

## MODEL

We use a standard New Keynesian model

$$\pi_t = \beta E_t \pi_{t+1} + \kappa x_t + e_t, \quad (1)$$

$$x_t = E_t x_{t+1} + \frac{1}{\sigma} (E_t \pi_{t+1} - i_t) + u_t. \quad (2)$$

Three different interest rate rule specifications:

$$i_t = \pi^T + \phi_1 (E_t \pi_{t+1} - \pi^T) + \phi_2 E_t x_{t+1}, \quad (3)$$

$$i_t = \pi^T + \phi_1 (\pi_t - \pi^T) + \phi_2 x_t, \quad (4)$$

$$i_t = \pi^T + \phi_1 (\pi_{t-1} - \pi^T) + \phi_2 x_{t-1}, \quad (5)$$

Expectations are **heterogeneously** distributed around fundamental RE values  $\bar{\pi}$  and  $\bar{x}$ . Agents learn from the past and adjust their predictions.

$$E_t x_{t+1} = \bar{x} + \sum_{h=1}^H b_h n_t^h \quad (6)$$

$$n_t^h = \frac{e^{-\omega(x_{t-1} - b_h - \bar{x})^2}}{\sum_{h=1}^H e^{-\omega(x_{t-1} - b_h - \bar{x})^2}}, \quad (7)$$

with  $\omega$  the **intensity of choice**, determining how fast agents switch to other prediction biases  $b_h$ .

## LARGE TYPE LIMIT ( $H \rightarrow \infty$ )

For a **continuum** of prediction values ( $N(0, s^2)$ ):

$$E_t x_{t+1} = \frac{\bar{x}}{2\omega s^2 + 1} + \frac{2\omega s^2}{2\omega s^2 + 1} x_{t-1}, \quad (8)$$

$$E_t \pi_{t+1} = \frac{\bar{\pi}}{2\omega s^2 + 1} + \frac{2\omega s^2}{2\omega s^2 + 1} \pi_{t-1}. \quad (9)$$

For (3) stability then requires  $\phi_1 >$

$$1 - \frac{1 + (1 - \beta)2\omega s^2}{2\omega s^2 + 1} \left( \frac{\sigma}{2\omega s^2 \kappa} + \frac{\phi_2}{\kappa} \right).$$

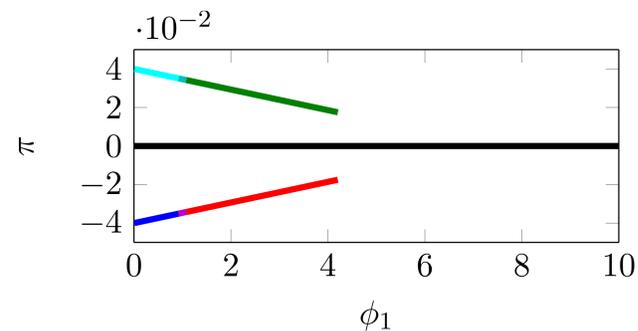
For (4) and (5) stability requires  $\phi_1 >$

$$\frac{2\omega s^2}{2\omega s^2 + 1} - \frac{1 + (1 - \beta)2\omega s^2}{2\omega s^2 + 1} \left( \frac{\sigma}{(2\omega s^2 + 1)\kappa} + \frac{\phi_2}{\kappa} \right)$$

For  $\omega s^2 \rightarrow \infty$  both conditions reduce to the **Taylor principle**. For smaller  $\omega s^2$  (implying that **aggregate expectations** are more strongly **anchored** to  $\bar{\pi}$  and  $\bar{x}$ ), the conditions become weaker.

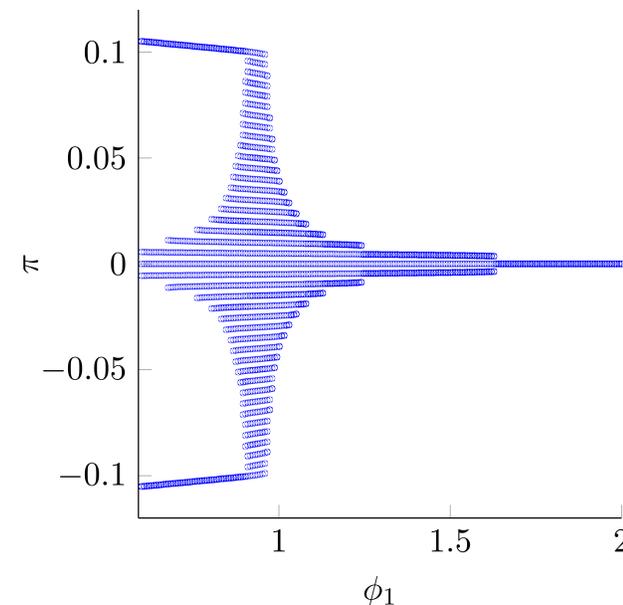
## DISCRETE EXPECTATIONS AND MULTIPLE STEADY STATES

When expectations are **discrete** (e.g because of digit preference) multiple stable steady states can arise due to (almost) **self-fulfilling expectations**. In a stylized model with fundamentalists ( $E_t \pi_{t+1} = \bar{\pi}$ ), optimists ( $E_t \pi_{t+1} = \bar{\pi} + b$ ), and pessimists ( $E_t \pi_{t+1} = \bar{\pi} - b$ ), the CB must be **aggressive** enough in order to prevent the existence of non-fundamental steady states.



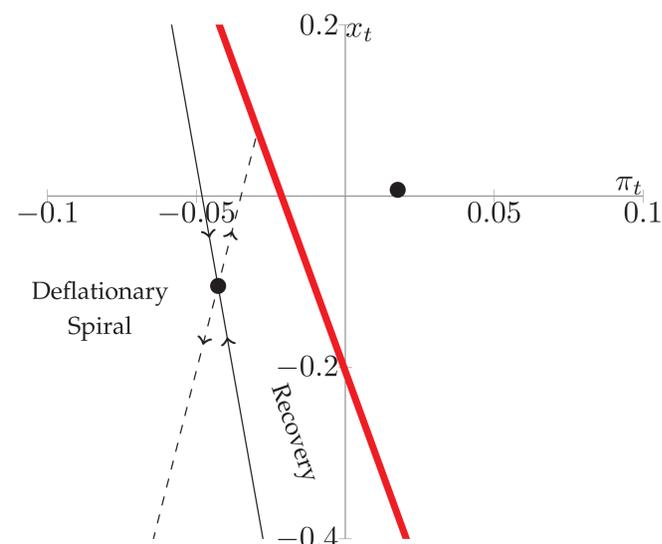
In a more realistic model where all multiples of 0.5% between  $-10\%$  and  $+10\%$  are allowed as ex-

pectations (giving a total of 41 prediction values), almost self-fulfilling expectations can also lead to multiple steady states.



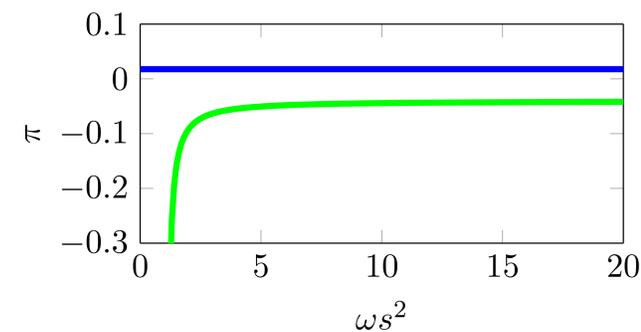
## ZERO LOWER BOUND (ZLB): LARGE TYPE LIMIT

With a continuum of prediction values, the ZLB on the nominal interest rate creates an extra steady state. This ZLB steady state is an unstable **saddle-point**.



- Above the red line the ZLB is not binding and convergence to the target steady state can occur.

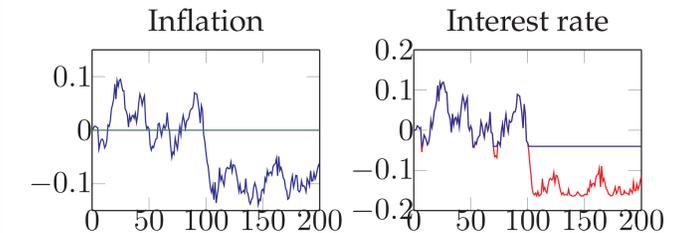
- Below the red line, but above the stable eigenvector of the ZLB steady state (black line) **recovery** to positive interest rate occurs.
- Below the stable eigenvector of the ZLB steady state, inflation and output gap start decreasing and a **deflationary spiral** arises.
- The **size of the recovery region** depends on the position of the ZLB steady state, which depends on the **anchoring** of aggregate expectations ( $\omega s^2$ ). See green line below.



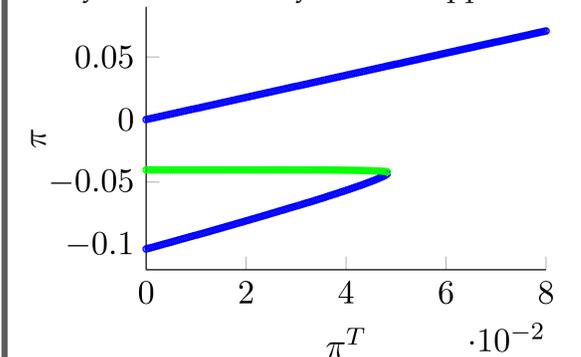
- The **size of the recovery region** can also be increased with a raised **inflation target** ( $\pi^T$ ).

## ZLB: DISCRETE EXPECTATIONS

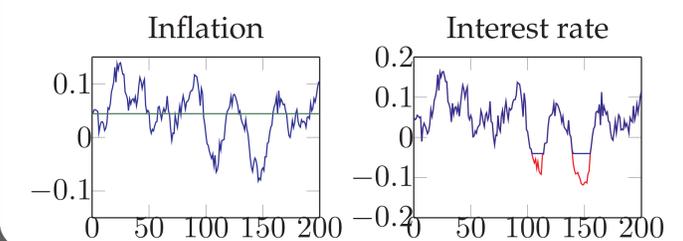
Under discrete expectations (41 prediction values) shocks can induce coordination on a stable pessimistic ZLB steady state.



For a higher  $\pi^T$  (or lower  $\omega s^2$ ) the basin of attraction of the ZLB steady state is reduced and eventually the ZLB steady state disappears.



Coordination on pessimistic expectation then no longer occurs, even when the ZLB is binding.



## POLICY RECOMMENDATIONS

- The Taylor principle is only a necessary condition for local stability when aggregate expectations are **unanchored**.
- When expectation values are **discrete**, the CB must be more aggressive in order to prevent coordination on almost self-fulfilling steady states.
- Under the ZLB, coordination on pessimistic expectations (or even a deflationary spiral) can arise when expectations are unanchored and  $\pi^T$  is too low.