The Science of Monetary Policy: An Imperfect Knowledge Perspective

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Motivation

- Monetary policy under rational expectations engendered a substantial quantitative literature. Quantitative models used to evaluate
  - Welfare implications of different instrument rules
  - Robustness of different instrument rules to uncertainty about the transmission mechanism of monetary policy
    - Bryant, Hooper and Mann (1993), Taylor (1993, 1999), ...
- Less emphasis given to target criteria — despite being a better description of actual central banking practice
- Relatively little serious quantitive analysis of such issues under learning dynamics

The Task

- Review what theory tells us about optimal policy under learning dynamics

- Proceed in two steps
  - Policies optimal from the perspective of rational expectations
    - Robustness: appropriate use of forecasts in monetary policy
    - How should optimal policy condition on these internal/external forecasts
  - Characterize fully optimal policy under non-rational expectations

- What characteristics should be evaluated quantitatively in any future research program

The Intellectual Framework

- Microfoundations: New Keynesian model
- Will insist on the following properties
  - Agents are optimizing: anticipated utility solution
  - Beliefs formed using some filtering algorithm
    - Not much hinges on this
  - Zero lower bound on interest rates not a relevant constraint

The New Keynesian Model

- Under arbitrary beliefs optimal decisions imply

$$
\begin{aligned}
x_t &= \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \left[(1-\beta)\, x_{T+1} - \sigma^{-1} \left(i_T - \pi_{T+1} - r_T^n \right) \right] \\
\pi_t &= \hat{E}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} \left[(1-\alpha)\, \beta \pi_{T+1} + \kappa w_T + u_T \right]
\end{aligned}
$$

where

$$\kappa = \frac{(1-\alpha\beta)(1-\alpha)}{\alpha}$$

  - Introduce cost-push shock — inefficient variations in marginal costs
  - Assume exogenous disturbances $\{r_t^n, u_t\}$ are iid

The Policy Problem

- The policymaker minimizes the loss function

$$\mathbb{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \left(\pi_T^2 + \lambda_x x_T^2 \right) \tag{1}$$

  where $\lambda_x \equiv \kappa/\theta > 0$ determines the relative weight given to output gap versus inflation stabilization

  - This welfare-theoretic loss function represents a second-order approximation to household utility

  - Feasible sequences of inflation and the output gap must satisfy the aggregate demand and supply equations

The Policy Problem II

- Under rational expectations the optimal commitment solution requires satisfaction of the first-order conditions

$$
\begin{aligned}
\frac{\lambda_x}{\kappa} x_t + \Lambda_t + \sum_{j=1}^{\infty} \alpha^j \Lambda_{t-j} &= 0 \\
\pi_t + \Lambda_t - \frac{(1-\alpha)}{\alpha} \sum_{j=1}^{\infty} \alpha^j \Lambda_{t-j} &= 0
\end{aligned}
$$

  for all t where Λ_t is the Lagrange multiplier attached to the aggregate supply equation

  - Assume optimality from the timeless perspective
  - Feasible sequences of inflation and the output gap therefore satisfy

$$
\pi_t = -\frac{\lambda_x}{\kappa} (x_t - x_{t-1})
$$

Dynamics under Optimal Policy

- Optimal state-contingent paths given by

$$\pi_t = (1-\mu)\frac{\lambda_x}{\kappa}x_{t-1} + \mu u_t$$

$$x_t = \mu x_{t-1} - \frac{\kappa}{\lambda_x}\mu u_t$$

and

$$i_t = \frac{\lambda_x - \sigma\kappa}{\kappa}(1-\mu)\mu x_{t-1} + \frac{\lambda_x - \sigma\kappa}{\kappa}\mu(\mu-1)u_t + \sigma r_t$$

  - Where $0 < \mu < 1$ is the model's only stable eigenvalue
  - History dependence; complete stabilization of demand shocks

Dynamics under Optimal Policy II

- Equivalent target criterion under rational expectations

$$p_t = \bar{k} - \frac{\lambda_x}{\kappa} x_t$$

- In terms of the price level, optimal state-contingent paths given by

$$\begin{aligned} p_t - \bar{k} &= \mu \left(p_{t-1} - \bar{k}\right) + \mu u_t \\ x_t &= -\mu \cdot \frac{\kappa}{\lambda_x} \left(p_{t-1} - \bar{k}\right) - \frac{\kappa}{\lambda_x} \mu u_t \end{aligned}$$

and

$$i_t = -\frac{\lambda_x - \sigma\kappa}{\kappa} (1 - \mu) \mu \left(p_{t-1} - \bar{k}\right) + \frac{\lambda_x - \sigma\kappa}{\kappa} \mu (\mu - 1) u_t + \sigma r_t$$

Beliefs

- Agents use linear forecasting model

$$z_t = \omega_{0,t-1} + \omega_{1,t-1} z_{t-1} + e_t$$

where $z_t = (\pi_t, x_t, i_t)'$, e_t is the usual error-vector term

  - The conformable matrices $\{\omega_{0,t}, \omega_{1,t}\}$ are parameters to be estimated
  - In the case of price level rules assume prices not inflation is forecast

Beliefs II

- Forecasts can then be computed as

$$\hat{E}_t z_T = \left(I_3 - \omega_{1,t-1}\right)^{-1} (I_3 - \omega_{1,t-1}^{T-t}) a_{t-1} + \omega_{1,t-1}^{T-t} z_t$$

  - Forecasting function is predetermined — use historical data available up to period $t-1$
  - But: forecasts are not predetermined as they depend up period t equilibrium realizations of z_t

Beliefs III

- Beliefs are updated according to

$$\omega_t = \omega_{t-1} + g_t t^{-1} R_t^{-1} y_{t-1} \left(y_t - \omega_{t-1}' y_{t-1} \right)'$$

$$R_t = R_{t-1} + g_t t^{-1} \left(y_{t-1} y_{t-1}' - R_{t-1} \right)$$

where $y_t = (1, z_t)'$

- Different assumptions about g_t deliver different gains in the filtering problem
  - When $g_t = 1$ recursive least squares, which gives equal weight to all data
  - When $g_t = \bar{g}t$ recursive updating is given by a constant-gain algorithm, implying past observations are discounted more heavily
    * An observation n periods old receives a weight of $(1 - \bar{g})^n$.

Beliefs IV

- In the case of a constant-gain algorithm
  - Beliefs do not converge to rational expectations equilibrium
  - For $\bar{g}$ small enough: ergodically distributed around REE
    * Evans and Honkapohja (2001)
  - E-Stability therefore important to an understanding of such dynamics

Candidate Targeting Rules

- Recalling $\lambda_x \equiv \kappa/\theta$, two proposed rules:

$$\pi_t = -(x_t - x_{t-1})/\theta$$

$$p_t = \bar{k} - x_t/\theta$$

- Properties:
  - Determine an EQ which minimizes loss in the set of all EQ's s.t $\pi_0 = \bar{\pi}_0$ for appropriate choice of $\bar{\pi}_0$
  - Ensure a unique bounded REE; same EQ response to disturbances
  - The minimum-state-variable REE solution for $\{\pi_t, x_t, i_t\}$ is linear in $\{x_{t-1}, u_t, r_t\}$ and $\{p_{t-1}, u_t, r_t\}$ respectively

Candidate Targeting Rules II

- Target criteria are restrictions on one or more endogenous variables

- Requires the policy maker to have a model of the economy
  - Does it matter what model is employed to construct projections
  - To what extent are target criteria robust to model uncertainty — i.e. uncertainty about the model of the transmission mechanism
  - Are there reasons to prefer policies cast in terms of the inflation rate or the price level?
    - What about nominal income targeting — level or growth?

Candidate Targeting Rules III

- Under rational expectations robust to a range of views about the transmission mechanism
  - Changes in the nature of current and projected disturbances
  - Changes in market structure that affect real rigidities — affects κ
  - Changes in the degree of price friction — affects κ
  - Changes in aggregate demand — the presence of habit formation
  - Changes in agents' projections of the economy??

Projection under Rational Expectations

- Suppose the central bank projects the evolution of the economy assuming rational expectations.

- Observes the history of exogenous disturbances; lagged endogenous variables. Then it solves:

$$\pi_t = -(x_t - x_{t-1})/\theta$$

$$\pi_t = \kappa x_t + \beta E_t \pi_{t+1} + u_t$$

$$x_t = E_t x_{t+1} - \sigma^{-1}(i_t - E_t \pi_{t+1} - r_t^n)$$

Projection under Rational Expectations II

- Decision procedure implies the instrument setting

$$i_t = \frac{\lambda_x - \sigma\kappa}{\kappa} (1 - \mu)\, \mu x_{t-1} + \frac{\lambda_x - \sigma\kappa}{\kappa} \mu\, (\mu - 1)\, u_t + \sigma r_t$$

  - "Fundamentals-based" reaction function
    - ∗ Evans and Honkapohja (2003, ReStud)
  - No response to beliefs that deviate from desired equilibrium — potential source of instability
  - Result: A sufficient condition for instability

$$\theta > \sigma^{-1}$$

  - Uses the property that $\lambda = \kappa/\theta$ in the underlying microfoundations

Some Calibration

- Is $\theta > \sigma^{-1}$ reasonable?
  - Chari, Kehoe and McGrattan: $\theta = 10$
  - Rotemberg-Woodford (1998) estimates: $\sigma^{-1} = 6.25$
- Most argue $0 < \sigma^{-1} < 2$ on the basis of a range of micro and macro data
  - Restriction likely to be satisfied

Projection using Private Sector Forecasts

- Suppose the central bank projects the evolution assuming the Euler equations

$$\pi_t = \kappa x_t + \beta E_t \pi_{t+1} + u_t$$

$$x_t = E_t x_{t+1} - \sigma^{-1} (i_t - E_t \pi_{t+1} - r_t^n)$$

  are valid

  - Solves for a rational expectations equilibrium as a function expectations
  - Implies the instrument rule

$$i_t = \sigma \left[\hat{E}_t x_{t+1} - \frac{\lambda_x}{\lambda_x + \kappa^2} x_{t-1} + \left(\frac{\beta\kappa}{\lambda_x + \kappa^2} + \frac{1}{\sigma} \right) \hat{E}_t \pi_{t+1} + \frac{\kappa}{\lambda_x + \kappa^2} u_t + r_t^n \right]$$

- This "expectations-based" reaction function has long-run coefficients

$$\begin{aligned}\Phi_\pi &= \frac{\sigma\beta\kappa}{\lambda_x + \kappa^2} + 1 \\ \Phi_x &= \frac{\sigma\kappa^2}{\lambda_x + \kappa^2}\end{aligned}$$

  - Satisfaction of the Taylor Principle delivers a unique bounded rational expectations equilibrium

  - Evans and Honkapohja (2003, ReStud)

Projection using Private Sector Forecasts II

- Literature suggest such rules might be problematic. For example for

$$i_t = \phi_\pi E_t \pi_{t+1}$$

  stability, in the neighborhood of flex-price equilibiurm, requires

$$\phi_\pi > \frac{1}{1-\beta}$$

- Expectations-based reaction function leads to instability
  - That is for all values: $\beta, \alpha, \kappa, \sigma$ are 0.99, 0.66, $(0,1]$, $(0,7]$
  - Note: the procedure would imply stability for all parameters values if the Euler equations did in fact describe the true dynamics — not robust to the model of the transmission mechanism

Projection using the True Model

- If the central bank understands the structural relations describing spending and pricing plans in can implement the target criterion

$$\pi_t = -\left(x_t - x_{t-1}\right)/\theta$$

by setting interest rates according to

$$\begin{aligned} i_t &= -\sigma \hat{x}_t + \\ &\quad \sigma \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \left[(1-\beta) x_{T+1} - \sigma^{-1} (\beta i_{T+1} - \pi_{T+1} - r_T^n) \right] \end{aligned}$$

where $\hat{x}_t$ is the value of x_t satisfying both

$$\begin{aligned} \pi_t &= -\left(x_t - x_{t-1}\right)/\theta \\ \pi_t &= \hat{E}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} \left[\kappa x_T + (1-\alpha)\beta \pi_{T+1} + u_T \right] \end{aligned}$$

Projection using the True Model II

- Result: the inflation target criterion is expectationally stable for all maintained parameter values
  - Robust to the nature of expectations formation: will implement the optimal equilibrium for arbitrary beliefs
  - Requires the central bank to monitor expectations
  - Will also lead to convergence if the Euler equations describe the true transmission mechanism
  - Not all uses of forecasts equivalent

Intuition

- The interest-rate setting procedure satisfies the long-run Taylor principle

- Can show the long-run coefficient on inflation is

$$\Phi_\pi = 1 + \frac{\kappa\sigma\,(1-\beta)\,(1-\alpha)\,\beta}{\left(\lambda_x + \kappa^2\right)(1-\alpha\beta)} > 1$$

  using $\lambda_x = \kappa/\theta$

  - Procedure has the property that interest rates adjust to offset the effects of expectations

  - Period by period the central bank attempts to ensure satisfaction of the target criterion — not the rule itself

Price-level Targeting Rule

- Now suppose the central bank implements

$$p_t = \bar{k} - \theta^{-1} x_t$$

projecting the evolution of the economy under rational expectations observing only exogenous and lagged endogenous variables

$$i_t = -\frac{\lambda_x - \sigma\kappa}{\kappa}(1-\mu)\mu\left(p_{t-1} - \bar{k}\right) + \frac{\lambda_x - \sigma\kappa}{\kappa}\mu(\mu - 1)u_t + \sigma r_t$$

  - Result: This rule leads to stability for all benchmark parameter values
  - Compare: Inflation targeting criterion required $\theta < \sigma^{-1}$ for stability

Price-level Targeting Rule: Responding to Expectations

- Now suppose the central bank implements

$$p_t = \bar{k} - \theta^{-1} x_t$$

projecting the evolution of the economy under rational expectations observing only exogenous and lagged endogenous variables

$$i_t = \frac{\sigma\kappa - \lambda_x}{\left(\kappa^2 + \lambda_x + \beta\lambda_x\right)} (p_{t-1} + u_t) + \sigma \left(\hat{E}_t x_{t+1} + r_t\right) + \frac{\beta\kappa\sigma + (\kappa^2 + \lambda_x)}{\left(\kappa^2 + \lambda_x + \beta\lambda_x\right)} \hat{E}_t p_{t+1}$$

  - Result: This rule leads to stability for most parameter values (depends on the relative magnitudes of σ^{-1} and κ. For $\sigma^{-1} < 2$ always stable)
  - Equivalent rule under inflation targeting unstable for all such values

Intuition

- Price-level targeting has the advantage that it does not inherit past policy mistakes
  - Each period the central bank attempts to achieve the target criterion afresh

- The target criterion

$$\pi_t = \theta^{-1} (x_t - x_{t-1})$$

  does not have this property

  - Past errors affect the evolution of the output gap: distorts objectives
  - Propagates policy errors which is destabilizing

Aoki and Nikolov (2003)

- Provides an example of the kind of quantitative work that would be useful to see more of. Consider central bank and agent learning about structural parameters: κ and σ. Compare
  - Optimal non-inertial rule
  - The optimal history-dependent rule
  - Optimal price level rule
- Price level rules best: example of integral control. Robust to
  - Persistent mismeasurement of natural rates
  - Errors induced by history dependence

Conclusions on Target Criteria

- Target criteria are robust to pirvate agent belief formation if they have knowledge of the true model

- Absent this, target criteria cast in terms of the price level evince greater robustness to model misspecification

- Responding to forecasts desirable, but has to be the right kind of dependency

- Inertial policy desirable, but has to be of the right kind
  - Responding to the lagged price level seems desirable

Optimal Policy under Learning

- Important issue since we may be interested in
  - The transition dynamics and specifically how policies perform in transition to rational expectations
  - Beliefs may not converge to rational expectations

- Consider the class of targeting rules

$$\pi_t + \psi^{-1} x_t = 0$$

  where ψ^{-1} is chosen to maximize agents welfare

  - Call a simple rule since inconsistent with optimal commitment equilibrium
  - The parameter ψ^{-1} determines the relative stabilization weight over inflation and output

Optimal Policy under Learning II

- Then the central bank seeks to minimize the loss function

$$\mathbb{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \left(\pi_T^2 + \lambda_x x_T^2 \right)$$

  by choice of ψ^{-1} subject to the aggregate supply equation

  - Assume the exogenous disturbances are iid for simplicity
  - Implies the only uncertainty that agents face concerns the means of each variable

Beliefs

- Assume agents have the forecasting model

$$z_t = \bar{\omega}_{0,t-1} + e_t,$$

where $z_t = [\pi_t \; x_t]'$

- The unobserved drift, $\bar{\omega}_{0,t}$, is believed to evolve according to a random-walk

$$\bar{\omega}_{0,t} = \bar{\omega}_{0,t-1} + \eta_t$$

with innovation η_t. Assume estimate of $\bar{\omega}_{0,t}$ update according to the filter

$$\omega_{0,t} = \omega_{0,t-1} + \bar{g}\left(z_t - \omega_{0,t-1}\right)$$

True Data Generating Process

- Evaluating expectations in the Phillips curve provides

$$\pi_t = \frac{\alpha\beta\kappa}{(1+\psi\kappa)(1-\alpha\beta)}\omega_{t-1}^x + \frac{(1-\alpha)\beta}{(1+\psi\kappa)(1-\alpha\beta)}\omega_{t-1}^\pi + \frac{\kappa}{1+\psi\kappa}u_t.$$

- If agents understand the policy rule equilibrium beliefs satisfy $\omega_t^x = -\psi\omega_t^\pi$. Model dynamics are summarized by the equations

$$\begin{aligned} \pi_t &= T_\pi(\psi)\cdot\omega_{t-1}^\pi + \frac{\kappa}{1+\psi\kappa}u_t \\ \omega_t^\pi &= \omega_{t-1}^\pi + \bar{g}\left(\left[(T_\pi(\psi)-1)\,\omega_{t-1}^\pi + \frac{\kappa}{1+\psi\kappa}u_t\right]\right) \end{aligned}$$

where

$$T_\pi(\psi) = \frac{\beta}{(1-\alpha\beta)}\left(\frac{1}{(1+\psi\kappa)} - \alpha\right).$$

Observations

- Inflation dynamics comprise drift induced by beliefs and response to shocks
  - Response to shocks exactly that under optimal discretion
  - The drift in inflation determined by beliefs

- The strength of self-referentiality regulated by monetary policy
  - Larger values of ψ which imply greater concern for inflation stabilization, weaken self-referentiality
  - For sufficiently large ψ increase in inflation expectations leads to a fall in inflation!
  - Underscores the role of monetary policy under learning dynamics

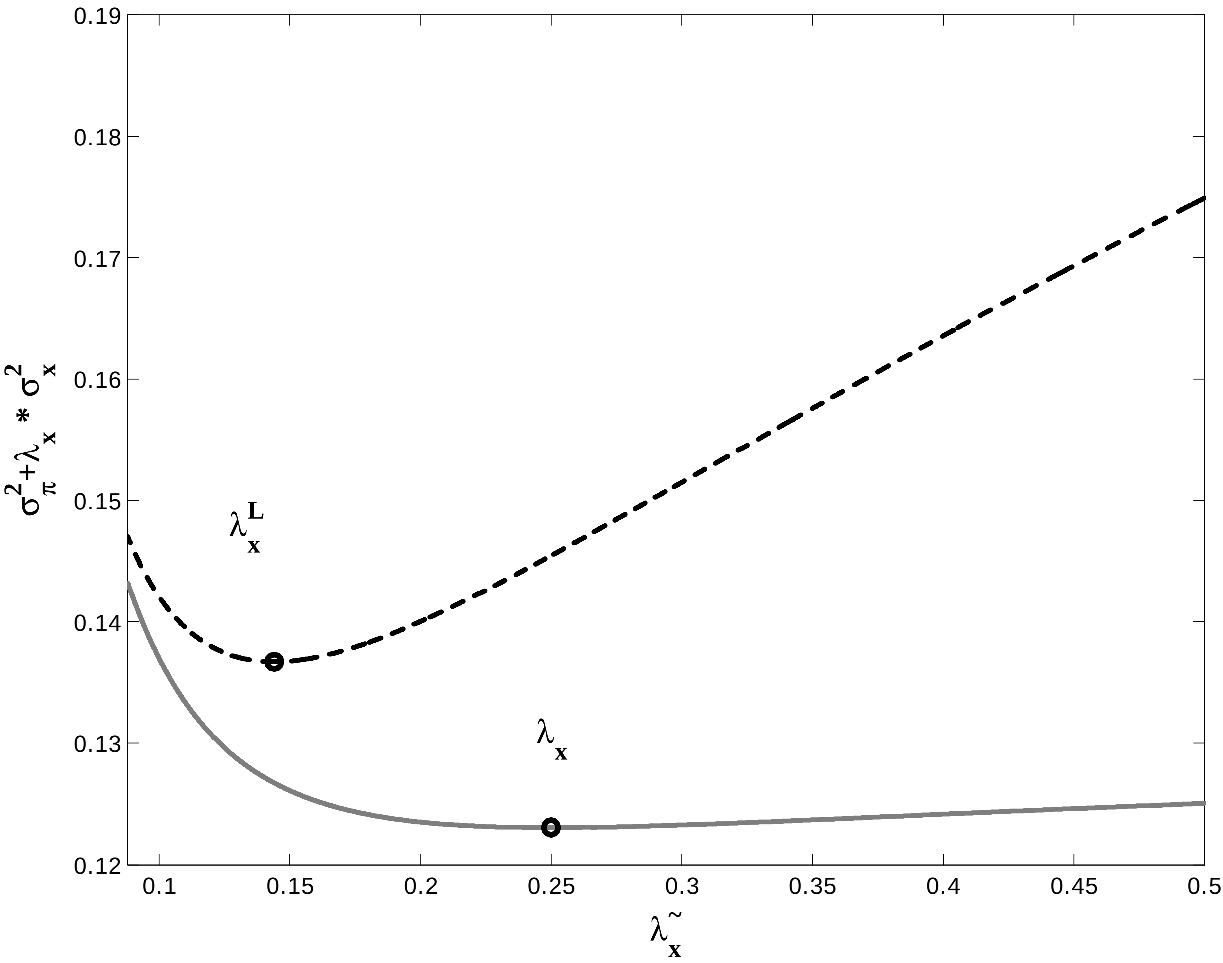

Figure 4: The figure shows the (weighted) volatilities out inflation and output gap associated to alternative targeting rules. The black dashed line corresponds to targeting the model with learning, while the grey soldi line show pertains the model under rational expectations.

Observations II

- Within this class of simple rule, optimal policy requires a more aggressive response to inflation than under rational expectations

- This insight comes from a number of papers in the literature
  - Bomfim, Tetlow, von zur Muehlen and Williams (1997)
  - Orphanides and Williams (2005a, 2005b, 2005c, 2007) and Ferrero (2007)
  - Molnar and Santoro (2013)

Optimal Policy under Full Control

- What is the optimal policy given learning dynamics?

- Assume the central bank

  - Has rational expectations

  - Knows the true structure of the economy including the evolution of beliefs

    * Best-case scenario: less knowledge implies more difficult control problem

  - Can directly control the output gap as the instrument of policy

    * Abstract from the transmission mechanism of monetary policy for the time being

Central Bank's Decision Problem: Molnar-Santoro (2013)

- Minimize the loss

$$\mathbb{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \left(\pi_T^2 + \lambda_x x_T^2\right)$$

  by choice of $\{\pi_t, x_t, \omega_t^{\pi}, \omega_t^{x}\}$subject to

$$\begin{aligned} \pi_t &= \hat{E}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} \left[\kappa \left(x_T + u_T\right) + (1-\alpha)\,\beta\pi_{T+1}\right] \\ \omega_t &= \omega_{t-1} + \bar{g}\left(z_t - \omega_{t-1}\right) \end{aligned}$$

  - Taking as given initial beliefs ω_{-1}. Because beliefs are state variables, no distinction between commitment and discretion

Central Bank's Decision Problem II

- In the neighborhood of flexible prices and using the optimal stabilization weight $\lambda_x = \kappa/\theta$ can show the first-order conditions imply

$$\pi_t = -\theta^{-1}\left(x_t - \bar{g}\beta^2\mathbb{E}_t\sum_{T=t}^{\infty}\left[\beta\left(1-\bar{g}\right)\right]^{T-t}x_{T+1}\right)$$

  - Effectively a target criterion for optimal policy under learning dynamics
    * When the gain goes to zero we have optimal discretion
  - More generally: when the central bank projects negative output gaps optimal policy is more aggressive relative to discretion
    * Generalizes the insights from simple optimal rules
    * Central bank faces an intertemporal trade-off from drift in inflation beliefs

Central Bank's Decision Problem III

- Optimal policy indues inertia — Gaspar, Smets and Vestin (2007, 2010)

- The source of this inertia fundamentally different to optimal commitment under rational expectations
  - Arises from the drift in inflation beliefs
  - Requires aggressive response to inflation expectations

Outstanding Issues

- Treatment of optimal policy makes two important abstractions
  - Abstracts from an inefficient steady state level of output: $x^* = 0$
  - Abstracts from modeling the transmission mechanism of monetary policy
- Do these omissions matter?

The Case of an Inefficient Steady State

- The optimal policy problem becomes

$$\mathbb{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \left(\pi_T^2 + \lambda_x \left(x_T - x^* \right)^2 \right)$$

by choice of $\left\{ \pi_t, x_t, i_t, \omega_t^{\pi}, \omega_t^{x}, \omega_t^{i} \right\}$ subject to

$$\begin{aligned}
\pi_t &= \hat{E}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} \left[\kappa \left(x_T + u_T \right) + (1-\alpha) \beta \pi_{T+1} \right] \\
x_t &= \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \left[(1-\beta) x_{T+1} - \sigma^{-1} \left(i_T - \pi_{T+1} - r_T^n \right) \right] \\
\omega_t &= \omega_{t-1} + \bar{g} \left(z_t - \omega_{t-1} \right)
\end{aligned}$$

  - Taking as given initial beliefs ω_{-1}. Aggregate demand is a constraint with arbitrary interest rate beliefs

The Case of an Inefficient Steady State II

- Evaluating beliefs in the aggregate supply equation permits the problem to be written as

$$\mathbb{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \left(\pi_T^2 + \lambda_x (x_T - x^*)^2 \right)$$

by choice of $\{\pi_t, x_t, \omega_t^\pi, \omega_t^x\}$ subject to

$$\begin{aligned}
\pi_t &= \kappa x_t + \kappa \frac{\alpha\beta}{1-\alpha\beta} \omega_{t-1}^x + \frac{(1-\alpha)\beta}{1-\alpha\beta} \omega_{t-1}^\pi \\
x_t &= -\frac{1}{1-\beta} \left(\beta a_{t-1}^i - a_{t-1}^\pi \right) + (1-\beta) a_{t-1}^w - \sigma^{-1} (i_t - r_t^n) \\
\omega_t &= \omega_{t-1} + \bar{g} (z_t - \omega_{t-1})
\end{aligned}$$

  – Taking as given initial beliefs ω_{-1}

The Case of an Inefficient Steady State III

- First-order conditions constitute a system of linear rational expectations in the variables

$$\left\{x_t, \pi_t, i_t, \omega_t^{\pi}, \omega_t^{x}, \omega_t^{i}, \lambda_{1,t}, \lambda_{2,t}, \lambda_{3,t}, \lambda_{4,t}, \lambda_{5,t}\right\}$$

  where λ_j are the Lagrange multipliers on the five constraints

  - Can prove the system has a unique bounded rational expectations equilibrium for all maintained parameter values and initial conditions ω_{-1}

  - Application of Giannoni and Woodford (2013)

Long-run Implications

- The existence of a unique bounded rational expectations implies dynamics converge to steady state

- This permits the following properties

$$\pi^{LR} = \lim_{T\to\infty} E_t \pi_T = \frac{\lambda_x \kappa x^*}{\kappa^2 \Xi + \lambda_x (1-\beta)}$$

$$x^{LR} = \lim_{T\to\infty} E_t x_T = 0$$

where

$$\Xi = \frac{g\beta + (1-\beta)(1-\alpha\beta)}{g\beta(1-\beta) + (1-\beta)(1-\alpha\beta)} \geq 1.$$

  - Optimal long-run inflation rate depends critically on beliefs and household patience

Long-run Implications II

- Two limiting cases of interest

$$\lim_{g \to 0} \pi^{LR} = \frac{\lambda_x \kappa x^*}{\kappa^2 + \lambda_x (1 - \beta)}$$

$$\lim_{\beta \to 1} \pi^{LR} = 0$$

  - These are the outcomes under commitment and discretion
  - In general price stability is not optimal in the long run

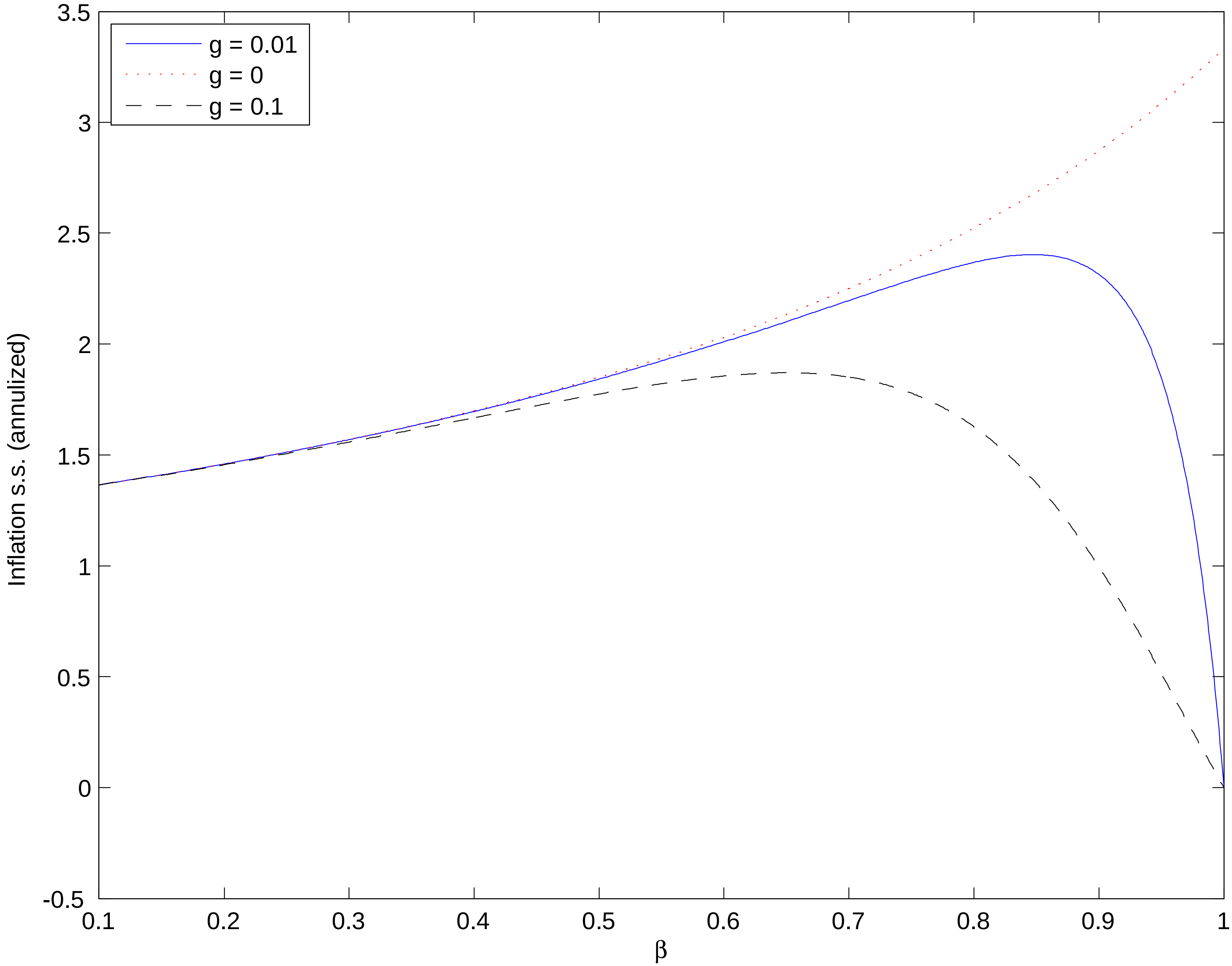

Figure 1: Steady state inflation as a function of the discount factor for different gains.

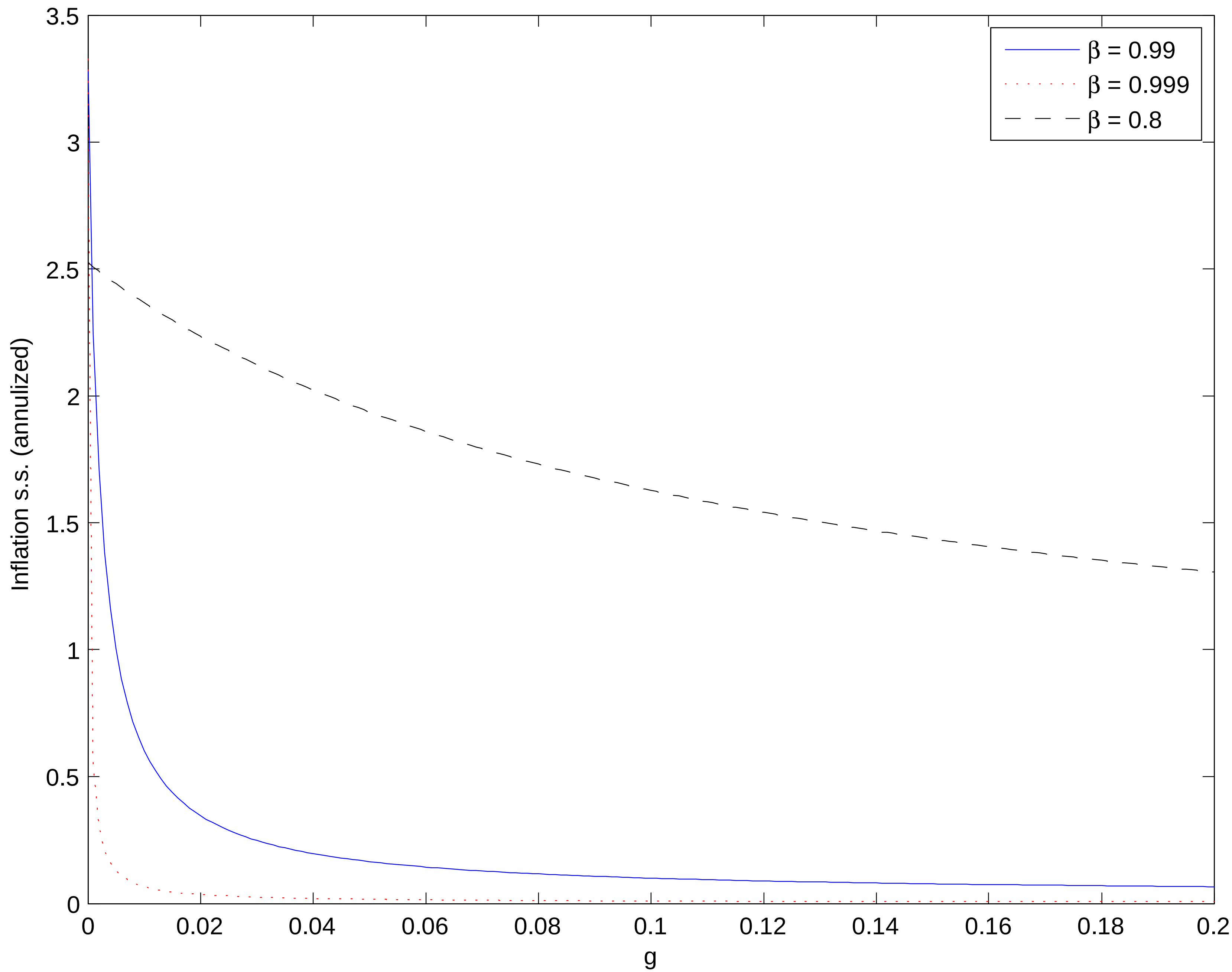

Figure 2: Steady state inflation as a function of the gain for different discount factors.

Intuition

- Two competing effects
  - As the discount factor rises, worsens short-run trade-off: as expectations exploited rising inflation expectations raise present discounted losses
    - Tends to lower the equilibrium inflation rate
  - As the gain falls, beliefs less sensitive to new information, more open to exploitation
    - Tends to raise the equilibrium inflation rate
- Optimal long-run inflation rate depends on these two forces
  - Bounded by the rational expectations outcomes when resolved in favor of one over the other

Short-run Responses to Disturbances

- Return to the question of how optimal policy responds to shocks
  - Assume $x^* = 0$ for simplicity
  - Assume only demand shocks temporarily

Optimal Policy under Rational Expectations Revisited

- Solving aggregate demand forward in any bounded equilibrium

$$
\begin{aligned}
x_t &= -\sigma^{-1} E_t \sum_{T=t}^{\infty} (i_T - E_t \pi_{t+1} - r_T^n) \\
\pi_t &= \kappa E_t \sum_{T=t}^{\infty} \beta^{T-t} x_T
\end{aligned}
$$

  - Having nominal interest rates track

$$E_t \pi_{t+1} + r_T^n$$

    delivers $x_t = \pi_t = 0$

  - Complete stabilization of demand shocks optimal
    - ∗ General result: Justiniano, Primiceri and Tambalotti (2012)

Policymaker Problem under Long-run Drift

- Rational policymaker minimizes

$$E_t \sum_{T=t}^{\infty} \beta^{T-t} \left[\pi_T^2 + \lambda_x x_T^2\right]$$

by choice of $\{x_t, \pi_t, i_t, a_t\}$ subject to

$$\begin{aligned}
\pi_t &= \frac{\kappa\alpha\beta}{1-\alpha\beta} a_{t-1}^w + \frac{(1-\alpha)\beta}{1-\alpha\beta} \omega_{t-1}^\pi + \kappa\sigma x_t \\
x_t &= -\frac{1}{1-\beta}\left(\beta a_{t-1}^i - a_{t-1}^\pi\right) + (1-\beta) a_{t-1}^w - \sigma^{-1}\left(i_t - r_t^n\right) \\
a_t &= a_{t-1} + \bar{g}\left(z_t - a_{t-1}\right)
\end{aligned}$$

  - Beliefs are state variables: no distinction between commitment and discretion
  - Giannoni and Woodford (2013): problem always admits a locally unique equilibrium

Policymaker Problem under Long-run Drift: $\bar{g} = 0$

- Rational policymaker minimizes

$$E_t \sum_{T=t}^{\infty} \beta^t \left[\pi_T^2 + \lambda_x x_T^2 \right]$$

subject to

$$\begin{aligned} \pi_t &= \kappa \sigma x_t \\ x_t &= -\sigma^{-1} (i_t - r_t^n) \\ a_t &= a_{t-1} \end{aligned}$$

  - Equivalent to optimal discretion: $i_t = r_t^n$

Optimal policy under drifting beliefs

- Expected policy path not fully anchored: aggregate demand becomes a constraint

- Ability to manage short-run trade-off depends on the nature of long-run drift in expectations

  - In this simple model: $\bar{g} > 2\,(1-\beta)$ impedes full stabilization

  - The closer beliefs are to being rational, the tighter is potential control of the central bank

Optimal policy under drifting beliefs

- Assume beliefs $\left\{a_t^\pi, a_t^y, a_t^i\right\}$ initially at rational expectations equilibrium — i.e. equal to zero; $\sigma = 1$.

- Consider shock in period t with beliefs at rational expectations equilibrium

$$\pi_t = \kappa x_t; \qquad x_t = -\left(i_t - r_t^n\right)$$

- Stabilization possible in period t
  - Nominal interest rate policy must track natural rate r_t^n — gives $\pi_t = x_t = 0$.

- But this implies subsequent movements in long-run interest-rate beliefs

$$a_t^i = a_{t-1}^i + \bar{g}\left(r_t^n - a_{t-1}^i\right)$$

Optimal policy under drifting beliefs

- The next period's stabilization problem — and every subsequent period — is

$$
\begin{aligned}
\pi_{t+1} &= \kappa x_{t+1} \\
x_{t+1} &= -\left(i_{t+1} - r_{t+1}^n\right) - \frac{1}{1-\beta}\beta a_t^i
\end{aligned}
$$

- Stabilization again possible — nominal interest rates track long-run expectations.

- But is this interplay sustainable?

Optimal policy under drifting beliefs

- Note that the aggregate demand constraint defines implicit policy reaction function.

- with $x_{t+1} = \pi_{t+1} = 0$:

$$i_{t+1} = r^n_{t+1} - \frac{\beta}{1-\beta} a^i_t$$

- Substituting into the updating rule for a^i_t

$$a^i_{t+1} = \left(1 - \frac{\bar{g}}{1-\beta}\right) a^i_t + \bar{g} r^n_{t+1}$$

- Stability: $\bar{g} > 2\,(1-\beta)$ implies explosive beliefs. Policy is not sustainable.

  – key role of self-referential dynamics.

Drifting Beliefs and Volatility

- Implications of limited adjustment of nominal interest rates is greater output gap volatility.
  - Consider "volatility frontiers" for the more general loss function

$$E_0^{RE} \sum_{t=0}^{\infty} \beta^t \left[\pi_t^2 + \lambda_x x_t^2 + \lambda_i i_t^2 \right]$$

where $\lambda_i \geq 0$ determines relative stabilization weight on interest-rate variability.

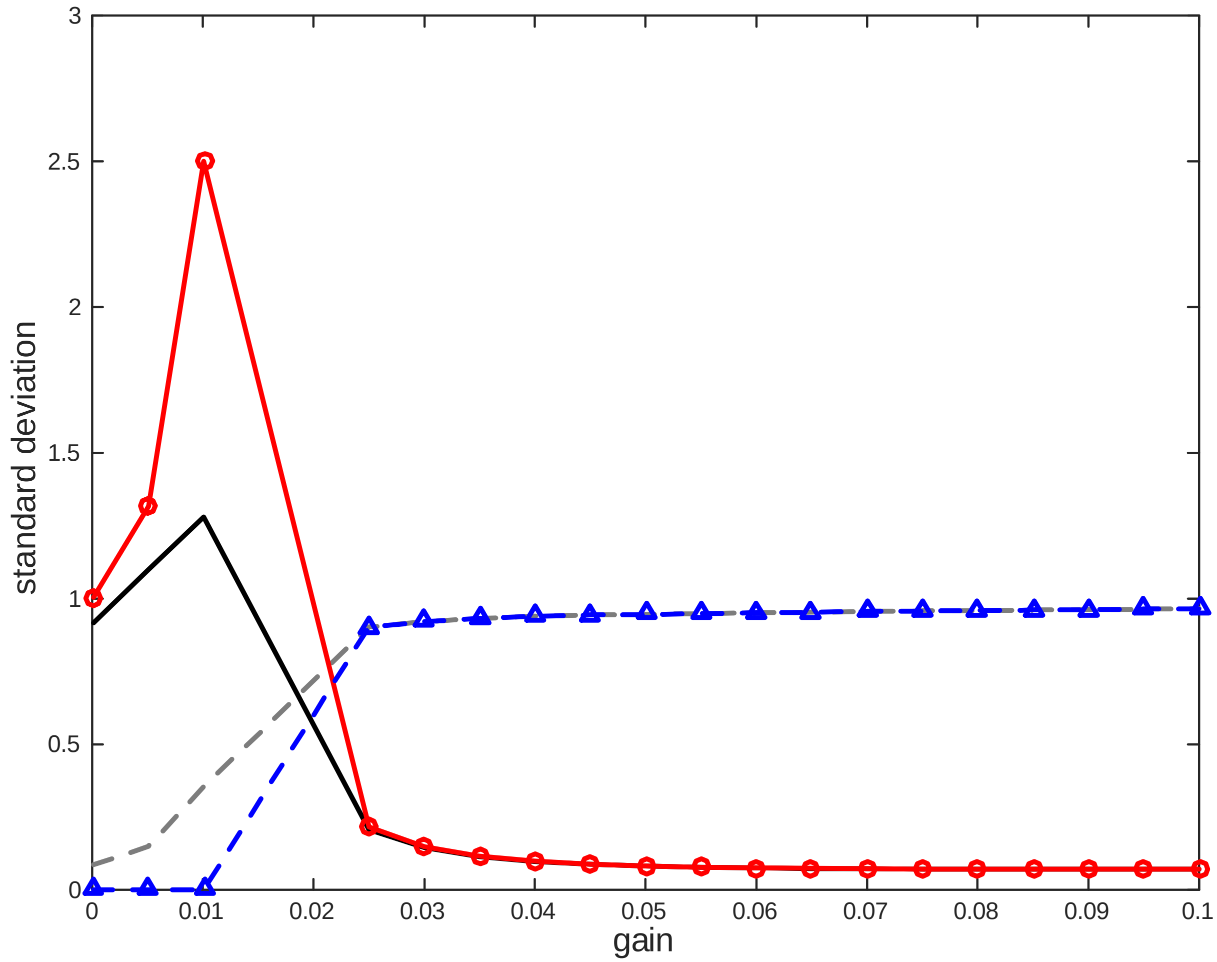

Figure 3: The volatility of output and interest rates as a function of the constant gan $\bar{g}$. The welfare theoretic loss gives the volatility of the interest rate (red circles) and the output gap (blue triangles); a policy maker with a concern of interest rate volatility delivers the interest rate (black line) and the output gap (grey dashed line).

Implications of Volatility Frontiers

- Higher gains imply increased drift of long-run interest-rate expectations in response to current interest-rate movements — creates volatility in aggregate demand

  - Optimal to adjusted current interest-rates little in responses to disturbances.

  - Output gap volatility must be higher relative to a more aggressive policy.

Further Insights

- Transmission mechanism places constraints on any policy framework

- Consider simple policy rule

$$i_t = \phi_\pi (\pi_t + \lambda_x x_t)$$

where $\phi_\pi \geq 1$

  - What policy response coefficients are consistent with stability for different gain coefficients?

  - This is an example of "robust stability" analysis proposed by Evans and Honkapohja (2009).

  - Note: optimal targeting criterion can be thought of a limiting case of this rule

$$\pi_t + \lambda_x x_t = \lim_{\phi \to \infty} \phi^{-1} i_t = 0.$$

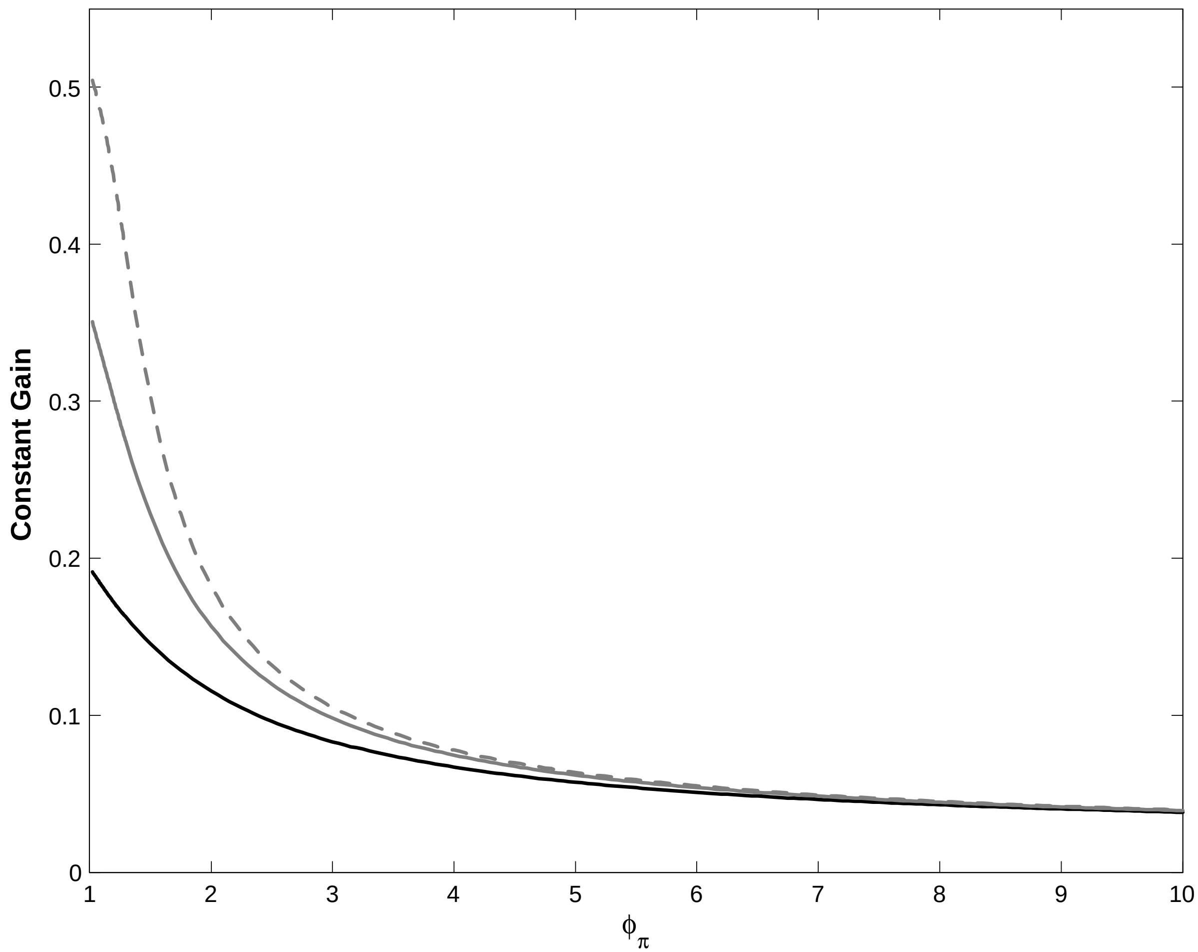

Figure 4: The figure shows stability frontiers corresponding to alternative Taylor rules. In particular $(\bar{g}, \phi_\pi)$ above the frontier correspond to locally unstable equilibria under constant-gain learning. The black solid line coresponds to the standard Taylor Rule. The sold (dashed) grey line corresponds to $\phi_x = \phi_x^*/2$ $(\phi_x = \phi_x^*/3)$.

Conclusions

- Conditioning policy on expectations requires care

- Good policy is inertial
  - Closer to optimal policy under any belief structure
  - Assists stability if of the right kind — responding to past price level appears desirable
  - This is also true when the zero lower bound is a constraint