Expectations and the empirical fit of DSGE models

Eric Gaus (Ursinus College) and Christopher G. Gibbs (UNSW Australia)

Objectives

This paper explores the empirical fit of and real-time out-of-sample forecasting performance of a New Keynesian DSGE model under different expectation assumptions. Below we list the objectives of our research

1. Determine the cause of improvements in empirical fit in models with non-rational expectations.
2. Conduct the first real-time out-of sample forecasting with learning.
3. Demonstrate expectational error and not persistence is a significant source of improvements in fit and forecasting.

Model & Analysis

Structural model following [3]:

$$
\begin{aligned}
x_t &= \bar{r} + E_t x_{t+1} - (i_t - E_t \pi_{t+1}) && (1)\\
&\quad + (1-\omega)(1-\rho_a) a_t && (2)\\
\pi_t &= (1-\beta)\bar{\pi} + \beta E_t \pi_{t+1} + \psi x_t - e_t && (3)\\
i_t &= \bar{r} + \bar{\pi} + \theta_\pi(\pi_t - \bar{\pi}) + \theta_x x_t + \epsilon_{i,t} && (4)\\
g_t &= \hat{y}_t - \hat{y}_{t-1} + \bar{g} + \epsilon_{z,t} && (5)\\
x_t &= \hat{y}_t - \omega a_t && (6)\\
a_t &= \rho_a a_{t-1} + \epsilon_{a,t} && (7)\\
e_t &= \rho_e e_{t-1} + \epsilon_{e,t}, && (8)
\end{aligned}
$$

Three cases with three reduced forms:

1. RE reduced form solution

$$\mathbf{y}_t = \bar{\mathbf{a}} + \bar{\mathbf{c}} \mathbf{v}_t + \varepsilon_t, \qquad (9)$$

where $\bar{\mathbf{a}} = (\mathbf{I} - \mathbf{B})^{-1}\mathbf{\Gamma}$ and $\text{vec}(\bar{\mathbf{c}}) = (\mathbf{I} - \rho' \otimes \mathbf{B})^{-1}\text{vec}(\mathbf{C})$.

2. Fixed Beliefs (FB) reduced form

$$\mathbf{y}_t = \mathbf{\Gamma} + \mathbf{B}\mathbf{a} + (\mathbf{B}\mathbf{c}\rho + \mathbf{C})\mathbf{v}_t + \mathbf{D}\varepsilon_t \qquad (10)$$

where

$$\hat{\mathbf{a}} = \mathbf{\Gamma} + \mathbf{B}\mathbf{a}, \;\; \hat{\mathbf{c}} = \mathbf{B}\mathbf{c}\rho + \mathbf{C}.$$

3. Constant gain learning (CGL) $\hat{\mathbf{a}}_\mathbf{t}$ and $\hat{\mathbf{c}}_\mathbf{t}$ are time-varying.

Two step-estimation of FB and CGL: Use estimates of $\bar{\mathbf{a}}$ and $\bar{\mathbf{c}}$ as the fixed belief function and initial beliefs in CGL. **Reestimate** structural parameters.

Goodness of Fit Results

Table 1: ML estimates of three cases

	RE	FB	CGL
θ_π	1.595	1.448	1.573
	[1.35 , 1.95]	[1.18 , 1.70]	[1.26 , 1.86]
θ_x	0.193	0.980	0.981
	[0.10 , 0.32]	[0.91 , 1.03]	[0.91 , 1.03]
ω	0.000	0.000	0.000
	[0.00 , 0.01]	[0.00* , 0.03]	[0.00* , 0.04]
$\bar{\pi}$ x 400	1.041	0.000	0.000
	[0.30 , 2.10]	[0.00* , 1.00]	[0.00* , 0.30]
ρ_a	0.931	0.987	0.990
	[0.92 , 1.00*]	[0.67 , 1.00*]	[0.63 , 1.00*]
ρ_e	0.980	0.822	0.773
	[0.92 , 1.00*]	[0.67 , 0.95]	[0.63 , 0.93]
σ_a x 100	1.686	0.878	0.845
	[0.97 , 1.69]	[0.79 , 0.88]	[0.76 , 0.84]
σ_e x 100	0.068	0.076	0.082
	[0.06 , 0.09]	[0.06 , 0.09]	[0.06 , 0.10]
σ_i x 100	0.551	1.430	1.411
	[0.48 , 0.68]	[1.21 , 1.71]	[1.19 , 1.69]
σ_z x 100	0.179	0.179	0.179
	[0.15 , 0.21]	[0.15 , 0.21]	[0.15 , 0.21]
γ_π	-	-	0.000
			[0.00* , 0.00]
γ_x	-	-	0.005
			[0.00* , 0.02]
Log Likelihood	1981.4	2014.6	2021.6
LR Statistic	-	66.4	100.4
P-value	-	0.00	0.00

Notes: ML estimates for the Ireland model under the three different assumptions for expectations. The forecasting function in the FB case and the initial beliefs in the CGL case are set to the RE solution implied by the estimates in the first column.

Table 2: Reduced from values

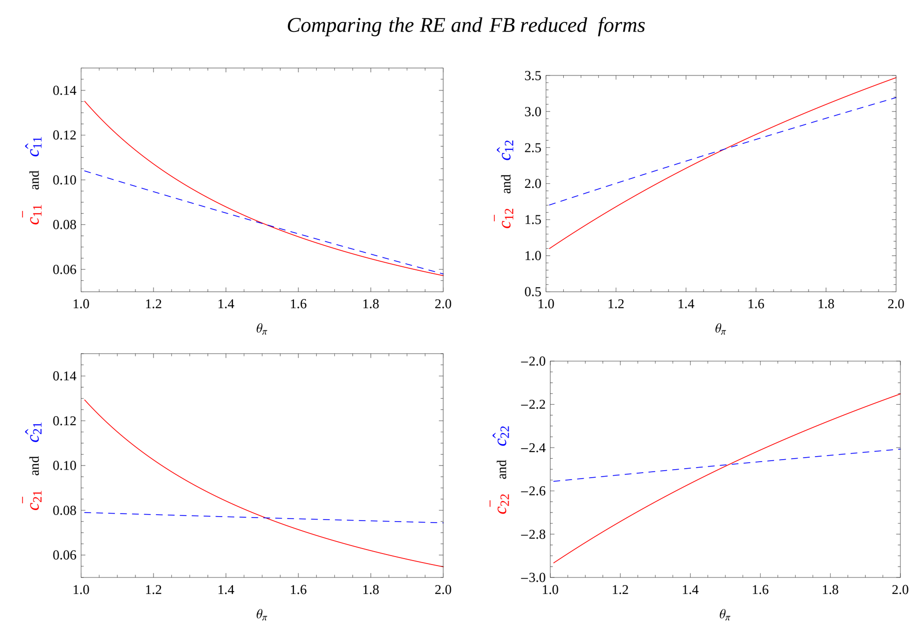
	a_1	a_2	c_{11}	c_{12}	c_{21}	c_{22}
RE	0.000	0.003	0.059	9.215	0.080	-3.181
RE Fixed	-0.001	0.003	0.017	4.792	0.081	-3.124
CGL Init.	-0.001	0.003	0.011	4.718	0.080	-2.975

Notes: The reduced form values of $\mathbf{a}$ and $\mathbf{c}$ implied by Equations (9) and (10) and the estimation results given in Table 1.

Table 1 shows that both FB and CGL show large and statistically significant increases in in-sample fit relative to the benchmark case, while the parameter estimates are mostly unchanged compared to RE. The relatively small changes observed in the estimated parameters is consistent with the findings of [1, 2].

Small changes in parameters, though, can imply large changes in the reduced form as shown in Table 2. Figure 1 illustrates the mechanism by plotting the four elements of $\bar{\mathbf{c}}$ and $\hat{\mathbf{c}}$ as θ_π is varied with the remaining parameters set to $\beta = 0.995$, $\psi = 0.1$, $\theta_x = 0.5$, $\rho_a = 0.9$, $\rho_e = 0.7$, and $\omega = 0.06$. Changes in θ_π have a much larger effect, *ceteris paribus* , under RE.

Figure 1: Comparison of the reduced form values of the elements of $\bar{\mathbf{c}}$ and $\hat{\mathbf{c}}$.



Forecasting Results

Table 3 shows the real-time out-of-sample results for the three cases. The FB and CGL cases generate consistent improvements over the RE model for forecasts of inflation with largest gains observed at short horizons. For GDP Growth, however, both FB and CGL do statically significantly worse than the RE model.

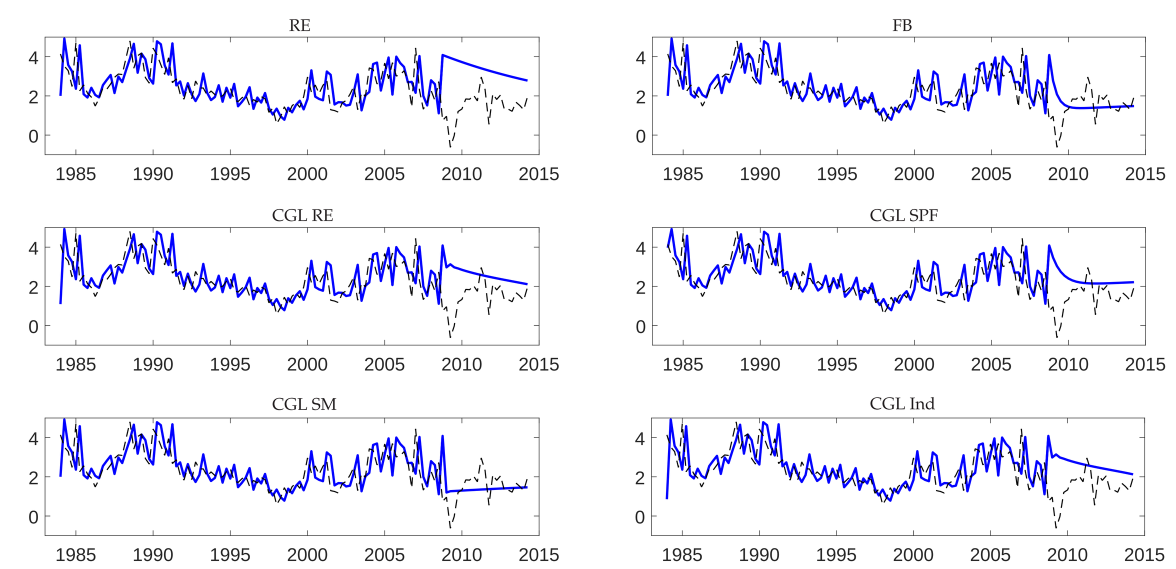
Table 3: Real-time forecast results

Inflation Annualized RMSFE	t (Nowcast)	t+1	t+4	t+6	GDP Annualized RMSFE	t (Nowcast)	t+1	t+4	t+6
RE	1.11	1.05	0.85	1.01	RE	2.38	2.12	1.75	1.80
Relative to RE					Relative to RE				
RE	1.00	1.00	1.00	1.00	RE	1.00	1.00	1.00	1.00
	-	-	-	-		-	-	-	-
FB	0.89**	0.85**	0.86*	0.88	FB	1.92	1.49	1.24	1.15
	(-2.03)	(-2.01)	(-1.61)	(-0.67)		(3.43)	(3.38)	(3.49)	(0.88)
CGL	0.91*	0.91**	0.93	0.93	CGL	1.94	1.44	1.19	1.12
	(-1.42)	(-1.69)	(-1.23)	(-0.98)		(3.99)	(4.12)	(4.30)	(1.85)

*** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$

Notes: Diebold and Mariano test statistic with a short sample and forecast horizon correction reported in parentheses.

Figure 2: Initial beliefs, fitted values, and forecasts for RE, FB, and CGL with four different initial beliefs



Notes: CGL RE is the base case. CGL SM sets $\theta_x = 0$ in initial beliefs. CGL Ind sets $\theta_\pi = 0.965$ and $\theta_x = 0.5$. CGL SPF sets initial inflation forecast to 4%.

Figure 2 shows the impact of different initial beliefs under CGL.

Abstract

Rational Expectations (RE) is compared to fixed non-rational beliefs and adaptive learning with a constant gain. We show that significant improvements in-sample fit are obtained in a fixed belief case that both nests RE and is nested by adaptive learning. The improvements in in-sample fit translate into modest out-of-sample forecast accuracy. The fixed belief case demonstrates that the nonlinear relationships imposed on the structural parameters of the model by RE significantly impair model fit and forecasting ability. The relaxation of the assumption explains a large portion of the improvement in fit of adaptive learning models which has nothing to do with generating persistence.

Conclusion

We investigate the mechanisms that underly observed improvements in empirical fit of DSGE models with boundedly rational expectations. We show that a majority of the improvement in fit and out-of-sample real-time forecast accuracy cannot be attributed to increased persistence. Specifically, we:

- Find that the FB case always leads to large increases in in-sample fit in the Ireland model, which explains the majority of the improvements in the CGL case.
- Can replicate result 1 using the model of Smets and Wouters [4].
- Find improvements in fit from FB also explain the out-of-sample forecast accuracy of CGL.

References

[1] Sergey Slobodyan and Raf Wouters. Learning in an estimated medium-scale dsge model. *Journal of Economic Dynamics and control*, 36(1):26–46, 2012.

[2] Stephen Cole and Fabio Milani. The misspecification of expectations in new keynesian models: A dsge-var approach. Technical report, 2014.

[3] Peter N Ireland. Technology Shocks in the New Keynesian Model. *The Review of Economics and Statistics*, 86(4):923–936, November 2004.

[4] Frank Smets and Rafael Wouters. Shocks and frictions in us business cycles: A bayesian dsge approach. *The American Economic Review*, pages 586–606, 2007.