The sources of aggregate persistence in an estimated DSGE model with real-time learning*

Pablo Aguilar[a,b] and Jesús Vázquez[b]

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ABSTRACT: We extend the adaptive learning model of Slobodyan and Wouters (2012a) by introducing the term structure of interest rates, which results in a model of multi-period forecasting. While retaining the feature of AL based on small forecasting models, our extension allows the term spread of interest rates to fully characterize the forward-looking variables of the model in real time (i.e. using only information truly available at the time of forming agents' expectations). We view the use of real time information as a crucial step forward in understanding the consequences of learning in estimated DSGE models. Our estimation results show that the importance of most endogenous sources of aggregate persistence (such as habit formation, Calvo probabilities, the elasticity of the cost of adjusting capital, and the elasticity of capital utilization adjustment cost) decline dramatically when multi-period forecasting is incorporated through the term structure of interest rates. Moreover, model expectations based on term structure information provides a sound and rather simple characterization of the consumption growth and inflation forecasts reported in the Survey of Professional Forecasters.

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a IRES, Université catholique de Louvain

b Universidad del País Vasco (UPV/EHU)

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1 Introduction

In a recent article, Eusepi and Preston (2011) show that adaptive learning (AL) based on forecasts beyond the one-period-ahead AL forecasts (i.e. considering maintained beliefs over multiple horizons) are crucial for understanding the persistence of macroeconomic dynamics. In particular, Eusepi and Preston (2011) show that the maintained beliefs hypothesis results in a huge amplification mechanism of technology shocks in a standard real business cycle model. Thus, independent and identically distributed technology shocks when combined with maintained beliefs under AL are able to reproduce the aggregate persistence observed in US data.

This paper studies the maintained beliefs hypothesis in the context of the medium-scale DSGE model suggested by Smets and Wouters (2007) (hereinafter called the SW model) in order to assess the relative importance of maintained beliefs in a model combining many sources of aggregate persistence. We consider three building block extensions for the SW model. Although these extensions have been already explored separately in the literature, here we study all of them together in order to analyze how they interact and thus assess their relative importance in the transmission process of shocks.

The first block deviates from the rational expectations assumption by assuming AL expectations based on small forecasting models, as in Slobodyan and Wouters (2012a,b), where the expectation of a forward looking variable is described as a least square projection on a small information set. By considering small forecasting models, we deviate from the minimum state variable AL approach followed by Eusepi and Preston (2011) and others (Orphanides and Williams, 2005a; and Milani, 2007, 2008, 2011) where agent's expectations are assumed to be based on a (linear) function of the state variables of the model. In contrast, small forecasting models assume that agents form their expectations based on the information provided by observable endogenous variables, such as those showing up in the Euler equations of a DSGE model.[1] An important finding in Slobodyan and Wouters (2012a) is that AL relying on small

[1] Considering small forecasting models based only on observable variables is arguably a more appealing

forecasting models results in lower estimates for the persistence of the exogenous shocks that drive price and wage dynamics than those obtained in an estimated RE version of the model. The intuition is simple: AL introduces a source of persistence through the learning dynamics that reduces the role of exogenous shock persistence. However, Slobodyan and Wouters' (2012a) AL approach only considers 1-period ahead forecasts, ignoring the contribution of maintained beliefs suggested in Preston (2005) and Eusepi and Preston (2011).

The second building block extends the Slobodyan and Wouters model by incorporating the term structure of interest rates. The motivation for including the term structure is three-fold. First, the term structure allows us to introduce the maintained beliefs hypothesis in the characterization of the household behavior without further manipulation of the optimality conditions, such as solving forward the intertemporal budget constraint as in Eusepi and Preston (2011) and Sinha (2015, 2016).[2] Thus, household decisions on bond holdings of different maturities depend explicitly upon the expected paths of both consumption and inflation (i.e. their maintained beliefs about consumption and inflation). Second, as discussed deeper below, the term structure contains relevant information observed in real time, which can be used in the learning process of economic agents. This extension was partially investigated in Aguilar and Vázquez (2015), but we also ignored the possibility of maintained beliefs learning. Aguilar and Vázquez (2015) found that when the term spread, a forward looking variable, is included in the AL model the sluggishness of the learning process decreases, which results in an increase of the parameters characterizing endogenous persistence

approach to AL in empirical applications than the minimum state variable approach since the latter requires that agents know what the state variables are and, in addition, they perfectly observe the realizations of all state variables. Other papers (Adam, 2005; Orphanides and Williams, 2005b; Branch and Evans, 2006; Ormeño and Molnár, 2015) have also provided support for the use of small forecasting models on several grounds such as their relative forecast performance and their ability to approximate well the Survey of Professional Forecasters. In particular, Ormeño and Molnár (2015) use a small forecasting model to characterize inflation expectations, but they rely on the minimum state variable approach to characterize the rest of the forward-looking variables of the SW model.

[2] As pointed out by Adam and Marcet (2011), there is an element of arbitrariness when analyzing AL. The issue is that first order conditions under the RE hypothesis can be written in many equivalent ways. However, depending on which RE version of the optimality condition is used one can end up with rather different outcomes under learning. By considering the (log-linear approximation of the) optimality condition directly, without further manipulation, we avoid this source of AL arbitrariness.

in order to mimic the persistence of actual aggregate data. Third, most estimated AL models typically use final revised data whereas actual learning dynamics are driven by real-time data available to agents when forming their expectations. By restricting small forecasting models to include only lagged term structure information, which is observed at the time expectations are formed in real time, we overcome this limitation associated with AL models. The idea of using only term structure information to predict business cycle conditions goes back at least to McCallum (1994), who emphasized that the term spread can be a simple, sufficient predictor regarding future macroeconomic conditions for defining monetary policy.

Finally, the third building block imposes that the learning process of agents in the estimated DSGE model is somehow disciplined by requiring that the deviations of agents' expectations from those reported in the Survey of Professional Forecasters (SPF) are stationary. In contrast to Ormeño and Molnár (2015), we do not impose the more restrictive assumption that agents' expectations match, up to a white noise error, those of the SPF. In short, we allow for persistent deviations between AL expectations and those reported in the SPF.

The estimated model shows that the importance of most endogenous sources of aggregate persistence (such as habit formation, Calvo probabilities, the elasticity of the cost of adjusting capital, and the elasticity of capital utilization adjustment cost) decline dramatically when the hypothesis of maintained beliefs is incorporated through the term structure of interest rates whereas the estimated persistence of structural shocks remains rather high. These results shows the importance of the maintained beliefs hypothesis in an estimated DSGE model, confirming and generalizing the results found by Eusepi and Preston (2011) in a prototype real business cycle model.

The rest of the paper is organized as follows. Section 2 introduces a DSGE model with real-time AL under the maintained beliefs hypothesis. Section 3 shows the estimation results and discusses their implications. Finally, Section 4 concludes.

2 A real-time adaptive learning DSGE model

This paper investigates the interaction of the maintained beliefs hypothesis with the term structure of interest rates - an important source of information observed in real time- in the characterization of the agent's learning process in an estimated DSGE model. Our model builds on the SW model and its AL extension studied by Slobodyan and Wouters (2012a). This standard medium-scale estimated DSGE model contains both nominal and real frictions affecting the choices of households and firms. We extend the AL medium-scale DSGE model in three directions. First, we extend the model to account for the term structure of interest rates. We follow a similar approach as the one considered in Aguilar and Vázquez (2015), but considering the standard consumption-based asset pricing equations associated with each maturity instead of imposing the term structure expectation hypothesis, which is obtained using the law of iterated projections. This feature allows us to consider the possibility that multiple-period-ahead expectations matter to agents' decisions in line with Sargent and Marcet (1989), Preston (2005) and Eusepi and Preston (2011), but without iterating forward optimality conditions involving one-period-ahead expectations. Second, only lagged term structure information observed in real time is used in the small forecasting models of all forward-looking variables of the medium-scale DSGE model. In this way, we study the importance of considering only data available to agents when forming their expectations in real time instead of revised data and/or state variables information, as is standard in the AL literature, which are hardly observable. Finally, AL expectations are disciplined by requiring that the deviations of estimated AL model expectations from the corresponding forecasts reported in the Survey of Professional Forecasters (SPF) are stationary.

We present these extensions of the model next. The remaining log-linearized equations of the model are presented in the Appendix.

2.1 The DSGE model

Our model is based on the standard SW model. Households maximize their utility that depends on their levels of consumption relative to an external habit component and leisure. Labor supplied by households is differentiated by a union with monopoly power setting sticky nominal wages à la Calvo (1983). Households rent capital to firms and decide how much capital accumulate depending on the capital adjustment costs they face. Intermediate firms decide how much capital they use depending on the capital utilization adjustment costs. Intermediate firms also decide how much differentiated labor they hire to produce differentiated goods and set their prices à la Calvo. In addition, both wages and prices are partially indexed to lagged inflation when they are not re-optimized, introducing another source of nominal rigidity. As a result, current prices depend on current and expected marginal cost and past inflation whereas current wages are determined by past and expected future inflation and wages. We deviate from the monetary policy rule in the SW model by assuming that the monetary authorities follow a Taylor-type rule reacting to expected inflation, and lagged values of output gap, output gap growth and a term spread as defined below. Finally, the model contains eight structural disturbances associated with technology, demand-side and price and wage markup shocks together with term structure shocks.

2.2 The term structure extension

This section introduces the term structure of interest rates in the SW model. This is an alternative approach of introducing the maintained beliefs hypothesis to solving the flow budget constraint forward as in Preston (2005) and Eusepi and Preston (2011).

Following De Graeve, Emiris and Wouters (2009) and Vázquez, María-Dolores and Londoño (2013), we extend the DSGE model by explicitly considering the interest rates associated with alternative bond maturities indexed by j (i.e. $j = 1, 2, ..., n$). From the first-order conditions characterizing the optimal decisions of the representative consumer, one can obtain the standard consumption-based asset pricing equation associated with each maturity:

$$E_t\left[\beta^j \frac{U_C(C_{t+j}, N_{t+j})exp(\xi_t^{\{j\}})(1+R_t^{\{j\}})^j}{U_C(C_t, N_t)\prod_{k=1}^{j}(1+\pi_{t+k})}\right] = 1,\ for\, j =\ 1,\ 2,\ ...,\ n,$$

where E_t stands for the RE or the AL operator depending on the estimated model analyzed below, U_C denotes the marginal utility consumption, and C_t, N_t, $R_t^{\{j\}}$ and π_t denote consumption, labor, the nominal return of a j-period maturity bond and the inflation rate, respectively. Assuming that the utility function is logarithmic in consumption,[3] after some algebra, the (linearized) consumption-based asset pricing equations can be written as

$$\left(\frac{1}{1-\frac{h}{\bar{\gamma}}}\right)c_t - \left(\frac{\frac{h}{\bar{\gamma}}}{1-\frac{h}{\bar{\gamma}}}\right)c_{t-1} =$$

$$E_t\left[\left(\frac{1}{1-\frac{h}{\bar{\gamma}}}\right)c_{t+j} - \left(\frac{\frac{h}{\bar{\gamma}}}{1-\frac{h}{\bar{\gamma}}}\right)c_{t+j-1}\right] - \left[jr_t^{\{j\}} - E_t\sum_{k=1}^{j}\pi_{t+k} + \xi_t^{\{j\}}\right],\ for j = 1, 2, ..., n,\quad (1)$$

where lower case variables denote log-deviations from balanced-growth values or, alternatively, deviations from steady state values. In particular, for $j = 1$ the last expression results in the standard IS-curve equation in the SW model assuming a logarithmic utility function in consumption:

$$c_t = \left(\frac{\frac{h}{\bar{\gamma}}}{1+\frac{h}{\bar{\gamma}}}\right)c_{t-1} + \left(\frac{1}{1+\frac{h}{\bar{\gamma}}}\right)E_t c_{t+1} - \left(\frac{1-\frac{h}{\bar{\gamma}}}{1+\frac{h}{\bar{\gamma}}}\right)\left(r_t^{\{1\}} - E_t\pi_{t+1} + \xi_t^{\{1\}}\right).$$

Under RE, the optimality conditions in (1) for $j > 1$ are somewhat redundant because long-term bonds are redundant assets in the equilibrium model. However, under AL agents face greater uncertainty and consequently long term bonds may help agents to hedge against future sources of uncertainty. Moreover, as is clear from the set of consumption-based asset

[3] This assumption largely simplifies the analysis by avoiding the characterization of labor supply expectations over multiple time horizons.

pricing equations (1), optimal consumption household decisions under AL involve explicitly the maintained beliefs hypothesis since current consumption depends, among other things, on the expected paths of both future consumption and inflation. In contrast to Eusepi and Preston (2011), the maintained beliefs hypothesis does not necessarily impose that today's consumption decision does depend on the entire (infinite path) of future consumption and inflation, but only on a finite number of periods, say n. We further assume that the risk premium shock $\xi_t^{\{1\}}$ follows an AR(1) process

$$\xi_t^{\{1\}} = \rho^{\{1\}} \xi_{t-1}^{\{1\}} + \eta_t^{\{1\}}, \tag{2}$$

whereas the term premium shocks $\xi_t^{\{j\}}$, for $j > 1$, follow AR(1) processes augmented with an additional term that allows for an interaction with the risk premium shock:

$$\xi_t^{\{j\}} = \rho^{\{j\}} \xi_{t-1}^{\{j\}} + \rho_\xi^{\{j\}} \xi_t^{\{1\}} + \eta_t^{\{j\}}. \tag{3}$$

That is, $\rho_\xi^{\{j\}}$ captures the interaction of the risk premium shock, $\xi_t^{\{1\}}$, with the term premium shock, $\xi_t^{\{j\}}$, associated with the j-period maturity bond.

2.3 Real-time adaptive learning

Most papers in the AL literature consider only revised aggregate data when estimating the linear forecasting models that agents (are assumed to) follow to update their expectations, the so-called "perceived law of motion" (PLM). This assumption is, at least, problematic because revised aggregate data is not available to economic agents when they are forming their expectations.[4] An exception is Aguilar and Vázquez (2015) that introduce the term spread between the 1-year constant maturity rate and the Fed funds rate -which is observed in real time- as an explanatory variable in the definition of the PLM in addition to revised

[4] See Croushore (2011) and references therein for an analysis of aggregate data revisions and their consequences.

aggregate data. In this paper, we take an additional step forward by considering only lagged term structure information available at the time agents are forming their expectations.[5]

We now proceed to a brief explanation of how AL expectation formation works.[6] A DSGE model can be represented in matrix form as follows:

$$A_0 \begin{bmatrix} y_{t-1} \\ w_{t-1} \end{bmatrix} + A_1 \begin{bmatrix} y_t \\ w_t \end{bmatrix} + A_2 E_t y_{t+n} + B_0 \epsilon_t = 0,$$

where y_t is the vector of endogenous variables at time t, $E_t y_{t+n}$ contains multi-period ahead expectations, and w_t is the exogenous driving force following a VAR(1):

$$w_t = \Gamma w_{t-1} + \Pi \epsilon_t,$$

where ϵ_t is the vector of innovations.

Under AL, the adaptive expectations of the forward-looking variables,$E_t y_{t+n}$, are defined as linear functions of lagged values of the variables, whose time-varying (learning) coefficients are updated as explained in the subsection below. Once the expectations of the forward-looking variables,$E_t y_{t+n}$, are computed they are plugged into the matrix representation of the DSGE model to obtain a backward-looking representation of the model as follows

$$\begin{bmatrix} y_t \\ w_t \end{bmatrix} = \mu_t + T_t \begin{bmatrix} y_{t-1} \\ w_{t-1} \end{bmatrix} + R_t \epsilon_t,$$

where the time-varying matrices μ_t, T_t and R_t are nonlinear functions of structural parameters (entering in matrices A_0, A_1, A_2 and B_0) together with learning coefficients discussed below.

[5] Preliminary attempts to include additional real-time data, such as inflation and consumption real-time data, suggest that including these real-time data in the PLM do not add much whenever the term spread information is already included.

[6] For a detailed explanation see Slobodyan and Wouters (2012a,b).

Forming and updating expectations

Agents are assumed to have a rather limited view of the economy under AL. More precisely, their PLM process is generally defined as follows:

$$y_{t+j} = X_{t-1}\beta_{t-1}^{\{j\}} + u_{t+j},\ for\ j =\ 1,\ 2,\ ...,\ n,$$

where y is the vector containing the k forward-looking variables of the model, X is the matrix of the kxn regressors, $\beta^{\{j\}}$ is the vector of the n updating parameters, which includes an intercept, and u is a vector of errors. These errors are linear combinations of the true model innovations, ϵ_{t+1}. So, the variance-covariance matrices, $\Sigma = E[u_{t+j}u_{t+j}^T]$, are non-diagonal.

Agents are further assumed to behave as econometricians under AL. In particular, it is assumed that they use a linear projection scheme in which the parameters are updated to form their expectations for each forward-looking variable:

$$E_t y_{t+j} = X_{t-1}\beta_{t-1}^{\{j\}}.$$

In line with Jordà (2005), we assume that agents make multi-period ahead forecasts using local projections conditional on the information set available at time $t-1$.[7] The updating parameter vector β, which results from stacking all the vectors $\beta^{\{j\}}$, is further assumed to follow an autoregressive process where agents' beliefs are updated through a Kalman filter. This updating process can be represented as in Slobodyan and Wouters (2012a) by the following equation:

$$\beta_t - \bar{\beta} = F(\beta_{t-1} - \bar{\beta}) + v_t,$$

[7] Among the numerous advantages of using local projections for characterizing multi-period ahead forecasts pointed out by Jordà (2005), we highlight two of them. First, they are easy to implement, which is a sensible approach when deviating from the RE hypothesis. Second, local projections are robust to model misspecifications, which is also a sensible approach when analyzing the forecasts of agents in a context where they face uncertainty about the true (highly non-linear) model economy.

where F is a diagonal matrix with the learning parameter $\mid \rho \mid \leq 1$ on the main diagonal and v_t are i.i.d. errors with variance-covariance matrix V. The Kalman filter updating and transition equations for the belief coefficients and the corresponding covariance matrix are given by

$$\beta_{t|t} = \beta_{t|t-1} + R_{t|t-1} X_{t-1} \Big[\Sigma + X_{t-1}^T R_{t|t-1}^{-1} X_{t-1} \Big]^{-1} \Big(y_t - X_{t-1} \beta_{t|t-1} \Big), \tag{4}$$

with $(\beta_{t+1|t} - \bar{\beta}) = F(\beta_{t|t} - \bar{\beta})$. $\beta_{t|t-1}$ is the estimate of β using the information up to time $t-1$ (but further considering the autoregressive process followed by β), $R_{t|t}$ is the variance-covariance matrix of X, $R_{t|t-1}$ is the estimate of matrix R based on the information at time $t-1$. Therefore, the updated learning vector $\beta_{t|t}$ is equal to the previous one, $\beta_{t|t-1}$, plus a correction term that depends on the forecast error, $\Big(y_t - X_{t-1} \beta_{t|t-1} \Big)$. Moreover, the mean-square error, $R_{t|t}$, associated with this updated estimate is given by

$$R_{t|t} = R_{t|t-1} - R_{t|t-1} X_{t-1} \Big[\Sigma + X_{t-1}^T R_{t|t-1}^{-1} X_{t-1} \Big]^{-1} X_{t-1}^T R_{t|t-1}^{-1}, \tag{5}$$

with $R_{t+1|t} = F R_{t|t} F^T + V$.

A PLM with only term structure information

We adapt our extended SW model with term structure information to the AL version of this model. As mentioned above, one of the key ingredients of a model with AL is the way agents' expectations formation is characterized (i.e. the PLM of agents). In our DSGE model with term structure, we consider a specific PLM motivated by the ability of term spreads to predict inflation (Mishkin, 1990) and real economic activity (Estrella and Hardouvelis, 1991, Estrella and Mishkin, 1997). Let us start considering the following PLM

$$E_t y_{t+j} = \theta_{y,t-1}^{\{j\}} + \sum_{j=1}^{n} \beta_{sp,y,t-1}^{\{j,k\}} sp_{t-1}^{\{k\}}.$$

where $sp_{t-1}^{\{k\}}$ denotes the term spread between the interest rate associated with a k-period maturity bond and the 1-period maturity bond at period $t-1$, which is already known at the beginning of period t when agents form their expectations at this period. The presence of the intercept $\theta_{y,t-1}^{\{j\}}$ relaxes the RE assumption of agents having perfect knowledge about a common deterministic growth rate and a constant inflation target assumed in the SW model. Thus, the consideration of a time-varying intercept coefficient allows expectations to trace trend shifts in the data as well as changes in the inflation target.

At first sight, one might think that considering the whole term structure of interest rates to characterize AL would be useful. However, considering term spreads associated with long-horizons bonds implies the need of defining the whole set of expectations of consumption and inflation from the 1-period horizon up to a long horizon. This task cannot be accomplished because, according to the consumption-based asset pricing equations (1), the number of parameters defining the PLM associated with these expectations dramatically increases with the number of expectational functions of consumption and inflation defined over alternative forecast horizons, which results in a curse of dimensionality problem. Furthermore, there is evidence (Mishkin, 1990) showing that at longer maturities than two quarters, the term structure of interest rates helps to anticipate future inflationary pressures. As a compromise, we focus our attention on the role of the 2-quarter and the 4-quarter term spread to characterize the PLM of forward-looking variables as follows:

$$\begin{cases} E_t y_{t+1} = \theta_{y,t-1} + \beta_{sp,y,t-1}^{\{2\}} sp_{t-1}^{\{2\}},, \quad for \;\; y = i,\ r^k,\ q,\ w \\ E_t y_{t+j} = \theta_{y,t-1}^{\{j\}} + \beta_{sp,y,t-1}^{\{j,2\}} sp_{t-1}^{\{2\}}, \quad for\; y = \ c,\ \pi \; and\; j = \ 0,\ 1,\ 2,\ 3 \\ E_t y_{t+j} = \theta_{y,t-1}^{\{j\}} + \beta_{sp,y,t-1}^{\{j,4\}} sp_{t-1}^{\{4\}}, \quad for\; y = \ c,\ \pi \; and\; j = 4 \end{cases} \tag{6}$$

where l, q, i, w, c, π and r^k stand for (in deviation from their respective steady-state values or detrended by its balanced growth rate) hours worked, Tobin's q, investment, real wage, consumption, inflation and the rental rate of capital, respectively; and $sp_{t-1}^{\{2\}} = r_{t-1}^{\{2\}} - r_{t-1}^{\{1\}}$, and $sp_{t-1}^{\{4\}} = r_{t-1}^{\{4\}} - r_{t-1}^{\{1\}}$ denote the term spreads associated with the 2-quarter and the 1-year term spread, respectively, measured with respect to the 1-quarter interest rate. According to the baseline PLM set, the PLM of all forward looking variables are characterized by the 2-quarter term spread, but those characterizing the expectations of consumption and inflation four-quarters ahead which are based on the the 1-year term spread.[8]

PLM disciplined by the Survey of Professional Forecasters (SPF)

AL is often criticized because it introduces additional degrees of freedom resulting in arbitrary model fit improvement. As a way of disciplining expectations, we assume that the deviations of agents' expectations of both inflation and consumption from the (observed) forecasts reported in the SPF follow stationary processes. We consider the expectations of consumption and inflation from the 1-quarter to the 4-quarter forecast horizon because we are restricting our attention to the short-term horizon of the yield curve (i.e. up to the 1-year maturity bond) in order to overcome the curse of dimensionality problem mentioned above. Formally, we assume

$$\epsilon_{\pi,t}^{\{j\}} = \rho_{\pi}^{\{j\}} \epsilon_{\pi,t-1}^{\{j\}} + \eta_{\pi,t}^{\{j\}}, \quad for\, j = \ 1,\ 2,\ 3,\ 4, \tag{7}$$

where $\epsilon_{\pi,t}^{\{j\}}$, denoting the deviation of the j-period ahead inflation model expectations $(E_t \pi_{t+j})$ from its observable counterpart reported in the SPF, is assumed to follow an AR(1). Similarly, the deviation of the j-period ahead consumption growth model expectations, $E_t\left(c_{t+j} - c_{t+j-1}\right)$, from its observable counterpart reported in the SPF, denoted by$\epsilon_{\Delta c,t}^{\{j\}}$, is assumed to follow an AR(1):

$$\epsilon_{\Delta c,t}^{\{j\}} = \rho_{\Delta c}^{\{j\}} \epsilon_{\Delta c,t-1}^{\{j\}} + \eta_{\Delta c,t}^{\{j\}}, \quad for\, j = \ 1,\ 2,\ 3,\ 4. \tag{8}$$

[8]The empirical results are rather robust to alternative specifications considering either only the 2-quarter term spread or only the 1-year term spread instead of the term-spread mix considered in (6).

2.4 Real-time monetary policy rule

In line with the limited information assumption considered in the paper, the monetary policy rule is assumed to be determined by inflation expectations and lagged values of output gap, output gap growth and the 1-year term spread, which are assumed to be available to the policymaker at the time of implementing monetary policy. Formally,

$$r_t^{\{1\}} = \rho_r r_{t-1}^{\{1\}} + (1-\rho_r)\left[r_\pi E_t \pi_{t+1} + r_y \hat{y}_{t-1}\right] + r_{\Delta y}\Delta\hat{y}_{t-1} + r_{sp} sp_{t-1}^{\{4\}} + \varepsilon_t^r, \tag{9}$$

where $\hat{y}_{t-1,t} = y_{t-1,t} - \Phi\varepsilon_t^a$. That is, following Slobodyan and Wouters (2012a) the output gap, $\hat{y}_{t-1,t}$, is defined as the deviation of output from its underlying neutral productivity process.[9] ε_t^r is assumed to follow an AR(1) process with persistence parameter denoted by ρ_R.

3 Estimation results

This section starts describing the data and the estimation approach. Subsequently, the model fit, estimation results, transmission of shocks and the PLM of the AL model are discussed.

3.1 Data and the estimation approach

Our AL model is estimated using US data for the Great moderation period running from 1984:1 until 2007:4.[10] The set of observable variables is identical to the set considered by Slobodyan and Wouters (2012a) (i.e. the quarterly series of the inflation rate, the Fed funds rate, the log of hours worked, and the quarterly log differences of real consumption, real

[9] In this way, we avoid the characterization of an important number of PLM associated with the frictionless economy, which describes the level of potential output needed to obtain the standard definition of the output gap.

[10] Our estimated model considers consumption growth forecasts from the SPF in addition to inflation forecasts used by Ormeño and Molnár (2015). While inflation forecasts are reported back to the late 1960's, the consumption growth forecast time series start at 1981:3. We decide to start our sample in 1984:1. In this way, we ignore the inflationary years right after the Volcker monetary experiment and focus on the Great moderation period.

investment, real wages and real GDP) with the addition of the 1-year Treasury constant maturity yield and the SPF forecast about inflation and consumption from 1- to 4-quarter horizons. GDP, consumption, investment and hours worked are measured in per-working age population terms. The measurement equation is

$$X_t = \begin{bmatrix} dlGDP_t \\ dlCONS_t \\ dlINV_t \\ dlWAG_t \\ dlP_t \\ lHours_t \\ FEDFUNDS_t \\ One\ year\ TB\ yield_t \\ dlCONS_t^{e\{j\}} \\ dlP_t^{e\{j\}} \end{bmatrix} = \begin{bmatrix} \overline{\gamma} \\ \overline{\gamma} \\ \overline{\gamma} \\ \overline{\gamma} \\ \overline{\pi} \\ \bar{l} \\ \overline{r} \\ \overline{r}^{\{4\}} \\ \overline{\gamma}^e \\ \overline{\pi}^e \end{bmatrix} + \begin{bmatrix} y_t - y_{t-1} \\ c_t - c_{t-1} \\ i_t - i_{t-1} \\ w_t - w_{t-1} \\ \pi_t \\ l_t \\ r_t \\ r_t^{\{4\}} \\ E_t\left(c_{t+j} - c_{t+j-1}\right) + \epsilon_{c,t}^{\{j\}} \\ E_t\pi_{t+j} + \epsilon_{\pi,t}^{\{j\}} \end{bmatrix},$$

where l and dl denote the log and the log difference, respectively. $\overline{\gamma} = 100(\gamma - 1)$ is the common quarterly trend growth rate for real GDP, real consumption, real investment and real wages, which are the variables featuring a long-run trend. $\bar{l}$, $\overline{\pi}$, $\overline{r}$ and $\overline{r}^{\{4\}}$ are the steady-state levels of hours worked, inflation, the Fed funds rate and the 1-year (four-quarter) constant maturity Treasury yield, respectively. The superscripts e and $\{j\}$ in the last two rows of the measurement equation denote actual forecasts from the SPF and the corresponding forecast horizon, respectively.

The estimation approach follows a standard two-step Bayesian estimation procedure. First, a maximization of the log posterior function is carried out by combining prior information on the parameters with the likelihood of the data. The prior assumptions are exactly the same as in Slobodyan and Wouters (2012a). Moreover, we consider rather loose priors for the parameters characterizing both the 1-year yield dynamics and the stationary processes characterizing the deviations of inflation and consumption growth model expectations from the corresponding forecasts reported in the SPF. The second step implements the Metropolis-Hastings algorithm, which runs a massive sequence of draws of all the possible realizations for each parameter in order to obtain its posterior distribution.[11]

3.2 Model fit

We estimate the RE and the AL versions of the model. The posterior log data density of the RE and AL models are 186.20 and 214.31, respectively. The difference between their log data densities is 28.11 points, which results in a very high posterior odd of 1.62E+12. This difference in favor of the AL learning specification is roughly half the difference found in Ormeño and Molnár (2015), but higher than the one obtained in Slobodyan and Wouters (2012a). Several reasons may explain these differences. On the one hand, our AL model introduces two type of stringent hypotheses. First, learning is based only on the information content of the (1-year) term spread which is observed in real time, whereas Slobodyan and Wouters (2012a) and Ormeño and Molnár (2015) used revised data when defining the PLM, which results in an unrealistic information set available to agents when forming their expectations. Second, we introduce the maintained beliefs hypothesis, which was ignored in the two previous papers. On the other hand, our learning process is less restrictive in one dimension than the one assumed in Ormeño and Molnár (2015). Namely, we impose that deviations of model expectations from their SPF counterparts are stationary instead of (the more restrictive assumption of) white noise processes as they assumed. Meanwhile, Slobodyan and Wouters (2012a) did not use data from the SPF as observables in their estimation procedure.

Beyond the overall model fit based on the posterior log data density, we also analyze the performance of the AL model to reproduce selected second-moment statistics obtained from actual data as shown in Table 1. We focus on three type of moments: standard deviations, contemporaneous correlations with inflation and first-order autocorrelations. Regarding the

[11] The DSGE models are estimated using Dynare codes gently provided by Slobodyan and Wouters with a few modifications to include term spread information in the PLM as described in equation (6).

actual size of fluctuations, we observe that the AL model is able to match reasonably well the standard deviation of inflation and the growth rates of real variables: output, consumption, investment and wages. A similar conclusion can be drawn by looking at the correlation of inflation with these real variables. Thus, the AL model is able to reproduce very well the negative and low correlations of inflation with the real variables. The AL model has more trouble in replicating quantitatively actual persistence: it generates too much persistence. This is particularly true in the case of inflation and to a lesser extent in the real macroeconomic variables studied. As shown below, this estimated feature is induced by the high persistence associated with the inflation and consumption growth forecasts reported in the SPF.

Table 1. Actual and simulated second moments

Actual data	Δc	Δinv	Δw	Δy	π
Standard deviation	0.51	1.68	0.62	0.54	0.24
Correlation with π	-0.30	-0.28	-0.29	-0.29	1
Autocorrelation	0.05	0.51	0.22	0.21	0.48
Simulated data	Δc	Δinv	Δw	Δy	π
Standard deviation	0.53	1.61	0.63	0.70	0.26
Correlation with π	-0.29	-0.26	-0.10	-0.30	1.0
Autocorrelation	0.26	0.70	0.57	0.48	0.97

3.3 Posterior estimates

Table 2 shows the estimation results for a selected group of parameters featuring both endogenous and exogenous persistence for the AL and the RE versions of the model. In general, most sources of endogenous persistence lose a great deal of importance. Thus, the estimates of the habit formation parameter, h, and the elasticity of the cost of adjusting capital, φ, are much smaller under AL (0.31 and 1.02, respectively) than under RE (0.92 and 8.88, respectively). Similarly, the price and wage probabilities as well as the elasticity of capital utilization adjusting cost, ψ, are much smaller under AL than under RE whereas the opposite occurs for the price and wage indexation parameters (ι_p and ι_w, respectively). Regarding exogenous sources of price and wage markup persistence, we found contrasting results. Thus, price markup shock persistence is lower under AL than under RE, whereas the opposite occurs for wage markup shock persistence. Overall we observe that AL under the maintained expectations hypothesis implied by multi-period forecasting results in lower estimates of the parameters characterizing both endogenous and exogenous sources of persistence than the ones implied by the RE hypothesis. These results are in line with those found in Eusepi and Preston (2011).

Table 2. Selected parameter estimates

	Priors			AL model		RE model	
	Distr	Mean	Std D	Mean	5%-95% CI	Mean	5%-95% CI
h: habit formation	Beta	0.7	0.10	0.31	(0.21,0.44)	0.92	(0.91,0.93)
φ: cost of adjusting capital	Normal	4.00	1.50	1.02	(0.69,1.37)	8.88	(8.46,9.50)
ψ: capital utilization adjusting cost	Beta	0.5	0.15	0.22	(0.14,0.29)	0.37	(0.31,0.43)
ξ_p: price Calvo probability	Beta	0.5	0.10	0.58	(0.51,0.66)	0.94	(0.93,0.95)
ξ_w: wage Calvo probability	Beta	0.5	0.10	0.60	(0.53,0.67)	0.75	(0.70,0.81)
ι_p: price indexation	Beta	0.5	0.10	0.85	(0.73,0.95)	0.11	(0.09,0.13)
ι_w: wage indexation	Beta	0.5	0.10	0.56	(0.39,0.77)	0.21	(0.15,0.27)
ρ_p: persistence of price markup shock	Beta	0.5	0.20	0.67	(0.41,0.91)	0.997	(0.994,0.999)
ρ_w: persistence of wage markup shock	Beta	0.5	0.20	0.94	(0.91,0.97)	0.83	(0.79,0.89)
log data density				216.70		186.20	

3.4 Impulse responses

Impulse responses to a term premium shock

Figure 1 shows the impulse responses to a term-spread innovation. The stability of learning coefficients associated with the PLM characterized only by the term spread implies that the impulse response functions barely change over time.[12] A positive term-spread shock increases the future interest rate relative to the contemporaneous interest rate, which brings forward consumption and investment decisions resulting in higher economic activity (output, consumption and investment), inflation and (short-term) nominal interest rate. The impulse responses of all variables are hump-shaped capturing the gradual process of learning. This hump-shaped feature is more pronounced in the nominal (inflation and nominal interest rate) variables than in the real variables (output and consumption). As emphasized in Aguilar and Vázquez (2015), the introduction of AL extended with term structure information allows for a feedback from the term structure to the macroeconomy through the learning dynamics that is missing under RE.

Figure 1. Impulse responses to a term-spread innovation

[12] This fact explains why we only report the average impulse responses instead of showing the time-varying impulse response functions. The stability of learning coefficients is also due to the Great moderation period studied in this paper.

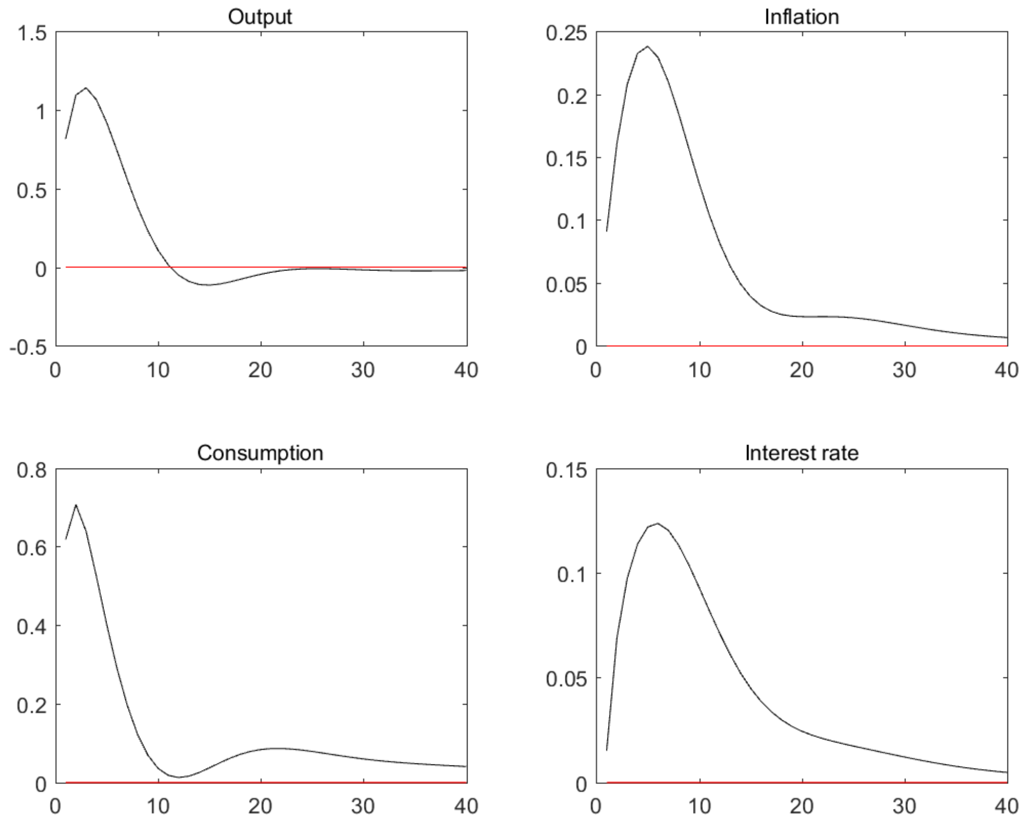
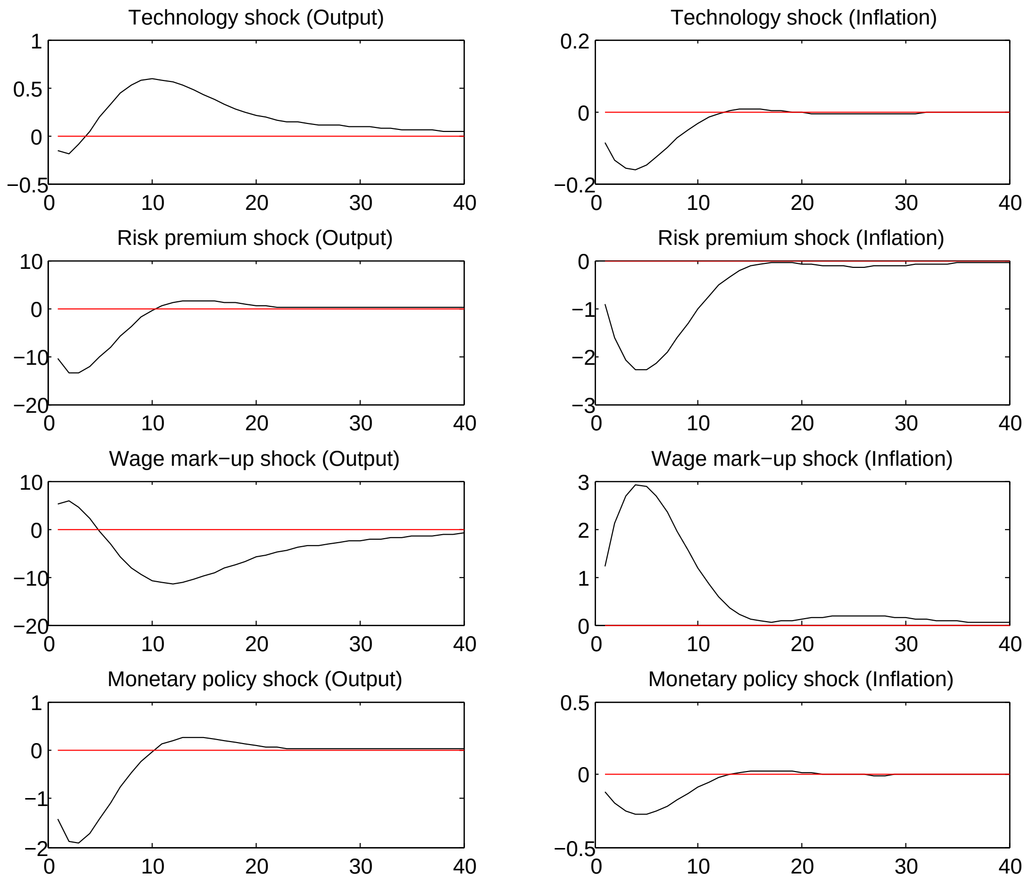

Output
1.5
1
0.5
0
-0.5
0
10
20
30
40
Inflation
0.25
0.2
0.15
0.1
0.05
0
0
10
20
30
40
Consumption
0.8
0.6
0.4
0.2
0
0
10
20
30
40
Interest rate
0.15
0.1
0.05
0
0
10
20
30
40

Figure 2. Impulse responses of output and inflation



Impulse responses of output and inflation to alternative shocks

Figure 2 shows the responses of output and inflation to a technology shock, a risk premium shock, a wage markup shock and a monetary policy shock. The backward-looking feature of AL dynamics results in hump-shaped (or alternatively U-shaped) impulse responses as occurred for the responses to a term-spread innovation studied above. A positive technology impulse decreases inflation and increases potential output more than output initially, which results in a lower output measured in deviations from the balanced growth path as shown in the top-left graph. A positive risk premium innovation decreases aggregate demand, which results in a fall in output and inflation. A positive wage markup shock initially stimulates aggregate demand by rising wages, which results in an increase in both output and inflation.

However, this increase in output does not last long as the higher labor costs lead to lower production. Finally, a positive interest rate shock results in both lower output and inflation as expected.

3.5 Analysis of the PLM

Figure 3 shows the evolution over time of the PLM coefficients for inflation and consumption expectations four quarters ahead. We focus on the four-quarter expectational horizon because the 1-year term spread is the only observable term spread considered in the PLM. The time-varying intercept of the PLM of inflation (consumption) shows how agents' perception about steady-state inflation (balanced consumption-growth) changes over time. Thus, the intercept of inflation expectations captures the fall of inflation expectations over the sample period, whereas the (roughly constant) intercept of consumption captures the fairly constant consumption growth expectations. The term-spread coefficients associated with these two PLM are negative, indicating that a higher 4-quarter bond yield today anticipate tighter financial conditions in the future resulting in lower inflation and consumption expectations. Notice that the term-spread coefficient of consumption's PLM is roughly eight times larger than the one associated with the PLM of inflation, which results in much larger swings in consumption growth expectations than in inflation expectations.

This last feature is consistent with the corresponding expectations four-quarters ahead reported by the SPF as shown in Figure 4. Moreover, the left graph of this figure shows that the AL expectations generated by the model (red line) reproduces the SPF (blue line) inflation downtrend. Nevertheless, there is a positive gap between the inflation expectations from the SPF and the inflation expectations from the AL model. The mean of this gap is partially explained by the difference between the average of the 4-quarter ahead inflation forecasts in the SPF (0.69) and the average of actual inflation (0.63) over the sample period, indicating that the AL inflation expectations generated by the model are closer to match actual inflation rather than SPF inflation (see the first panel of Table 3 below). The graph on the right of Figure 4 shows that the AL model (red line) does a great job reproducing both the timing and the large swings of consumption growth expectations reported by the SPF (blue line).

Figure 3. PLM of inflation and consumption expectations

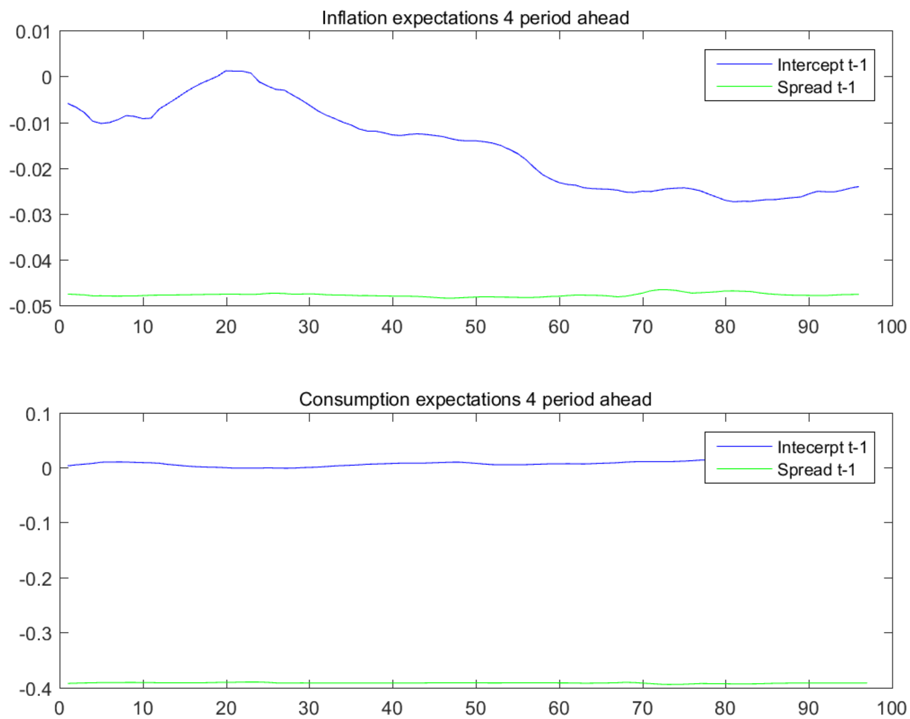


Figure 4. Model's expectations versus SPF's forecasts on inflation and consumption

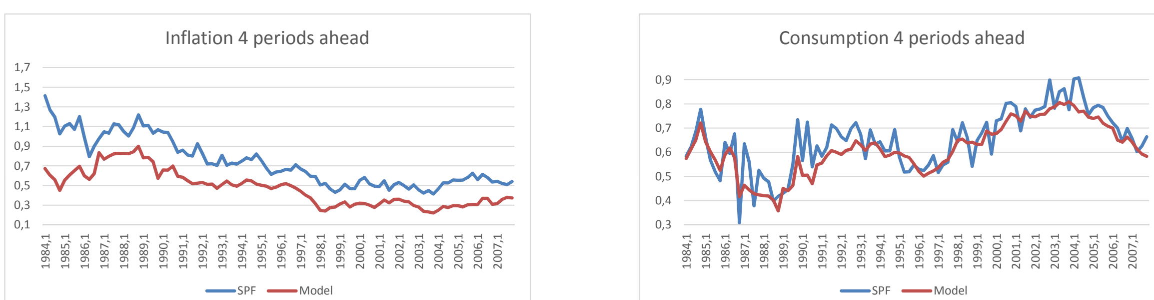


Table 3 shows descriptive statistics (mean, standard deviation and first-order autocorrelation) of inflation and consumption growth four-quarters ahead expectations from the SPF and the model together with the statistics of inflation and consumption growth from actual data and the model. The first panel shows that model's expectations of inflation and consumption growth are lower on average than those reported in the SPF and actual data, but model's expectations are closer to the latter. Regarding standard deviations, it is interesting to observe that the SPF forecasts and model's expectations of consumption growth show a similar volatility, but a much smoother behavior than the ones associated with both actual and simulated data. This last feature is not shared by inflation expectations. In this case, only inflation expectations from the model are slightly smoother than both actual data and the SPF inflation forecasts. Regarding persistence, both inflation and consumption expectations from the SPF and the model are much more persistent than actual data persistence. Moreover, both SPF forecasts and model's expectations share a similar degree of persistence. These last two features may explain, on the one hand, why the model is generating higher persistence than the one is observed as shown above in Table 1. On the other hand, they also suggest that using the SPF forecasts to disciplining model's expectations can be at least problematic because the former induces too much persistence in the latter, which results in highly persistent synthetic time series.

Table 3. Descriptive statistics of inflation and consumption growth

Mean	π	π^e_{t+4}	Δc	Δc^e_{t+4}
Data/SPF forecasts	0.63	0.73	0.57	0.65
Model	0.70	0.53	0.53	0.52
Standard deviation	π	π^e_{t+4}	Δc	Δc^e_{t+4}
Data/SPF forecasts	0.24	0.24	0.51	0.12
Model	0.26	0.16	0.53	0.10
Autocorrelation	π	π^e_{t+4}	Δc	Δc^e_{t+4}
Data/SPF forecast	0.48	0.96	0.05	0.70
Model	0.96	0.96	0.26	0.78

4 Conclusions

This paper considers multi-period forecasting (i.e. the maintained beliefs hypothesis) using only real-time information in an estimated DSGE model with adaptive learning (AL). We have extended the AL model of Slobodyan and Wouters (2012a) by introducing the term structure of interest rates, which results in multi-period forecasting. While retaining the feature of AL based on small forecasting models, our extension allows the term spread of interest rates to fully characterize the forward-looking variables of the model in real time. We view the use exclusively of real time information as a crucial step forward in the characterization of learning in estimated DSGE models. Moreover, the introduction of term structure information in small forecasting models results in more stable perceived law of motions for the forward-looking variables than those obtained when ignoring the term structure. This finding is important because AL schemes are often criticized for being arbitrary (see, for instance, Adam and Marcet (2011) and references therein), and potentially amplifying the size of fluctuations in an ad hoc manner. Furthermore, considering term structure information in the agents learning process results in model expectations about consumption growth and inflation that resemble those reported in the Survey of Professional Forecasters.

Our estimation results show that the importance of most endogenous sources of aggregate persistence (such as habit formation, Calvo probabilities, the elasticity of the cost of adjusting capital, and the elasticity of capital utilization adjustment cost) decline dramatically when the maintained beliefs hypothesis is incorporated through the term structure of interest rates in optimal decision making, whereas the estimated persistence of structural shocks remains rather high. These results are in line with those found by Eusepi and Preston (2011) in a calibrated real business cycle model.

Table A.1. Panel A: Priors and estimated posteriors of the structural parameters

	Priors			Posteriors					
				AL model			RE model		
	Distr	Mean	Std D.	Mean	5%	95%	Mean	5%	95%
φ: cost of adjusting capital	Normal	4.00	1.50	1.02	0.69	1.37	8.88	8.46	9.50
h: habit formation	Beta	0.70	0.10	0.31	0.21	0.44	0.92	0.91	0.93
σ_l : Frisch elasticity	Normal	2.00	0.75	1.47	0.63	2.21	1.23	1.11	1.35
ξ_p: price Calvo probability	Beta	0.50	0.10	0.58	0.51	0.66	0.94	0.93	0.95
ξ_w: wage Calvo probability	Beta	0.50	0.10	0.60	0.53	0.67	0.75	0.70	0.81
ι_w: wage indexation	Beta	0.50	0.15	0.56	0.39	0.77	0.21	0.15	0.27
ι_p: price indexation	Beta	0.50	0.15	0.85	0.73	0.95	0.11	0.09	0.13
ψ: capital utilization adjusting cost	Beta	0.50	0.15	0.22	0.14	0.29	0.37	0.31	0.43
Φ : steady state price mark-up	Normal	1.25	0.12	1.52	1.36	1.68	1.63	1.49	1.73
r_π: policy rule inflation	Normal	1.50	0.25	1.35	1.17	1.53	2.00	1.86	2.16
ρ_r: policy rule smoothing	Beta	0.75	0.10	0.90	0.87	0.94	0.81	0.77	0.85
r_y: policy rule output gap	Normal	0.12	0.05	0.15	0.09	0.23	0.07	0.05	0.08
$r_{\Delta y}$: policy rule output gap growth	Normal	0.12	0.05	0.04	0.01	0.06	0.06	0.04	0.07
r_{sp}: policy rule term spread	Normal	0.12	0.05	0.06	-0.01	0.14	0.16	0.12	0.21
π: steady-state inflation	Gamma	0.62	0.10	0.77	0.70	0.84	0.69	0.60	0.78
$100(\beta^{-1}-1)$: steady-state rate of disc.	Gamma	0.25	0.10	0.22	0.09	0.38	0.17	0.12	0.22
l: steady-state labor	Normal	0.00	2.00	1.01	0.07	1.78	0.17	-0.14	0.53
γ: one plus st-state rate of output growth	Normal	0.40	0.10	0.52	0.50	0.53	0.47	0.45	0.49
$\bar{r}^{\{4\}}$: steady-state 1-year yield	Normal	1.00	0.50	1.18	0.90	1.44	1.38	1.29	1.47
α: capital share	Normal	0.30	0.05	0.13	0.09	0.17	0.23	0.19	0.26
ρ: learning parameter	Beta	0.50	0.28	0.73	0.69	0.77	-	-	-

Table A.1. Panel B: Priors and estimated posteriors of the structural shock process parameters

	Priors			Posterior					
				AL model			RE model		
	Distr	Mean	Std D.	Mean	5%	95%	Mean	5%	95%
σ_a: Std. dev. productivity innovation	Invgamma	0.10	2.00	0.41	0.36	0.47	0.39	0.34	0.42
σ_b: Std. dev. risk premium innovation	Invgamma	0.10	2.00	0.99	0.71	1.34	5.16	4.81	5.48
σ_g: Std. dev. exogenous spending innovation	Invgamma	0.10	2.00	0.40	0.35	0.43	0.42	0.37	0.47
σ_i: Std. dev. investment innovation	Invgamma	0.10	2.00	1.38	1.23	1.50	0.32	0.27	0.37
σ_R: Std. dev. monetary policy innovation	Invgamma	0.10	2.00	0.10	0.09	0.12	0.11	0.09	0.12
σ_p: Std. dev. price mark-up innovation	Invgamma	0.10	2.00	0.20	0.18	0.22	0.21	0.18	0.23
σ_w: Std. dev. wage mark-up innovation	Invgamma	0.10	2.00	0.65	0.56	0.73	0.20	0.16	0.24
$\sigma_{\eta^{\{2\}}}$: Std. dev. 2-quarter yield innovation	Invgamma	0.10	2.00	2.86	1.32	4.21	0.06	0.03	0.08
$\sigma_{\eta^{\{3\}}}$: Std. dev. 3-quarter yield innovation	Invgamma	0.10	2.00	0.10	0.03	0.17	0.14	0.04	0.22
$\sigma_{\eta^{\{4\}}}$: Std. dev. 1-year yield innovation	Invgamma	0.10	2.00	0.46	0.40	0.54	0.33	0.29	0.36
ρ_a: Autoregressive coef. productivity shock	Beta	0.50	0.20	0.93	0.89	0.96	0.92	0.88	0.96
ρ_b: Autoregressive coef. risk-premium shock	Beta	0.50	0.20	0.93	0.91	0.95	0.19	0.16	0.22
ρ_g: Autoregressive coef. exog. spending shock	Beta	0.50	0.20	0.97	0.95	0.99	0.96	0.94	0.98
ρ_i: Autoregressive coef. investment shock	Beta	0.50	0.20	0.90	0.84	0.95	0.73	0.67	0.81
ρ_R: Autoregressive coef. monetary policy shock	Beta	0.50	0.20	0.56	0.46	0.65	0.37	0.29	0.46
ρ_p: Autoregressive coef. price markup shock	Beta	0.50	0.20	0.67	0.41	0.91	0.997	0.994	0.999
ρ_w: Autoregressive coef. wage markup shock	Beta	0.50	0.20	0.94	0.91	0.97	0.83	0.79	0.89
$\rho^{\{2\}}$: Autoregressive coef. 2-quarter yield shock	Beta	0.50	0.20	0.992	0.990	0.995	0.44	0.32	0.55
$\rho^{\{3\}}$: Autoregressive coef. 3-quarter yield shock	Beta	0.50	0.20	0.57	0.29	0.84	0.33	0.23	0.43
$\rho^{\{4\}}$: Autoregressive coef. 1-year yield shock	Beta	0.50	0.20	0.92	0.89	0.96	0.20	0.18	0.23
μ_p: MA coef. price mark-up shock	Beta	0.50	0.20	0.56	0.36	0.79	0.996	0.993	0.998
μ_w: MA coef. wage mark-up shock	Beta	0.50	0.20	0.28	0.13	0.41	0.54	0.46	0.65
ρ_{ga}: Interact. betw. product. and spending shocks	Beta	0.50	0.25	0.48	0.36	0.59	0.50	0.42	0.58
${\rho^{\{2\}}}_{\xi}$: Interact. betw. 1- and 2-quarter yield shocks	Beta	0.50	0.25	0.22	0.06	0.40	0.49	0.37	0.60
${\rho^{\{3\}}}_{\xi}$: Interact. betw. 1- and 3-quarter yield shocks	Beta	0.50	0.25	0.49	0.18	0.85	0.52	0.39	0.62
${\rho^{\{4\}}}_{\xi}$: Interact. betw. 1- and 4-quarter yield shocks	Beta	0.50	0.25	0.81	0.70	0.90	1.24	1.21	1.29

Table A.1. Panel C: Priors and estimated posteriors of parameters describing the deviations of model and SPF expectatio

	Priors			Posterior					
				AL model			RE model		
	Distr	Mean	Std D.	Mean	5%	95%	Mean	5%	95%
$\sigma_{\pi}^{\{1\}}$: Std. dev. 1-q ahead inflation expect. innov.	Invgamma	0.10	2.00	0.08	0.07	0.09	0.08	0.07	0.09
$\sigma_{\pi}^{\{2\}}$: Std. dev. 2-q ahead inflation expect. innov.	Invgamma	0.10	2.00	0.06	0.06	0.07	0.07	0.06	0.08
$\sigma_{\pi}^{\{3\}}$: Std. dev. 3-q ahead inflation expect. innov.	Invgamma	0.10	2.00	0.07	0.06	0.08	0.07	0.06	0.07
$\sigma_{\pi}^{\{4\}}$: Std. dev. 4-q ahead inflation expect. innov.	Invgamma	0.10	2.00	0.06	0.05	0.07	0.07	0.06	0.07
$\sigma_{\Delta c}^{\{1\}}$: Std. dev. 1-q ahead cons. growth expect. innov.	Invgamma	0.10	2.00	0.50	0.44	0.55	0.17	0.15	0.19
$\sigma_{\Delta c}^{\{2\}}$: Std. dev. 2-q ahead cons. growth expect. innov.	Invgamma	0.10	2.00	0.10	0.09	0.11	0.12	0.11	0.13
$\sigma_{\Delta c}^{\{3\}}$: Std. dev. 3-q ahead cons. growth expect. innov.	Invgamma	0.10	2.00	0.08	0.07	0.10	0.11	0.10	0.13
$\sigma_{\Delta c}^{\{4\}}$: Std. dev. 4-q ahead cons. growth expect. innov.	Invgamma	0.10	2.00	0.06	0.06	0.07	0.09	0.08	0.10
$\rho_{\pi}^{\{1\}}$: persist. 1-q ahead inflation expect. shock	Beta	0.50	0.20	0.90	0.85	0.94	0.63	0.54	0.73
$\rho_{\pi}^{\{2\}}$: persist. 2-q ahead inflation expect. shock	Beta	0.50	0.20	0.89	0.85	0.93	0.74	0.69	0.80
$\rho_{\pi}^{\{3\}}$: persist. 3-q ahead inflation expect. shock	Beta	0.50	0.20	0.92	0.88	0.96	0.76	0.70	0.82
$\rho_{\pi}^{\{4\}}$: : persist. 4-q ahead inflation expect. shock	Beta	0.50	0.20	0.89	0.86	0.92	0.82	0.74	0.90
$\rho_{\Delta c}^{\{1\}}$: persist. 1-q ahead cons. growth expect. shock	Beta	0.50	0.20	0.93	0.89	0.97	0.67	0.60	0.74
$\rho_{\Delta c}^{\{2\}}$: persist. 2-q ahead cons. growth expect. shock	Beta	0.50	0.20	0.18	0.05	0.30	0.59	0.51	0.68
$\rho_{\Delta c}^{\{3\}}$: persist. 3-q ahead cons. growth expect. shock	Beta	0.50	0.20	0.17	0.03	0.29	0.61	0.54	0.68
$\rho_{\Delta c}^{\{4\}}$: persist. 4-q ahead cons. growth expect. shock	Beta	0.50	0.20	0.23	0.07	0.39	0.66	0.57	0.75

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Appendix

Set of the remaining log-linearized dynamic equations:

- Aggregate resource constraint:

$$y_t = c_y c_t + i_y i_t + z_y z_t + \varepsilon_t^g, \tag{10}$$

where $c_y = \frac{C}{Y} = 1 - g_y - i_y$, $i_y = \frac{I}{Y} = (\gamma - 1 + \delta)\frac{K}{Y}$, and $z_y = r^k \frac{K}{Y}$ are steady-state ratios. As in Smets and Wouters (2007), the depreciation rate and the exogenous spending-GDP ratio are fixed in the estimation procedure at $\delta = 0.025$ and $g_y = 0.18$.

- Investment equation:

$$i_t = i_1 i_{t-1} + (1 - i_1) E_t i_{t+1} + i_2 q_t + \varepsilon_t^i, \tag{11}$$

where $i_1 = \frac{1}{1+\overline{\beta}}$, and $i_2 = \frac{1}{(1+\overline{\beta})\gamma^2\varphi}$ with $\overline{\beta} = \beta\gamma^{(1-\sigma_c)}$.

- Arbitrage condition (value of capital, q_t):

$$q_t = q_1 E_t q_{t+1} + (1 - q_1) E_t r_{t+1}^k - (R_t - E_t \pi_{t+1}) + c_3^{-1} \varepsilon_t^b, \tag{12}$$

where $q_1 = \overline{\beta}\gamma^{-1}(1-\delta) = \frac{(1-\delta)}{(r^k + 1 - \delta)}$.

- Log-linearized aggregate production function:

$$y_t = \Phi \left(\alpha k_t^s + (1-\alpha) l_t + \varepsilon_t^a \right), \tag{13}$$

where $\Phi = 1 + \frac{\phi}{Y} = 1 + \frac{\text{Steady-state fixed cost}}{Y}$ and α is the capital-share in the production function.[13]

- Effective capital (with one period time-to-build):

$$k_t^s = k_{t-1} + z_t. \tag{14}$$

[13] From the zero profit condition in steady-state, it should be noticed that ϕ_p also represents the value of the steady-state price mark-up.

- Capital utilization:

$$z_t = z_1 r_t^k, \tag{15}$$

where $z_1 = \frac{1-\psi}{\psi}$.

- Capital accumulation equation:

$$k_t = k_1 k_{t-1} + (1-k_1) i_t + k_2 \varepsilon_t^i, \tag{16}$$

where $k_1 = \frac{1-\delta}{\gamma}$ and $k_2 = \left(1 - \frac{1-\delta}{\gamma}\right)\left(1+\overline{\beta}\right)\gamma^2\varphi$.

- Marginal cost:

$$mc_t = (1-\alpha) w_t + \alpha r_t^k - \varepsilon_t^a. \tag{17}$$

- New-Keynesian Phillips curve (price inflation dynamics):

$$\pi_t = \pi_1 \pi_{t-1} + \pi_2 E_t \pi_{t+1} - \pi_3 mc_t + \pi_4 \varepsilon_t^p, \tag{18}$$

where $\pi_1 = \frac{\iota_p}{1+\overline{\beta}\iota_p}$, $\pi_2 = \frac{\overline{\beta}}{1+\overline{\beta}\iota_p}$, $\pi_3 = \frac{A}{1+\overline{\beta}\iota_p}\left[\frac{(1-\overline{\beta}\xi_p)(1-\xi_p)}{\xi_p}\right]$, and $\pi_4 = \frac{1+\overline{\beta}\iota_p}{1+\overline{\beta}\iota_p}$. The coefficient of the curvature of the Kimball goods market aggregator, included in the definition of A, is fixed in the estimation procedure at $\varepsilon_p = 10$ as in Smets and Wouters (2007).

- Optimal demand for capital by firms:

$$-\left(k_t^s - l_t\right) + w_t = r_t^k. \tag{19}$$

- Wage markup equation:

$$\mu_t^w = w_t - mrs_t = w_t - \left(\sigma_l l_t + \frac{1}{1-h/\gamma}\left(c_t - (h/\gamma)\, c_{t-1}\right)\right). \tag{20}$$

- Real wage dynamic equation:

$$w_t = w_1 w_{t-1} + (1-w_1)\left(E_t w_{t+1} + E_t \pi_{t+1}\right) - w_2 \pi_t + w_3 \pi_{t-1} - w_4 \mu_t^w + \varepsilon_t^w. \tag{21}$$

where $w_1 = \frac{1}{1+\overline{\beta}}$, $w_2 = \frac{1+\overline{\beta}\iota_w}{1+\overline{\beta}}$, $w_3 = \frac{\iota_w}{1+\overline{\beta}}$, $w_4 = \frac{1}{1+\overline{\beta}}\left[\frac{\left(1-\overline{\beta}\xi_w\right)(1-\xi_w)}{\xi_w((\phi_w-1)\varepsilon_w+1)}\right]$ with the curvature of the Kimball labor aggregator fixed at $\varepsilon_w = 10.0$ and a steady-state wage mark-up fixed at $\phi_w = 1.5$ as in Smets and Wouters (2007).