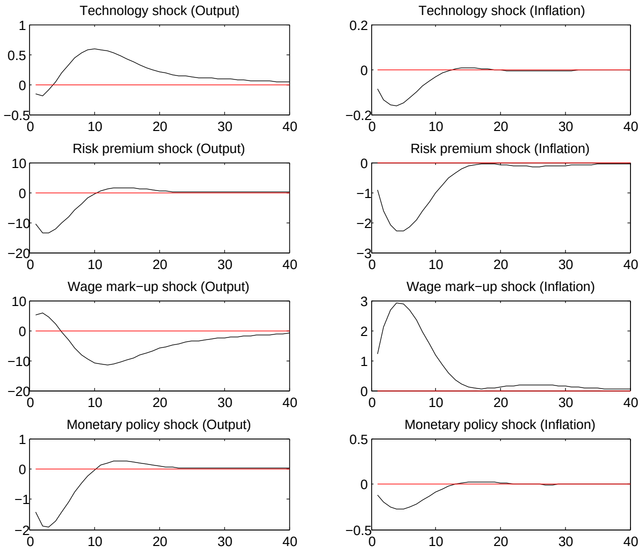
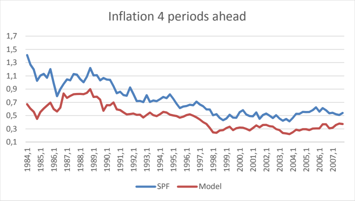


Figure 4. Model's expectations versus SPF's forecasts on inflation and consumption



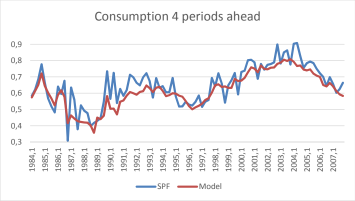


Table 2. Actual and simulated second moments

Actual data	Δc	Δinv	Δw	Δy	π
Standard deviation	0.51	1.68	0.62	0.54	0.24
Correlation with π	-0.30	-0.28	-0.29	-0.29	1
Autocorrelation	0.05	0.51	0.22	0.21	0.48

Simulated data	Δc	Δinv	Δw	Δy	π
Standard deviation	0.53	1.61	0.63	0.70	0.26
Correlation with π	-0.29	-0.26	-0.10	-0.30	1.0
Autocorrelation	0.26	0.70	0.57	0.48	0.97

Table 3. Descriptive statistics of inflation and consumption growth

Mean	π	π^e_{t+4}	Δc	Δc^e_{t+4}
Data/SPF forecasts	0.63	0.73	0.57	0.65
Model	0.70	0.53	0.53	0.52
Standard deviation	π	π^e_{t+4}	Δc	Δc^e_{t+4}
Data/SPF forecasts	0.24	0.24	0.51	0.12
Model	0.26	0.16	0.53	0.10
Autocorrelation	π	π^e_{t+4}	Δc	Δc^e_{t+4}
Data/SPF forecast	0.48	0.96	0.05	0.70
Model	0.96	0.96	0.26	0.78

Table 4. Parameter estimates with (***and without***) SPF forecasts

	AL model		RE model	
	Mean	5%-95% CI	Mean	5%-95% CI
h: habit formation	0.31	(0.21,0.44)	0.92	(0.91,0.93)
	0.35	***(0.30,0.40)***	***0.70***	***(0.63,0.77)***
φ: cost of adjusting capital	1.02	(0.69,1.37)	8.88	(8.46,9.50)
	2.34	***(2.19,2.49)***	***6.13***	***(4.38,7.85)***
ψ: capital utilization adjusting cost	0.22	(0.14,0.29)	0.37	(0.31,0.43)
	0.21	***(0.14,0.28)***	***0.78***	***(0.65,0.91)***
ξ_p: price Calvo probability	0.58	(0.51,0.66)	0.94	(0.93,0.95)
	0.62	***(0.54,0.69)***	***0.69***	***(0.63,0.76)***
ξ_w: wage Calvo probability	0.60	(0.53,0.67)	0.75	(0.70,0.81)
	0.60	***(0.52,0.69)***	***0.52***	***(0.43,0.62)***

Table 4. (Continued)

	AL model		RE model	
	Mean	5%-95% CI	Mean	5%-95% CI
ι_p: price indexation	0.85	(0.73,0.95)	0.11	(0.09,0.13)
	0.46	***(0.28,0.62)***	***0.13***	***(0.05,0.22)***
ι_w: wage indexation	0.56	(0.39,0.77)	0.21	(0.15,0.27)
	0.19	***(0.07,0.35)***	***0.45***	***(0.21,0.69)***
ρ_p: persistence of price markup shock	0.67	(0.41,0.91)	0.997	(0.994,0.999)
	0.83	***(0.73,0.93)***	***0.94***	***(0.89,0.99)***
ρ_w: persistence of wage markup shock	0.94	(0.91,0.97)	0.83	(0.79,0.89)
	0.96	***(0.93,0.99)***	***0.97***	***(0.96,0.99)***
log data density	216.70		186.20	
	-474.92		***-375.77***	
log data density difference	691.62		561.97	

- Term structure of interest rates is incorporated in a DSGE model with AL
- We show that the term spread innovations and the maintained beliefs hypothesis are two important sources of aggregate persistence under AL
- Our estimation results show that the importance of most endogenous sources of aggregate persistence decline dramatically when multi-period forecasting is incorporated
- Model expectations based on term structure information provides a sound characterization of the consumption growth and inflation forecasts reported in the SPF
- It does a good job when reproducing U.S. business cycle features

When considering longer maturity bonds we end up having a curse of dimensionality problem: there are many more expectational parameters to be identified in the PLM (2) with just a few more observables. We address this issue by defining the following two simple recursive rules:

$$\begin{cases} E_t c_{t+j} = \beta^c E_t c_{t+j-1}, & \textit{for } j > 4 \\ E_t \pi_{t+j} = \beta^\pi E_t \pi_{t+j-1}, & \textit{for } j > 4 \end{cases} \tag{3}$$

Yield curve extension

Table 5. Model fit using additional bond yields as observables

	1-year	+ 3-year	+ 5-year	+ 7-year	+ 10-year
AL model	216.70	221.07	273.68	512.11	727.80
RE model	186.20	126.38	328.04	375.77	235.29
Difference	30.50	94.69	-54.36	136.34	492.51

Table 6. Parameter estimates up to 1-year versus ***10-year*** yield

	AL model		RE model	
	Mean	5%-95% CI	Mean	5%-95% CI
h: habit formation	0.31	(0.21,0.44)	0.92	(0.91,0.93)
	0.37	***(0.33,0.41)***	***0.85***	***(0.82,0.87)***
φ: cost of adjusting capital	1.02	(0.69,1.37)	8.88	(8.46,9.50)
	1.19	***(1.01,1.44)***	***7.60***	***(6.14,9.20)***
ψ: capital utilization adjusting cost	0.22	(0.14,0.29)	0.37	(0.31,0.43)
	0.01	***(0.00,0.01)***	***0.81***	***(0.68,0.91)***
ξ_p: price Calvo probability	0.58	(0.51,0.66)	0.94	(0.93,0.95)
	0.56	***(0.53,0.59)***	***0.92***	***(0.90,0.94)***
ξ_w: wage Calvo probability	0.60	(0.53,0.67)	0.75	(0.70,0.81)
	0.57	***(0.52,0.63)***	***0.88***	***(0.82,0.92)***

Table 6. (Continued)

	AL model		RE model	
	Mean	5%-95% CI	Mean	5%-95% CI
ι_p: price indexation	0.85	(0.73,0.95)	0.11	(0.09,0.13)
	0.26	***(0.17,0.31)***	***0.07***	***(0.03,0.12)***
ι_w: wage indexation	0.56	(0.39,0.77)	0.21	(0.15,0.27)
	0.43	***(0.34,0.52)***	***0.28***	***(0.09,0.48)***
ρ_p: persistence of price markup shock	0.67	(0.41,0.91)	0.997	(0.994,0.999)
	0.95	***(0.92,0.99)***	***0.03***	***(0.00,0.06)***
ρ_w: persistence of wage markup shock	0.94	(0.91,0.97)	0.83	(0.79,0.89)
	0.99	***(0.98,0.99)***	***0.63***	***(0.34,0.99)***
log data density	216.70		186.20	
	727.80		***235.29***	
log data density difference	511.10		49.09	